Enhancing Environmental Protection Tax Enforcement via Multi-Agent Game Theory in an Integrated E-commerce Online Supervision Platform: A Comprehensive Assessment

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Abstract

Over the past five years since the enactment of the environmental protection tax, a measure has been implemented that has harmonized the conflicting interests among the government, E-businesses, and the E-market. A substantial body of game-theoretic analysis has focused exclusively on the interactions between government entities, producers (firms), and the general populace (consumers). This paper introduces an OSP (Online Supervision Platform) into the game framework, which assists government in sharing ecological environmental guardianship responsibilities, concurrently stimulating society and the E-market to explore environmental protection tax loopholes. It constructs a four-party evolutionary game model. The analysis assesses the stability of strategic choices for each participant in the game. Furthermore, a stability analysis of potential equilibrium points in the system is conducted based on the first law of Lyapunov stability. Simulation delves into the impact of key elements on the E-commerce's evolution, scientifically examines the influence of OSP on stakeholders during the environmental protection tax implementation, providing a more comprehensive decision-making basis for environmental protection.

Keywords: Online supervision platform, Lyapunov, Environmental protection

1 Introduction

Accelerated advancements in information technology have enabled a more efficient channel for gathering public opinions on policy implementation, OSP (Online Supervision Platform) [1]. It not only addresses the limitations of traditional offline methods, such as mail correspondence, public hearings, and social organizations which often suffer from extended response times, limited reach, and restricted departmental engagement [2], but is increasingly becoming a vital conduit for public sentiment.

Since the introduction of the environmental protection tax [3-4], a degree of equilibrium has been achieved in the dynamic interplay of interests among government, corporations, and the marketplace. This balance has facilitated the government's delicate task of weighing environmental protection against economic development towards double carbon [5], corporations' endeavor to reconcile environmental stewardship and economic returns, and the market's ability to adjust supply and demand prices in a timely fashion [6-7].

By integrating game theory into the process of collecting feedback for the implementation of the environmental protection tax [8-10], we are able to analyze the impact of this tax on all stakeholders in a more comprehensive and scientifically rigorous manner. However, the existing research primarily focuses on the discussion of the three-party game and does not take into account the impact of online platform reviews.

This approach ultimately provides a more robust and scientifically-sound foundation for decision-making in environmental protection, which incorporates entity-OSP. In addition, it also explores the implementation of online communication as a feedback mechanism within the effectiveness of the environmental protection tax. This innovative method assists government agencies in sharing the onus of ecological supervision, while also motivating societal and market forces to further probe potential deficiencies in environmental protection. This process aims to foster a positive cycle of balanced benefits for all stakeholders.

This research principally explores the following inquiries:

How can online communication channels collaborate with government environmental protection departments to implement environmental regulations effectively, aiming to significantly reduce the public living pollution index?

What methods can be employed to refine the environmental tax collection system, thereby motivating both individuals to improve their living environment quality and businesses to voluntarily adopt low-carbon, clean production practices in online consumption?

How can we ensure the effectiveness of online communication channels, even in the face of potential administrative complacency within pertinent departments, to guarantee that digital feedback truly reflects public opinions, fosters continuous improvement of the environ-

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mental protection tax, safeguards the standard of residents' living environments, and maintains government credibility?

Most of the above-mentioned data comes from academic statistics and lack of objective evaluation by the people, so the analysis results are not representative and complete [11-12]. In this paper, we take the third-party, online opinions and online supervision platform into consideration (hereinafter OSP) and uses the multi-party evolutionary game for analysis.

2 Problem Description

Under the backdrop of the 'Double Carbon' policy, the implementation of environmental tax constitutes a strategic game among the government, firms, and the consumer [13]. In this game, the government needs to strike a balance between environmental protection and economic development, while enterprises need to reconcile the trade-off between environmental protection and economic gains. Meanwhile, the market needs to adjust prices according to market supply and demand. To evaluate the benefits of environmental tax implementation and analyze its impact on the interests of all parties, game theory is introduced to provide a scientific decision-making basis [14]. As showed in Figure 1.

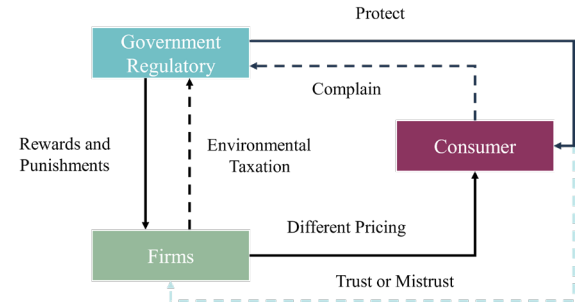

Figure 1. Three-party evolutional game relation

Our study addresses the challenge of environmental tax collection in the presence of pollution and governance externalities by proposing the integration of an OSP into the environmental governance supervision mechanism [15-16]. By leveraging environmental protection OSP [17], part of the ecological environment supervision responsibility can be delegated, promoting a win-win opportunity under the government compensation mechanism. Figure 2 shows the improved muti-agent evolutional game relationship.

