Optimizing Grover's Algorithm for Routing in Quantum Wireless Communication Networks

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Abstract

This paper presents a novel approach to optimizing Grover's algorithm for addressing the routing problem in quantum wireless communication networks. We begin by conducting a comprehensive analysis of key quantum communication technologies, including quantum teleportation and quantum entanglement swapping, and subsequently construct a quantum wireless communication network model grounded in these principles. Building on this foundation, we design and implement a multi-level quantum wireless network information transmission mechanism, incorporating quantum channels for identity authentication to ensure secure communication. Focusing on the routing challenges in quantum wireless communication networks, we propose innovative applications and optimizations of Grover's quantum search algorithm. Specifically, we introduce an improved algorithm that efficiently searches for routing paths with maximal metrics within a limited number of hops. This enhancement not only significantly increases routing efficiency but also mitigates the risk of quantum channel disconnection caused by the depletion of entangled quantum pairs, thereby improving the overall success rate of communication. Simulation and experimental results demonstrate that the proposed optimization algorithm substantially enhances the routing performance of quantum wireless communication networks while maintaining robust security measures. This study offers a new solution to the routing challenges in quantum wireless communication networks and significantly optimizes Grover's algorithm, contributing to the development of more efficient and reliable quantum communication networks.

Keywords: Quantum wireless communication, Quantum routing algorithm, Quantum teleportation, Quantum entanglement

1 Introduction

The rapid advancement of information technology has exposed the limitations of traditional wireless communication networks in meeting the growing demands for high-speed, large-capacity, and low-latency communication. As a promising emerging technology, quantum wireless communication offers novel solutions to these challenges by leveraging unique quantum properties, such as quantum teleportation and quantum entanglement swapping. Quantum wireless communication networks not only retain the functionalities of traditional communication networks but also enable more efficient information transmission while significantly enhancing communication security.

In quantum wireless communication networks, the routing algorithm is crucial in determining communication efficiency and quality. To optimize these algorithms for quantum communication, it is essential to consider the unique characteristics of quantum channels and the limited quantum resources available. Traditional routing algorithms often fail to deliver optimal results in quantum environments. Grover's algorithm, a well-known quantum search algorithm with a square-root speedup, can rapidly identify target items within large datasets. However, the original Grover's algorithm faces certain limitations within the specific context of quantum wireless communication, such as potential instability in quantum channels, low success rates, and high computational demands.

Quantum teleportation is one of the key technologies in quantum communication. In reference [1], Ren et al. conducted in-depth research on the preparation of continuous variable quantum teleportation and remote states between two spatially separated local networks. Quantum key distribution plays an important role in quantum communication security. In reference [2], Nikolopoulos and Fischlin proposed a quantum key distribution post-processing method driven by physically unclonable functions. Dupuy et al. reviewed the quantum entanglement routing protocol in reference [3] and pointed out the challenges it faces in wide area networks. In terms of secure quantum communication, establishing fault-tolerant remote quantum entanglement is crucial. Tsai and Lin conducted a study on this in reference [4]. In order to improve the performance of quantum communication, a relay based quantum communication protocol has been proposed. In reference [5], Ghosal et al. proposed a relay based quantum communication protocol to maximize the fidelity of teleportation and minimize entanglement consumption. In wireless sensor networks, Wang et al. proposed an energy-efficient, clustering based routing protocol based on fuzzy logic and quantum annealing algorithm in reference [6]. In addition, long-distance continuous variable quantum key distribution is also a research direction. In quantum hybrid

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networks, Zhang and Liu proposed a concurrent multi-path quantum entanglement routing based on segmented routing in reference [7]. For quantum search algorithms, Li and Li proposed a flexible fixed phase quantum search algorithm in reference [8] for searching unordered databases of any size.

In summary, the routing search problem in quantum wireless communication networks is a challenging research topic. This article proposes a method based on optimized Grover algorithm to improve the routing search performance of quantum wireless communication networks and provide new solutions for the development of quantum wireless communication networks.

2 Quantum Wireless Communication Network Technology

2.1 Quantum Teleportation

Quantum teleportation is a technology that exploits the principles of quantum mechanics to transfer information (specifically, quantum state information) from one location to another without physically transmitting the particles themselves. The core concept is based on quantum entanglement and quantum measurement, allowing an unknown quantum state to be "teleported" from one particle to another, even if the two particles are spatially separated. During this process, the quantum state of the original particle is destroyed, while the particle at the receiving end acquires an identical quantum state. The mechanism of quantum teleportation involves transmitting an unknown quantum state by decomposing its information into two components: classical information and quantum information. These components are then sent to a distant receiver via classical and quantum channels, respectively. The classical information is obtained through a measurement process performed by the sender, while the quantum information is derived from the entanglement property of Einstein-Podolsky-Rosen (EPR) pairs. By utilizing these two types of information, the receiver can accurately reconstruct the quantum state of the original particle, thus enabling long-distance transmission of quantum information [9].

Preparing an Entangled Pair: In quantum teleportation, the process begins with Alice and Bob sharing a pair of entangled quantum bits (qubits). Alice possesses photon 1, which is in the quantum state to be teleported:

$$\left|\psi\right\rangle_1 = a\left|0\right\rangle + b\left|1\right\rangle \tag{1}$$

Simultaneously, Alice prepares a shared pair of photons, labeled as photons 2 and 3, in an EPR entangled state:

$$\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right) \tag{2}$$

In this setup, Alice retains photon 2 and sends photon 3 to Bob, who is at a distant location. Alice then performs a Bell measurement on the two quantum bits she possesses: photon 1 (the quantum state to be transmitted) and photon 2 (one of the entangled pair). At this stage, the three photons collectively form a mixed state, represented by the following equation:

$$\begin{aligned}\left|\psi\right\rangle_{123} = \frac{1}{2}\Big[&\left|\psi^-\right\rangle_{12}\left(-a\left|0\right\rangle - b\left|1\right\rangle\right)_3 + \left|\psi^+\right\rangle_{12}\left(-a\left|0\right\rangle + b\left|1\right\rangle\right)_3 \\ &+ \left|\phi^-\right\rangle_{12}\left(b\left|0\right\rangle + a\left|1\right\rangle\right)_3 + \left|\phi^+\right\rangle_{12}\left(-b\left|0\right\rangle + a\left|1\right\rangle\right)_3\Big]\end{aligned} \tag{3}$$

After performing the Bell-state measurement on photons 1 and 2, Alice publicly shares the measurement results. According to the equations above, photon 3 in Bob's possession immediately collapses to a corresponding quantum state following Alice's measurement. Bob can then apply an appropriate quantum transformation to photon 3 to reconstruct the original quantum state of photon 1. For example, if Alice's measurement yields the result $|\psi^-\rangle_{12}$, then photon 3 collapses to the state －a|0−b|1. Bob can restore this to the original state a|0+b|1 by applying the unitary transformation: $U_1=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$. Similarly, if Alice's measurements yield the states $|\psi^+\rangle_{12}$, $|\Phi^-\rangle_{12}$, and $|\Phi^+\rangle_{12}$, the corresponding unitary transformations Bob should apply to recover the transmitted quantum state are:

$$U_2 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix},\ U_3 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ U_4 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \tag{4}$$

Quantum teleportation leverages the non-local nature of quantum entanglement, allowing quantum states to be "teleported" from one location to another without physically transmitting the particles. Although the quantum state itself is not directly sent to Bob, the combination of entanglement and classical communication enables Bob to reconstruct an identical copy of the original quantum state on his particle. This process preserves the integrity and security of the quantum information, even in the presence of potential eavesdroppers. By integrating shared quantum entangled pairs with classical communication, quantum teleportation not only provides a novel method for transmitting quantum information but also plays a crucial role in the development of quantum computing and quantum communication technologies.

2.2 Quantum Entanglement Swapping Technology

Quantum Entanglement Swapping (QES) is a technique that leverages the properties of quantum entanglement to establish entanglement between different quantum systems that were not initially entangled. The fundamental concept involves creating entanglement between two previously unentangled quantum systems via an intermediate node, without the need for direct quantum state transmission. This is achieved through quantum measurements and classical communication [10].

Quantum entanglement swapping, often referred to as the hyperspace transport of quantum entanglement, allows for the correlation between two or more quantum systems by utilizing two already entangled quantum pairs. Through

Bell-state measurements on these pairs, QES effectively "entangles" two otherwise independent quantum particles. This process enables the establishment of entanglement between two quantum particles, even in the absence of a direct quantum channel, facilitating the storage and transmission of quantum information across different locations. The implementation scheme is as follows.

Entangled particle pairs are first prepared at different nodes within the network. These pairs are typically generated via specific physical processes (such as parametric downconversion) and are then distributed to various nodes through quantum channels.

For instance, photon 1 and photon 2 are prepared in an entangled state, denoted as $|\psi^-\rangle$ (initial state), while photon 3 and photon 4 form another entangled state, $|\psi^-\rangle$ (initial state). Initially, there is no entanglement between these two pairs of photons. Suppose the communicating parties, Alice and Bob, each possess different photons: Alice holds photon 1 and photon 3, while Bob holds photon 2 and photon 4. The initial state of the entire system can be represented as:

$$\begin{aligned}|\Psi\rangle_{1234} &= |\Psi^-\rangle_{12} \otimes |\Psi^-\rangle_{34} \\ &= \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2) \otimes \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)\end{aligned} \quad \textbf{(5)}$$

Once the entangled particle pairs are prepared, they must be distributed across the network nodes while maintaining their entanglement properties. This typically requires the use of quantum error-correcting codes and quantum relaying techniques to overcome noise and loss within the quantum channel. When it becomes necessary to establish entanglement between two remote nodes, the entangled particle pairs at an intermediate node can serve as a "bridge" to facilitate entanglement swapping through a series of local operations and classical communication. These operations may include Bell-state measurements and single-qubit measurements, with the results communicated to the other node via classical channels.