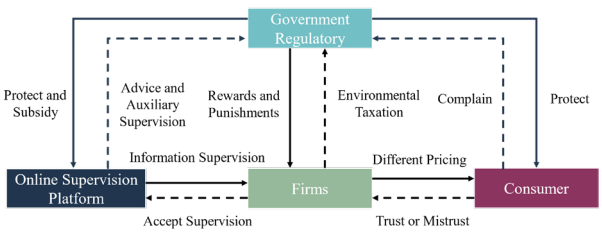

Figure 2. Muti-agent evolutional game relation

3 Model Assumption

In the pursuit of constructing a game-theoretic model to analyze the dynamics among various stakeholders in the implementation of environmental taxation within the context of a dual-carbon framework for network regulatory participation [18-19], this study posits the ensuing assumptions while delving into the evaluative strategy of environmental tax implications vis-à-vis consumer involvement in fostering a consciousness centered on the "Double Carbon" paradigm.

3.1 ASSUM 1

Assuming the selection of four entities as game participants: the governmental taxation department, OSP, environmental protection enterprises, and consumers. The environmental protection enterprises exhibit a propensity r to disclose product quality reports post-environmental tax payment conscientiously and comprehensively, while a probability of $1-r$ is attributed to the dissemination of inaccurate quality reports [20]. The publication of quality assessment reports necessitates utilization of an OSP, where the impartiality and equitable oversight of product environmental quality by the internet platform is represented by the probability p, with the probability of direct reporting without verification being $1-p$. The populace is endowed with the probability q to rationally discern the veracity of quality reports, while a probability of $1-q$ is associated with an outright belief in quality reports. Under the backdrop of the "Double Carbon" paradigm, consumer sensitivity towards environmental quality reports of enterprises is heightened [21]. Should product quality nonconformity with "Double Carbon" policies provoke consumer protests or a sequence of adverse behaviors indicative of mistrust in the government, the regulatory authority is compelled to opt for pre-publication oversight of product environmental quality reports [22-23]. The probability of rigorous regulatory intervention is s, whereas the probabilities of lax supervision or regulatory malfeasance are $1-s$, where r, p, q, s are confined to the interval [0,1]. The aforementioned assumptions are extrapolated to encompass the entire societal cohort, with each probability indicating the proportion at which each game participant within the collective opts for a specific strategy. All four entities exhibit risk neutrality, guided by the aim of maximizing individual interests.

3.2 ASSUM 2

Upon the successful oversight of environmental quality reports by the internet regulatory platform, environmental protection enterprises will experience an augmented consumer trust level, leading to an increase in product sales revenue, enhanced market valuation, brand recognition, other benefits denoted as V. In alignment with environmental tax policies, environmental protection enterprises undertake research entailing heightened personnel and time investments, as well as the acquisition of specialized instrumentation, incurring research costs C_{px}.

Moreover, costs C_{nr} arise when quality reports are disclosed without full and exhaustive prior investigation. Given the interdependence between internet platform oversight gains, consumer negative feedback, and governmental subsidies, particularly within the context of the "dual-carbon" paradigm, the internet regulatory platform engages more actively in supervising the environmental quality of products from various enterprises [24-25], thereby accruing gains K. C_{pt} pertains to confirming the authenticity of environmental protection enterprise product reports, while C_{pf} corresponds to the cost of directly reporting without verification, where $C_{pt} > C_{pf}$.

Assuming the influence of the internet environmental regulatory platform is characterized by μ, with μ constrained to the interval [0,0.5], signifying that μ proportion of online users pay attention to information disseminated by the platform and allow it to influence their ultimate decisions. On an individual level, this implies the probability of an individual being influenced by reported information is μ, operating under the presumption that consumers exhibit rational behavior. Typically, after becoming aware of platform reports, the majority of consumers do not immediately place trust in them, instead, they exercise rational discernment. This discernment involves consulting reference materials, seeking information from multiple sources, and expending additional time and effort to ascertain the credibility of internet regulatory platform reports. The associated cost of this discernment process is designated as C_p.

Furthermore, the costs associated with governmental regulatory interventions manifest as C_{gs} for stringent supervision and C_{gc} for regulatory malfeasance.