Alice performs a Bell-state measurement on the photons 1 and 3 in her possession, which results in a spectral decomposition and collapse of the quantum state. The entire system's state can then be rewritten in terms of four Bell-state decompositions:

$$|\Psi\rangle_{1234} = \frac{1}{2}\begin{bmatrix} |\Phi^+\rangle_{13}|\Phi^+\rangle_{24} - |\Phi^-\rangle_{13}|\Phi^-\rangle_{24} \\ -|\Psi^+\rangle_{13}|\Psi^+\rangle_{24} + |\Psi^-\rangle_{13}|\Psi^-\rangle_{24} \end{bmatrix} \quad \textbf{(6)}$$

The system will randomly collapse into one of these four possible states. For example, if Alice's measurement yields the result $|\Phi^+\rangle_{13}$ (final state), she can communicate this outcome to Bob via classical communication. Upon receiving this information, Bob recognizes that the two photons in his possession are now entangled due to the correlated collapse, resulting in the state $|\Phi^+\rangle_{24}$ (final state).

From Equation (6), it is clear that photons 2 and 4 do not directly interact; their entanglement is established indirectly through the interaction of photons 1 and 3 during Alice's Bell-state measurement. This effectively entangles photons 2 and 4, demonstrating the principle of quantum entanglement swapping.

After the entanglement swapping is completed, it is essential to confirm and purify the newly established entanglement. Confirmation can be achieved through methods such as quantum state tomography, while purification involves enhancing the quality of the entanglement and mitigating the effects of noise and errors. Once high-quality entanglement is established between remote nodes, this entanglement resource can be utilized for various quantum communication and quantum computing tasks, including quantum teleportation, quantum key distribution, and remote quantum state preparation.

3 Establishment of a Quantum Wireless Communication Network Model

3.1 Quantum Synchronous Communication

In quantum wireless communication, the principle of quantum teleportation can be utilized to develop a quantum synchronous communication system. Apart from the time needed to establish the quantum channel and perform classical communication, the convergence of the quantum state is instantaneous. By exploiting this property, a quantum synchronous communication system can be effectively constructed.

As illustrated in Figure 1, Alice serves as the sender, and Bob is the receiver. A quantum channel is established between Alice and Bob for communication purposes.

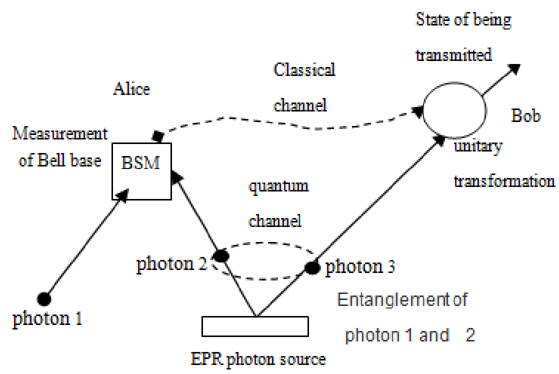

Figure 1. Quantum transmission model

Assume that the photon carrying message M, which Alice intends to send, is following:

$$|\psi\rangle_M = \frac{1}{\sqrt{2}}\left(|0\rangle_M + b|1\rangle_M\right) \quad \textbf{(7)}$$

Where b=1 and b=−1 correspond to the message $M(i)$=0 and $M(i)$=1, respectively. Bob prepares a pair of entangled photons (A, B) in the Bell state as follows:

$$|\beta_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right) \quad \textbf{(8)}$$

The mixed state of the three photons is then represented as:

$$\begin{aligned}|\psi_0\rangle &= |\psi\rangle_M \otimes |\beta_{00}\rangle_{AB} \\ &= \frac{1}{2}\left[|\psi\rangle_M \left(|00\rangle_{AB} + |11\rangle_{AB}\right) + b|1\rangle_M \left(|00\rangle_{AB} + |11\rangle_{AB}\right)\right]\end{aligned} \quad \textbf{(9)}$$

In this configuration, the first two quantum bits belong to Alice, while the third quantum bit belongs to Bob. Alice performs a C_{not} gate transformation on her quantum bits, as described by the following equations:

$$|00\rangle \xrightarrow{C-NOT} |00\rangle, \quad |01\rangle \xrightarrow{C-NOT} |01\rangle$$
$$|10\rangle \xrightarrow{C-NOT} |11\rangle, \quad |11\rangle \xrightarrow{C-NOT} |10\rangle \quad \textbf{(10)}$$

This operation results in the following state:

$$|\psi_1\rangle = \frac{1}{2}\left[|0\rangle_M \left(|00\rangle_{AB} + |11\rangle_{AB}\right) + b|1\rangle_M \left(|10\rangle_{AB} + |01\rangle_{AB}\right)\right] \quad \textbf{(11)}$$

Next, Alice performs a Hadamard transformation on the first quantum bit, resulting in:

$$|0\rangle \xrightarrow{H} |+\rangle = \frac{1}{\sqrt{2}}\left[\left(|0\rangle + |1\rangle\right)\right]$$
$$|1\rangle \xrightarrow{H} |-\rangle = \frac{1}{\sqrt{2}}\left[\left(|0\rangle - |1\rangle\right)\right] \quad \textbf{(12)}$$

This operation produces the following state:

$$|\psi_2\rangle = \frac{1}{2\sqrt{2}}\begin{bmatrix}\left(|0\rangle + |1\rangle\right)_M \left(|00\rangle_{AB} + |11\rangle_{AB}\right) \\ +b\left(|0\rangle - |1\rangle\right)_M \left(|10\rangle_{AB} + |01\rangle_{AB}\right)\end{bmatrix} \quad \textbf{(13)}$$

Through the classical channel, Alice performs a single-qubit measurement on the first two quantum bits of $|\psi_2\rangle$, with the possible outcomes being $|\beta_{00}\rangle$, $|\beta_{01}\rangle$, $|\beta_{10}\rangle$, and $|\beta_{11}\rangle$. The state $|\psi_2\rangle$ will collapse into one of the four superposition states with equal probability. When Alice communicates the results of her measurements to Bob, he can perform the corresponding inverse transformations—I^{-1}, X^{-1}, Z^{-1}, and Y^{-1}—on his photon B. This allows Bob to replicate the quantum state carrying the message M on photon B. Bob can then measure his photon to determine b=1 or b=−1, thereby deriving the corresponding message $M(i)$=0 or $M(i)$=1. This method, based on the principle of free-space quantum teleportation, can be successfully extended to long-distance wireless networks.

3.2 Quantum Wireless Communication Network Model

The process of constructing a quantum wireless communication network and transmitting information is depicted in Figure 2. This model leverages quantum communication technology, particularly the principles of quantum entanglement and quantum teleportation, to ensure secure and reliable information transmission. The core concept is to transmit quantum information from the source to the destination via a quantum channel while optimizing the transmission path through a self-organized network structure and routing protocol, thereby enhancing both efficiency and reliability.

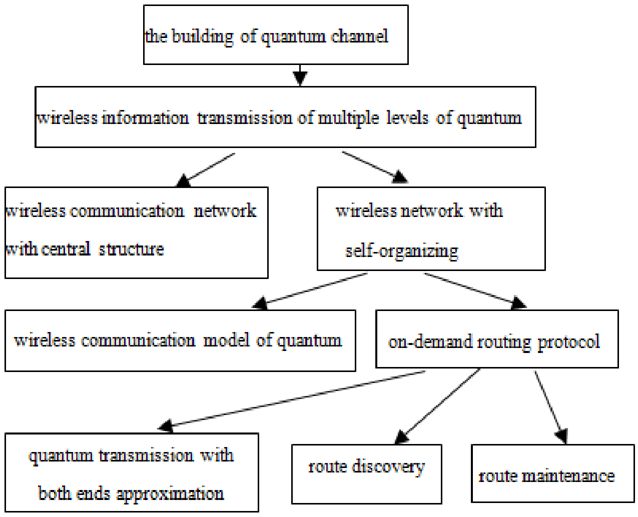

Figure 2. Quantum wireless communication model

The quantum wireless communication network starts with the construction of a quantum channel, which is the cornerstone of the entire system, ensuring stable quantum information transmission. Information is efficiently transmitted through multiple levels within the channel, highlighting the inherent flexibility of quantum communication. This information is then integrated into a centralized wireless network, combining quantum and classical technologies for wider dissemination. Subsequently, the data enters a self-organizing wireless network that dynamically optimizes routing to enhance transmission efficiency and reliability. Within this network, information is relayed back to the quantum communication model, the technical core of the network. This model employs advanced technologies, such as quantum error correction and key distribution, to manage complex transmission tasks and ensure network security and stability. The quantum channel serves as a robust transmission pathway, leveraging quantum properties like superposition and entanglement. Each transmission level uses different quantum states and coding methods, significantly enhancing efficiency and security. The integration of quantum and classical technologies broadens the reach of quantum information, while the self-organizing network structure maintains efficient and reliable transmission.

The technical routes adopted in this model include [11]:

1. Quantum Channel Technology: Utilizing cutting-edge technologies such as quantum entanglement and quantum teleportation to construct a stable and efficient quantum channel.

2. Multi-Level Transmission Strategy: Employing quantum state superposition and entanglement techniques to enable multi-level transmission and coding of information,

thereby enhancing transmission efficiency.