3.3 ASSUM 3

Amidst the backdrop of 'Double Carbon' imperatives, the implementation is orchestrated. The government and OSP function as purveyors of empirically grounded and authentic points of reference, thereby augmenting the ecological consciousness of consumers within the 'Double Carbon' framework [26]. This endeavor contributes to the elevation of the government's credibility. Let the affirmative utilities accrued for consumers and the government be denoted as B_{cp} and B_{gp}, respectively. However, in the event that the government and OSP disseminate fallacious signals to consumers or engage in regulatory malfeasance, consumers are poised to voice their dissent against products that fall short of conforming to environmental policies. It may culminate in the stagnation of product demand and could potentially lead to environmental degradation and tumultuous fluctuations in market prices. Suppose the adverse utilities incurred by consumers and the government be represented as N_c, N_g.

Table 1. Payoff matrix

Strategic decision		Online platform	Governmental taxation department			
			Strict regulation s		Regulatory malfeasance $1-s$	
			Online users appraise q	Online users trust $1-q$	Online users appraise q	Online users trust $1-q$
Firms	r	p	$V-C_{px}$	$V-C_{px}$	$V-C_{px}$	$V-C_{px}$
			$K-C_{pt}$	$K-C_{pt}$	$K-C_{pt}$	$K-C_{pt}$
			$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$
			$B_{cp}-C_p$	C_p	$B_{cp}-C_p$	C_p
		$1-p$	$V-C_{px}$	$V-C_{px}$	$V-C_{px}$	$V-C_{px}$
			$K-C_{pf}$	$K-C_{pf}$	$K-C_{pf}$	$K-C_{pf}$
			$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$	$B_{gp}-C_{gs}$
			$B_{cp}-C_p$	C_p	$B_{cp}-C_p$	C_p
	$1-r$	p	$-C_{nr}-C_{pt}$	$-C_{nr}-C_{pt}$	$-C_{nr}-C_{pt}$	$-C_{nr}-C_{pt}$
			(0, 0)	(0, 0)	(0, 0)	(0, 0)
		$1-p$	$-C_{nr}-P_f$	$-C_{nr}-P_f$	$-C_{nr}-P_l$	$V-C_{nr}-\beta P_f$
			$-C_{pf}-P_s$	$-C_{pf}-P_s$	$-C_{pf}-R_s$	$K-C_{pf}-\beta R_w$
			$P_f+P_s-C_{gs}$	$P_f+P_s-C_{gs}$	$-C_{gs}$	$-C_{gs}-N_g-\beta T_g$
			(0, 0)	(0, 0)	$-C_p$	$-N_c+\beta T_c$

3.4 ASSUM 4

In the context where government regulatory authorities proactively monitor the environmental quality of eco-enterprises and conduct assessments of the potential societal ramifications before the submission of their quality reports, promptly intervening to prevent their release. Furthermore, they apply punitive actions and warnings to both the environmental enterprises and the internet regulatory media platforms [27], with penalty magnitudes designated as P_f and P_s, respectively, and with P_f being significantly greater than P_s. This strategic approach can effectively mitigate the adverse societal consequences

arising from the sensational reporting of non-compliant pollution products.

3.5 ASSUM 5

When non-environmentally friendly product quality reports are made public, if consumers are capable of discerning rationally, the credibility of eco-enterprises diminishes, and their research and development capabilities come under scrutiny, resulting in a reputation loss denoted as R_l. Similarly, the internet regulatory platforms experience a reduction in trustworthiness, a decrease in attention and click-through rates, leading to a loss denoted as R_w. These platforms collaborate with the government in facilitating the implementation of environmental taxes. If consumers readily trust the quality reports provided by eco-enterprises, falsified reports could exert detrimental effects on the public and society [16]. On the other hand, if the government itself acts against fraudulent environmental quality reports, both eco-enterprises and internet oversight platforms would suffer reputational damage. Let the likelihood of the government's relevant departments debunking third-party environmental regulatory reports be represented as β, where $\beta \in [0,1]$. The government's efforts in debunking can mitigate the losses incurred by consumers and society due to false environmental reporting, yielding positive utilities denoted as T_c and T_g, respectively.

The efficacy assessment of environmental taxation implementation incorporates a four-party game payment matrix pertaining to the eco-enterprises' environmental quality reports involving governmental regulation and OSP, as depicted in Table 1.