3. Converged Communication Technology: Integrating quantum information with existing communication networks using a centralized scheduling mechanism to achieve deep convergence between quantum and classical communication technologies.

4. Self-Organized Network Optimization: Implementing dynamic routing and adaptive algorithms to optimize the structure of self-organized networks and improve transmission performance.

5. Quantum Communication Model Innovation: Continuously researching and developing new technologies such as quantum error correction and key distribution to strengthen the core capabilities of the quantum communication model.

4 Wireless Communication Network Information Transmission Realization

Multilevel quantum wireless network information transmission leverages quantum channels to facilitate communication between multiple nodes, allowing information to be transmitted over longer distances using technologies like multilevel relaying and quantum entanglement swapping. This method enhances the coverage and transmission efficiency of quantum communication networks by utilizing the unique principles of quantum mechanics, such as quantum entanglement and quantum teleportation.

4.1 Establishment of Quantum Channel

In quantum communication networks, the quantum channel is the backbone of quantum information transmission. A quantum channel can be simply understood as a communication link formed by pairs of entangled photons. There are two primary methods for establishing such a channel:

Using an Entangled Photon Pair Source: An entangled photon pair source generates pairs of entangled photons and distributes them to relevant nodes within the network, thereby establishing a quantum channel between these nodes. These entangled photon pairs are then used for transmitting quantum information.

Utilizing Quantum Entanglement Swapping Technology: When two pairs of EPR (Einstein-Podolsky-Rosen) photons, which are not initially entangled with each other, undergo specific operations, quantum entanglement swapping can be achieved among the four photons to generate new pairs of entangled photons. These newly established pairs create quantum channels that can prevent interruptions caused by the depletion of entangled photon pairs [12].

Regardless of the method, establishing a reliable quantum channel is crucial for secure information transmission in quantum communication networks. Quantum channels enable nodes to perform critical functions like quantum key distribution and quantum encryption, ensuring the confidentiality and integrity of information during transmission.

The evolution equation of the quantum state within the channel is described by the equation (14):

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle \tag{14}$$

where $|\psi(t)\rangle$ is the quantum state, H is the Hamiltonian operator, and $\hbar$ is the reduced Planck constant.

After successfully identifying a path, the source and destination nodes begin to align and eventually establish a stable quantum channel, often with the help of intermediate nodes. For instance, when a quantum measurement is performed at an intermediate node B with a measurement result of $|00\rangle_B$, the quantum state between the source node A and the destination node C forms an entangled state $|\phi>_{AC}=\frac{1}{\sqrt{2}}(|0>_A|0>_C + |1>_A|1>_C)$. This entangled state becomes a critical resource in quantum communication, allowing for direct quantum state transfer between nodes A and C.

In addition to the $|00\rangle_B$ measurement, there are other three possible quantum measurements with corresponding management strategies to ensure that an entangled quantum pair is generated between the source node A and the destination node C, irrespective of intermediate node measurements. Once this entangled quantum pair is established, nodes A and C can communicate directly without needing to route through an intermediate node.

The flow of establishing a quantum channel involves nine critical steps, as outlined in Figure 3:

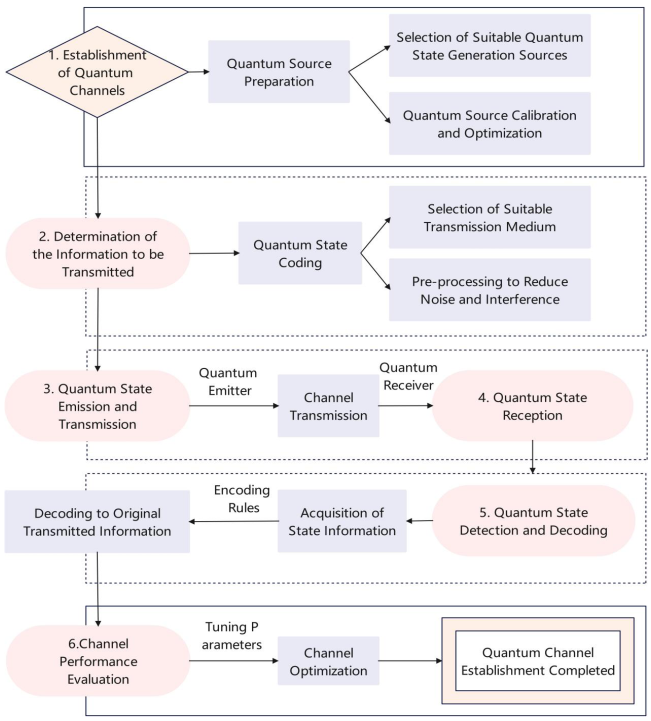

Figure 3. Quantum channel establishment workflow algorithm

1. Quantum Source Preparation: Choose a stable quantum source, such as a single photon source or an entangled

photon pair source. Perform precise calibration to ensure the quantum state's stability and reliability.