4 Analysis on Strategy of Online Feedback System

4.1 Proof 1

The network supervision system chooses to verify the expected return of reporting or direct reporting. The replication dynamic equation and first-order derivative equation of the verification strategy are as follows:

$$Q_p = \mu\left(\gamma K - C_{pt}\right) \tag{1}$$

$$Q_{1-p} = \mu\begin{cases}\left[\gamma + (1-\gamma)(1-s)(1-q)\right. \\ K - (1-\gamma)(1-s) \\ \left.\left[q + (1-q)\beta\right]R_w - (1-\gamma)sP_s - C_{pf}\right\}\end{cases} \tag{2}$$

$$F(p) = dp/dt = p\left(Q_p - \bar{Q}\right) \tag{3}$$

$$\begin{aligned} F(p) = {} & \mu p(1-p)(1-\gamma)(1-s)\left[q + (1-q)\beta\right]R_w \\ & + (1-\gamma)sP_s - (1-\gamma)(1-s)(1-q)K \\ & + C_{pf} - C_{pt} \end{aligned} \tag{4}$$

$$F'(p) = \mu(1-2p)\begin{Bmatrix}(1-\gamma)(1-s)\left[q + (1-q)\beta\right]R_w \\ + (1-\gamma)sP_s - (1-\gamma)(1-s)(1-q)K \\ + C_{pf} - C_{pt}\end{Bmatrix} \tag{5}$$

By the stability theorem of differential equations, the environmental online feedback system in a stable state must satisfy:

$$F(p) = 0 \text{ and } F'(p) < 0. \tag{6}$$

4.2 Proposition 1

$s = s_0$, the stability strategy cannot be determined.

$s < s_0$, the stabilization strategy is for online monitoring platforms to directly monitor and publish reports.

$s > s_0$, online monitoring platforms need to be cautious about environmental reports on this product and are advised not to publish reports directly. If it is released directly, it may cause consumer dissatisfaction and even public opinion. The thresholds value s_0 goes as follows:

$$\begin{aligned} Z_1 = {} & \left[q + (1-q)\beta\right]R_w \\ & \left[(1-\gamma)(1-q)K + (1-\gamma)P_s - (1-\gamma)\right] \\ & \left[q + (1-q)\beta\right]R_w \end{aligned} \tag{7}$$

$$s_0 = \left[C_{pt} - C_{pf} + (1-\gamma)(1-q)K - (1-\gamma)Z_1\right]^{-1} \tag{8}$$

$$\begin{aligned} G(s) = {} & (1-\gamma)(1-s)\left[q + (1-q)\beta\right]R_w \\ & + (1-\gamma)sF(p) - (1-\gamma)(1-s)(1-q)K \\ & + C_{pf} - C_{pt} \end{aligned} \tag{9}$$

$$\partial G(s)/\partial(s) > 0 \tag{10}$$

If $s < s_0$, $G(s) < 0$, $F(p)|_{p=0} = 0$, $F'(p)|_{p=0} < 0$, $p = 0$ is the ESS (Evolutionary Stable Strategy) point.

If $s > s_0$, $G(s) > 0$, $F(p)|_{p=1} = 0$, $F'(p)|_{p=1} < 0$, $p = 1$ is the ESS point.

If $s = s_0$, $G(s) = 0$, $F(p) = 0$, $F'(p) = 0$. Thus, $p \in [0,1]$ is under stable state, cannot determine stable strategy.

Proposition shows that if environmental-protection regulatory authority implements stringent oversight, the operational approach of the online supervision platform will shift from immediate disclosure to circumspect reporting. Conversely, in the event of lax supervision by government regulatory authority, the operational strategy of the online supervision platform will transition from circumspect reporting to immediate disclosure. This underscores the imperative for government regulatory authorities to enforce rigorous regulatory measures on online supervision platform to avert significant societal repercussions arising from the dissemination of reports pertaining to environmentally friendly products.

The phase diagram illustrating the feedback report verification strategy of the online supervision platform for environmentally friendly products, which is presented in Figure 3.

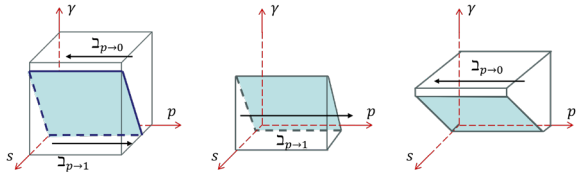

Figure 3. Replicator dynamic phase diagram of OSP

In Figure 3, the volume associated with part $\beth_{p\to 1}$ denotes the probability of meticulous reporting by the online supervision platform. Conversely, the volume of $\beth_{p\to 0}$ signifies the probability of unsupervised direct reporting by the online regulatory platform. The inference derived from this analysis is as follows:

$$\beth_{p\to 1}=\int_0^1\int_0^{1-\frac{C_{pt}-C_{pf}}{P_s}}\frac{(1-\gamma)P_s-C_{pt}+C_{pf}}{(1-\gamma)(1-q)K+(1-\gamma)P_s-(1-\gamma)\left[q+(1-q)\beta\right]R_w}d\gamma dp \tag{11}$$

$$\beth_{p\to 0}=\frac{C_{pt}-C_{pf}+(1-q)K-\left[q+(1-q)\beta\right]R_w-\left(C_{pt}-C_{pf}\right)\ln\frac{C_{pt}-C_{pf}}{P_s}}{(1-q)K-\left[q+(1-q)\beta\right]R_w+P_s} \tag{12}$$

5 Results and Discussion

When public confidence in online monitoring platforms is high, the likelihood of these platforms validating reports related to environmental protection products exhibits a negative correlation with both the magnitude of governmental sanctions and the erosion of trust. In cases where inaccurate reports on environmentally detrimental substandard products have been submitted without genuine feedback, and there is evidence of comment manipulation leading to the circulation of such products in the market, an escalation in government-imposed penalties can effectively enhance the likelihood of authentic reporting. Therefore, the establishment of an effective punitive mechanism by government regulators can incentivize online monitoring platforms to undertake thorough validation of reports. As credibility progressively improves, the probability of online regulatory platforms conducting report verification significantly rises, thereby enabling the public to rely on the genuine feedback provided by these platforms.

5.1 Analysis of Strategy Portfolio Stability

In the replicated dynamic system involving a multi-agent game among government environmental-protection regulators, manufacturers, online supervision platforms, and consumers, the stability of strategy combinations employed by these entities can be evaluated using Lyapunov's first law. Within the realm of the multi-agent evolution game, a stable solution is characterized as a strict Nash equilibrium. Given that a strict Nash equilibrium requires a pure strategy, this study seeks to analyze the stability of the equilibrium point within the framework of pure strategy in the multi-agent evolution game.

According to the replicated dynamic equations for each game subject, the Jacobian matrix of the replicated dynamic system is calculated as follows:

$$J=\begin{pmatrix}\partial F(\gamma)/\partial(\gamma)\partial F(\gamma)/\partial(p)\partial F(\gamma)/\partial(s)\partial F(\gamma)/\partial(q)\\ \partial F(p)/\partial(\gamma)\partial F(p)/\partial(p)\partial F(p)/\partial(s)\partial F(p)/\partial(q)\\ \partial F(s)/\partial(\gamma)\partial F(s)/\partial(p)\partial F(s)/\partial(s)\partial F(s)/\partial(q)\\ \partial F(q)/\partial(\gamma)\partial F(q)/\partial(p)\partial F(q)/\partial(s)\partial F(q)/\partial(q)\end{pmatrix} \tag{13}$$

When the government environmental protection regulatory department adopts a strategy of neglecting supervision, the asymptotic stability analysis of the equilibrium point for the replicated dynamic system is presented in Table 1.

Conversely, when the stabilization strategy of the government environmental protection regulatory department is strict supervision, the asymptotic stability analysis of the equilibrium point of the replicated dynamic system is shown in Table 2, Table 3.

5.2 Simulation Analysis

To elucidate more intuitively the influence of pivotal components within a replicated dynamic system on the evolution process and outcomes of multi-party games, this study employs MATLAB 2019a for the numerical simulation of the evolution trajectories for each participant in the game.

Table 2. Asymptotic stability of the equilibrium point within the replicated dynamic system, considering the absence of oversight by the government environmental protection department

ESS	Eigenvalues	Stability
(0,0,0,0)	$\mu\left(\beta R_l + C_{n\gamma} - C_{px}\right), \mu\left(\beta R_w + C_{pf} - C_{pt} - K\right),$ $\mu\left(P_f + P_s + C_{gc} - C_{gs} - \beta T_g\right), \mu(N_c - \beta T_c - C_p)$	ESS*
(1,0,0,0)	$\mu(C_{px} - \beta R_l - C_{n\gamma}),\ \mu\left(C_{pf} - C_{pt}\right), \mu\left(C_{gc} - C_{gs}\right), -\mu C_p$	ESS**
(0,1,0,0)	$\mu\left(V + C_{n\gamma} - C_{px}\right), \mu\left(K + C_{pt} - \beta R_w\right), 0, 0$	Unstable
(0,0,0,1)	$\mu\left(V + C_{n\gamma} - C_{px} + R_l\right), \mu\left(R_w + C_{pf} - C_{pt}\right),$ $\mu\left(P_f + P_s + C_{gc} - C_{gs}\right), \mu\left(\beta T_c + C_p - N_c\right)$	Not Exist
(0,1,0,1)	$\mu\left(V + C_{n\gamma} - C_{px}\right), \mu\left(C_{pt} - C_{pf} - R_w\right), 0, 0$	Unstable
(1,1,0,0)	$\mu\left(-V - C_{n\gamma} + C_{px}\right), -\mu\left(C_{pf} - C_{pt}\right), \mu\left(C_{gc} - C_{gs}\right), -\mu C_p$	Unstable
(1,0,0,1)	$\mu\left(-V - C_{n\gamma} + C_{px} - R_l\right), \mu\left(C_{pf} - C_{pt}\right), \mu\left(C_{gc} - C_{gs}\right), \mu C_p$	Unstable
(1,1,0,1)	$\mu\left(-V - C_{n\gamma} + C_{px}\right), -\mu\left(C_{pf} - C_{pt}\right), \mu\left(C_{gc} - C_{gs}\right), \mu C_p$	Unstable