2. Quantum State Encoding: Encode the information to be transmitted into the selected quantum state. This encoding could involve different qubit states or specific configurations of quantum entangled states.

3. Channel Initialization: Select an appropriate transmission medium, such as optical fiber or free space. Pre-process the transmission medium to minimize the adverse effects of environmental noise and interference.

4. Quantum State Transmission: Send the encoded quantum state through the quantum transmitter into the channel.

5. Channel Transmission: During transmission, the quantum state may be affected by attenuation, scattering, noise, and other factors.

6. Quantum State Reception: Use a specialized quantum receiver to capture the transmitted quantum state.

7. Quantum State Detection and Decoding: Detect and measure the received quantum states to obtain their state information. Decode the measurement results into the original transmission information using predetermined coding rules.

8. Channel Performance Evaluation: Analyze the differences between the received quantum state and the transmitted quantum state, evaluating performance metrics such as Bit Error Rate (BER) and fidelity.

9. Channel Optimization: Adjust the parameters of the quantum source, coding method, transmission medium, or receiver based on the performance evaluation results to optimize the channel and improve performance.

Upon completing these steps, a stable and efficient quantum channel is established, enabling secure and high-speed quantum communication across the network.

4.2 From Quantum Entanglement to Channel Performance Optimization

The process implementation of this algorithm thoroughly covers all aspects, from quantum entanglement generation to channel performance optimization, ensuring efficient and reliable quantum state transmission. We start by utilizing an advanced quantum entanglement generation algorithm. This step involves carefully preparing high-quality entangled photon pairs through techniques like parametric downconversion. The probability of generating these pairs can be represented as:

$$P_{ent} = \frac{\eta}{1+\eta}\left(\frac{P_p}{P_{th}}\right)^2 \quad \textbf{(15)}$$

where η is the nonlinear coefficient of the crystal. P_{th} is the threshold power. This equation shows that optimizing the wavelength λ_p and the power P_p of the pump light can significantly enhance the generation efficiency and purity of the entangled photon pairs. This step is crucial for building a stable quantum channel, laying a solid foundation for subsequent quantum state transmission.

Quantum states are vulnerable to noise and interference during transmission. To mitigate these effects, we introduce a novel error correction algorithm tailored for quantum channels. This algorithm includes the development of robust quantum error correction codes, which effectively resist noise and interference, thus ensuring that the original quantum state information is accurately restored at the receiving end [13].

To enhance the robustness of quantum communication, it is essential to develop error correction codes specifically designed for quantum channels. These codes must be capable of dynamically adjusting their parameters in response to real-time conditions, thereby adapting to varying levels of channel noise. Leveraging quantum algorithms, efficient decoding methods—such as the quantum version of the Belief Propagation (BP) algorithm—can be employed to improve both decoding speed and accuracy. Precise modeling of noise and interference within the quantum channel is crucial, as it provides the necessary channel information to optimize the error correction algorithm. This optimization enhances the error correction threshold and efficiency of quantum error correction codes. The error correction threshold for these codes is detailed in Equation (16).

$$\in_{th} = \frac{1}{2} - \frac{1}{2\sqrt{M}} \quad \textbf{(16)}$$

where $\in_{th}$ is the error correction threshold. M is the minimum Hamming distance of the quantum error correction code.

Further improving channel performance involves applying a channel parameter optimization algorithm. By monitoring and analyzing the quantum channel's characteristics in real-time—such as channel fading, noise level, and other key parameters—the algorithm dynamically adjusts parameters like transmission power and modulation mode to optimize performance. Quantum channel monitoring equipment is employed to collect key parameters, including channel attenuation and noise levels, in real time. This data is then used to dynamically adjust parameters such as transmit power and modulation mode, enabling the system to adapt to channel variations based on the monitored characteristics. The performance of these adjustments is evaluated by metrics such as the quantum bit error rate (QBER) and channel capacity. This adaptive adjustment mechanism allows for flexible responses to channel changes, ensuring the efficiency and reliability of quantum states during transmission. The channel fading model is presented in Equation:

$$P_{out} = P_{in} \cdot 10^{\frac{\alpha L}{10}} \quad \textbf{(17)}$$

where P_{in} is the input power. P_{out} is the output power. α is the fading coefficient. L is the channel length.

The noise level in the channel is given by:

$$N = k_B \cdot T \quad \textbf{(18)}$$

where N is the noise power. k_B is the Boltzmann constant. T is the temperature.

This comprehensive algorithm, through the combination of quantum entanglement generation, error correction, and channel parameter optimization, significantly enhances the transmission efficiency and purity of quantum states. It also strengthens the anti-interference capability and reliability of quantum communication systems, providing a robust foundation for the ongoing development of quantum communication technologies.