Note: ESS* denotes ESS point exist, if and only if $\beta R_l + C_{n\gamma} - C_{px} < 0$, $N_c - \beta T_c - C_p < 0$.
ESS** denotes ESS point exist, if and only if $C_{px} - \beta R_l + C_{n\gamma} < 0$.

Table 3. Asymptotic stability of the equilibrium point within the replicated dynamic system, considering strict supervision by the government environmental protection department

ESS	Eigenvalues	Stability
(0,0,1,0)	$\mu\left(V + P_f + C_{n\gamma} - C_{px}\right),\ \mu\left(P_s + C_{pf} - C_{pt}\right)$ $\mu\left(C_{gs} - P_f - P_s - C_{gc} + \beta T_g - N_g\right), 0$	Unstable
(0,0,1,1)	$\mu(V + P_f + C_{n\gamma} - C_{rh}),\ \mu\left(P_s + C_{pf} - C_{pt}\right), \mu\left(C_{gs} - P_f - P_s - C_{gc}\right), 0$	Unstable
(0,1,1,0)	$\mu\left(V + C_{n\gamma} - C_{px}\right), \mu\left(-P_s - C_{pf} + C_{pt}\right), 0, 0$	Unstable
(1,0,1,0)	$\mu\left(V + C_{n\gamma} - C_{px} + R_l\right), \mu\left(R_w + C_{pf} - C_{pt}\right), \mu\left(P_f + P_s + C_{gc} - C_{gs}\right),$ $\mu\left(\beta T_c + C_p - N_c\right)$	Not Exist
(0,1,1,1)	$\mu\left(V + C_{n\gamma} - C_{px}\right), \mu\left(C_{pt} - C_{pf} - P_s\right), 0, 0$	Unstable
(1,0,1,1)	$\mu\left(-V - C_{n\gamma} + C_{px} - P_f\right), \mu\left(C_{pf} - C_{pt}\right), -\mu\left(C_{gc} - C_{gs}\right), \mu C_p$	Not Exist
(1,1,1,0)	$\mu\left(-V - C_{n\gamma} + C_{px}\right), -\mu\left(C_{pf} - C_{pt}\right), -\mu\left(C_{gc} - C_{gs}\right), -\mu C_p$	Not Exist
(1,1,1,1)	$\mu\left(-V - C_{n\gamma} + C_{px}\right), -\mu\left(C_{pf} - C_{pt}\right), -\mu\left(C_{gc} - C_{gs}\right), \mu C_p$	Not Exist

The governmental supervision department imposes a cost of C_{gs} = 6 for stringent supervision and C_{gc} = 6 for lax supervision. In cases where the OSP furnishes subpar feedback, both the product manufacturer and the platform face penalties, denoted as P_f = 10, P_s = 6, respectively. The likelihood of the public placing trust in the aforementioned feedback is represented by μ = 0.5, with the cost of judicious identification denoted as C_p = 10. In instances of false environmental reports, individuals with discerning judgment refrain from believing them, fostering skepticism towards the network supervision platform. This skepticism incurs reputational losses for both entities, with R_l = 25 and R_w = 15. Simultaneously, substandard environmental feedback contributes to losses incurred by the public and society, quantified as N_c = 12 and N_g = 20, respectively. The initial strategy choices of each agent are γ = 0.5, p = 0.3, s = 0.2, q = 0.3.

5.3 The Impact of Strict Government Regulation on Costs

Set C_{gs} = {16,12,6}, the evolution process and results of the main strategies of the four-party game are shown in Figure 1.

Figure 4, Figure 5, Figure 6 illustrates that the cost associated with rigorous government oversight not only influences the strategic evolution trajectory of government departments but also exerts an impact on the strategic evolution trajectories of the remaining three parties. Notably, the most pronounced alteration occurs in the strategic evolution trajectory of environmentally friendly product manufacturers. As the cost of stringent supervision by governmental regulatory bodies diminishes, there is a noteworthy rise in the likelihood of merchants engaging in high-caliber pharmaceutical research. Concurrently, the duration required to stabilize on the pure strategy of furnishing high-quality research reports extends. Consequently, reducing the costs of stringent monitoring can incentivize businesses to contribute high-quality environmental research and elucidations.

Figure 7, Figure 8, Figure 9 reveals that, with prompt and accurate feedback from the online supervision platform, there is a substantial escalation in the likelihood of exposing environmentally conscious product merchants who furnish substandard reports. This heightened probability, in turn, leads to an incremental increase in the likelihood of disseminating high-quality research reports. Ultimately, the system stabilizes at the provision of superior environmental testing reports. Concurrently, governmental regulatory authorities can gradually ease supervision, resulting in savings on regulatory costs. Therefore, fostering a culture of commentary and feedback on erroneous environmental product testing reports by online supervision platforms proves effective in mitigating the likelihood of reporting inferior environmentally friendly products. This approach serves to safeguard the interests of the public and society while bolstering the government's influence.

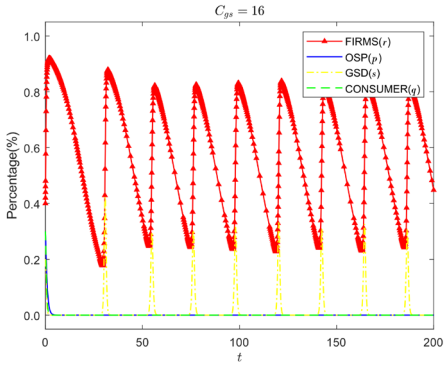

Figure 4. C_{gs} = 16

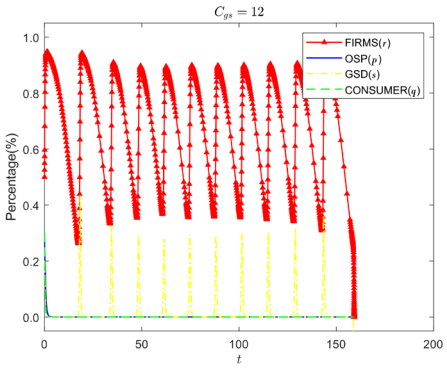

Figure 5. C_{gs} = 12

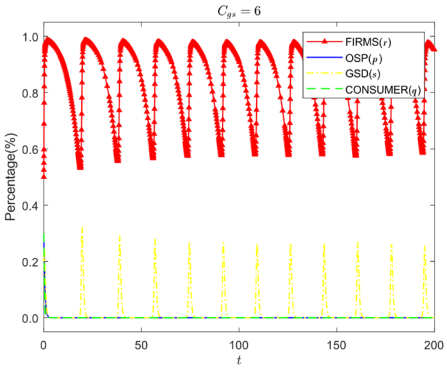

Figure 6. C_{gs} = 6

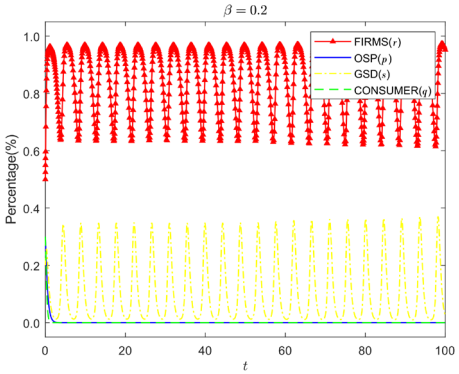

Figure 7. β = 0.2

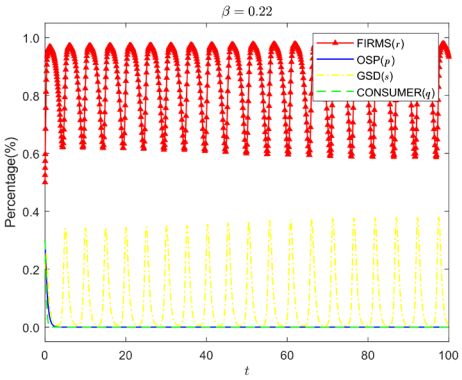

Figure 8. $\beta = 0.22$

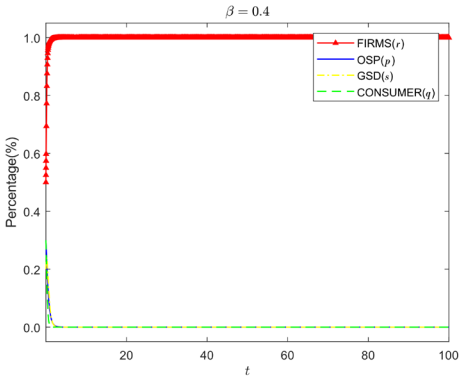

Figure 9. $\beta = 0.4$

The impact of the establishment of government regulatory mechanisms on the evolution of strategies of all parties.