4.3 Transmission of Information in Multilevel Quantum Wireless Networks

The transmission of information in multilevel quantum wireless networks is an advanced method that significantly enhances the capacity and efficiency of quantum communication. This approach leverages the principles of quantum mechanics, specifically the superposition and entanglement of quantum states, to encode and transmit vast amounts of information simultaneously. By utilizing multiple energy levels of quantum states or multiple entangled states, this method allows for the transmission of more data within the same spatial and temporal constraints.

In terms of the technical route, specialized equipment is required to generate multilevel quantum states that are stable and suitable for transmission. Information is then mapped onto the quantum states using sophisticated coding schemes and advanced quantum modulation techniques. During transmission, maintaining the stability and integrity of the quantum channel is paramount, which requires real-time monitoring and adaptive control. At the receiving end, highly sensitive detectors are used to measure the quantum states accurately to ensure that the received quantum states can be accurately mapped back to the original information [14-15].

Multilevel quantum wireless information transmission can be applied using the controlled quantum teleportation method proposed by Karlsson et al. In this approach, an unknown quantum state is transmitted from the sender to the receiver, but the receiver can only successfully obtain the transmitted state with the assistance of a third party. The key players in this system are Alice, the sender; Bob, the receiver; and Charlie, the controller, who each possess one of three particles (particles 1, 2, and 3) that are in a three-body entangled state. Alice wishes to transmit information carried by particle M, which has an unknown quantum state, to Bob, but this process requires Charlie's assistance. The unknown quantum state of the particle M that Alice wants to send to Bob is represented as follows.

$$|\phi\rangle_M = \left(\alpha|0\rangle + \beta|1\rangle\right)_M, \text{ where } |\alpha|^2 + |\beta|^2 = 1 \tag{19}$$

This information-carrying particle M, along with particles 1, 2, and 3, which are already in an entangled state, forms a total quantum state that can be described by the following expression:

$$\begin{aligned}|\psi_{\mathrm{M123}}\rangle = |\varphi\rangle_M \otimes |\xi\rangle_{123} = \left(\left(\alpha|0\rangle + \beta|1\rangle\right)_M\right) \\ \otimes \frac{1}{2}\left(|000\rangle + |110\rangle + |011\rangle + |101\rangle\right)_{123}\end{aligned} \tag{20}$$

When Charlie agrees to facilitate the teleportation between Alice and Bob, he performs measurements on his own particle (particle 3) based on $\{|0\rangle,|1\rangle\}$ and informs both Alice and Bob of the results via a classical communication channel. Alice then performs Bell-state measurements on her particles (particle M, and particle 1), and informs Bob of the results via a classical communication channel. Bob then uses the information from Charlie's and Alice's measurements to determine the quantum state he possesses. With this information, Bob can apply the corresponding quantum unitary transformation to reproduce the quantum state $|\varphi\rangle$ that Alice originally wanted to transmit on particle 2.

Based on this principle, we can construct a quantum communication network containing two relay sites. In this scenario, particle M carries quantum information at the starting point, and its quantum state needs to be transmitted to Bob at the endpoint through two relay sites—Alice and Charlie. By employing the aforementioned controlled quantum teleportation mechanism, the necessary quantum operations can be performed to transmit quantum information from the starting point to the endpoint.

Comparing the existing single-relay system with the dual-relay system proposed in this paper, it can be inferred that in a system containing N relay nodes, it is still possible to utilize quantum principles such as entanglement swapping and teleportation to establish a quantum communication link between the source and destination. With knowledge of the qubit state at each relay point and the source, the original information can be recovered by appropriately converting the quantum state at the endpoints, thereby realizing the wireless transmission of quantum information through a network containing N relay nodes.

5 Improved Grover's Algorithm for Routing Mechanism Design

5.1 Grover's Algorithm

Grover's algorithm is a cornerstone of quantum computing, renowned for its ability to search for specific items in an unstructured database with significantly improved efficiency compared to classical algorithms, particularly in solving large-scale unstructured search problems. This algorithm leverages quantum superposition and interference to encode all possible items in a search space into a quantum state. Through a series of quantum operations, the algorithm amplifies the probability amplitude of the target item, allowing it to be found with high probability [16].

In classical computing, finding a target item in an unordered database of N elements typically requires, on average, $N/2$ searches. In contrast, Grover's algorithm accomplishes the same task with a time complexity of $O(\sqrt{N})$, offering a substantial speedup, compared to classical algorithms.

The implementation steps of Grover's Algorithm are as follows:

Initialization: Begin by encoding all items into a uniform superposition state. Initialize the quantum state to an equal amplitude superposition state, where each possible state has the same probability amplitude.

This is achieved by initializing n qubits to the state |0⟩^{\otimes n}, and then applying a Hadamard gate to each qubit, transforming the system into a uniform superposition state: $|s\rangle=|+\rangle^{\otimes n}=\frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle$, where $N=2^n$ represents the size of the search space.