Figure 10, Figure 11, Figure 12 illustrates that when $g = 0$, indicating the absence of government regulatory oversight on environmental testing reports, the stable strategy of environmental product manufacturers is precarious due to various influences such as false comments on online platforms and the risk of reputation loss. Notably, the strategic inclination of the network supervision platform predominantly leans towards refraining from verification to promptly submit environmental protection research reports, aiming to maximize profits. In contrast, when g = 0.8, signifying the establishment of an environmental protection research report supervision and review mechanism by the government regulatory authorities, the system lacks a stable point. However, maintaining a certain level of supervisory probability by the authorities significantly enhances the likelihood of manufacturers delivering high-quality environmental testing reports, stabilizing the system at the provisioning of such reports. During this phase, the public can opt to place trust in the reports provided by the Internet supervision platform.

This study is grounded in the contemporary Internet landscape of our nation, where government supervision mechanisms intersect with OSP to assure the quality of environmental product reporting. The ensuing key conclusions are delineated below.

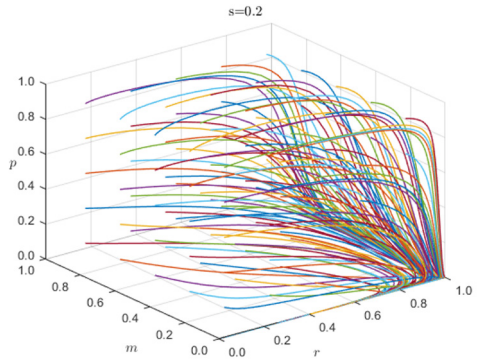

Figure 10. $s = 0.2$

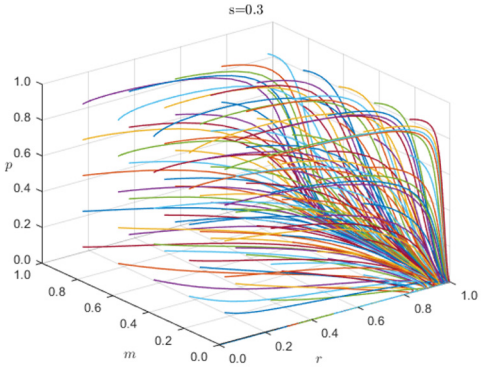

Figure 11. $s = 0.3$

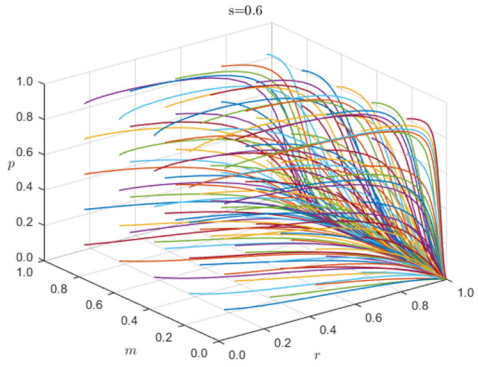

Figure 12. $s = 0.6$

6 Conclusion

While establishing a product environmental testing and supervision system, government regulatory authorities should also improve the punishment mechanism and increase the punishment for manufacturers that provide low-quality reports. They should also severely punish Internet regulatory platforms that publish false comments for profit. We can supervise illegal institutions through fines, notifications, and criticisms, severely crack down on socially irresponsible behavior, and adopt a “zero tolerance” attitude to deal with online inducement and rumor-mongering behaviors that cause social chaos in special periods. As penalties increase, manufacturers will take the initiative to conduct rigorous environmental

testing reports, and network supervision platforms will correspondingly increase the probability of verifying reports.

This study is grounded in the theoretical foundations of rational economic man and expected utility. However, in real-world scenarios, the decision-making processes of individual subjects are often swayed by ideas propagated through network recommendations. Consequently, in subsequent research endeavors, the integration of artificial intelligence or deep learning technology within network supervision platforms will be explored. This exploration aims to assess the influence of content recommendations and other inducing factors on the system's evolution process and outcomes in the context of multi-party repeated games.

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