The initialized quantum state is shown in Equation (21):

$$|\psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{x=0}^{N-1}|x\rangle \tag{21}$$

Here, N is the size of the database, and $|x\rangle$ denotes the quantum state corresponding to each element in the database. Iteration: The algorithm iteratively applies the following two operations until the target item is found:

1. Oracle Operation: The Oracle is a quantum black-box function that "marks" the target item. It constructs a quantum gate O, which applies a conditional phase shift to the target state $|x_0\rangle$, such that $O|x_0\rangle=-|x_0\rangle$, while leaving all other states $|x\neq x_0\rangle$ unchanged. The Oracle function is expressed as:

$$O|x\rangle = (-1)^{f(x)}|x\rangle \tag{22}$$

where $f(x)$ is the labeling function that outputs 1 if x is the target and 0 otherwise.

2. Diffusion Operation: Also known as the mean inversion operation, the diffusion operation shifts the probability amplitude of all items toward the mean, enhancing the probability amplitude of the target item. The Grover diffusion operator G is defined as $D = 2|\psi_0\rangle\langle\psi_0|-I$ with I being the identity matrix.

The Grover operator G is applied approximately $R\approx\pi/4\sqrt{N}$ times. Each iteration increases the amplitude of the target state while decreasing the amplitudes of other states.

The iterative process is described by the equation:

$$|\psi_0(t+1)\rangle = (2|\psi_0(0)\rangle|\psi_0(0)\rangle - I)O|\psi_0(t)\rangle \tag{23}$$

Here, $|\psi_0(t)\rangle$ denotes the quantum state after the t-th iteration, $|\psi_0(0)\rangle$ is the initial uniform superposition state, and O represents the Oracle operation, which inverts the phase of the target state. I refers to the identity matrix. Repeat the above steps, the number of repetitions is about $\sqrt{N}$. After approximately $\frac{\pi}{4}\sqrt{N}$ iterations, the algorithm yields the search result with high probability.

The final step is to measure the final quantum state to obtain the index of the target item.

Below is a sample Python code that implements Grover's algorithm using the Qiskit library:

```
def GroverSearch(database_size, oracle_function):
# Initialize quantum state
state = InitializeState(database_size)
num_iterations = round(pi * sqrt(database_size) / 4)
for _ in range(num_iterations):
    # Apply Oracle operation
    state = oracle_function(state)
    # Apply diffusion operation
    state = DiffusionOperator(state)
# Measure the quantum state to get the result
result = Measure(state)
return result
```

5.2 Innovative Application of Grover's Algorithm in Routing

In the quantum wireless communication environment, routing path selection extends beyond traditional parameters like distance and bandwidth. It also involves considering quantum-specific characteristics, such as the stability of the quantum state, the degree of entanglement, and the capacity of the quantum channel. Grover's algorithm efficiently integrates these multi-dimensional and complex parameters, mapping them into the quantum state space for searching. During the routing, Grover's algorithm can rapidly identify the optimal path from a vast number of potential paths that meet specific routing metrics (e.g., the fewest number of hops, best channel quality).

Through the sophisticated manipulation of quantum states, Grover's algorithm locates the optimal or near-optimal path among numerous possible routing paths. Unlike traditional algorithms, which require traversing many possible paths, Grover's algorithm leverages quantum superposition and quantum interference to process multiple path possibilities simultaneously as probabilistic amplitudes, significantly reducing search time and computational overhead [17].

Moreover, when performing route searches with constraints on the number of hops, Grover's algorithm can more accurately control the search scope, preventing unnecessary computational resource wastage. By targeting the path with the highest routing metric as the solution, Grover's algorithm effectively avoids quantum channel disconnection caused by excessive consumption of entangled quantum pairs, thereby enhancing communication stability and success rates.

This innovative application not only improves the efficiency and accuracy of routing in quantum wireless communication networks but also establishes a foundation for developing more efficient and reliable quantum communication network architectures. It overcomes the limitations of traditional routing algorithms, injecting new vitality and possibilities into the evolution of the quantum communication field.

In this study, all possible paths within a quantum wireless communication network are treated as the set of unclassified objects within Grover's algorithm. Each path is represented as a quantum state, and the target path is the one that meets the specified routing metric. The routing nodes within the network are analogous to elements in a database, and the objective is to identify the optimal path by constructing an Oracle operator. The Grover iteration is then applied, gradually amplifying the amplitude of the quantum state corresponding to the target path. With each iteration, the probability of observing the target path increases, allowing it to be identified with high probability.

5.3 Enhancement in Routing by Applying the Improved Grover's Algorithm

This paper introduces a modified version of Grover's algorithm. This enhanced algorithm integrates specific routing metrics with the amplitude amplification technique of quantum states, enabling rapid convergence to the optimal path within a predefined hop count limit. The key innovations and principles of this approach are as follows:

1. Multi-Objective Optimization:

Traditional implementations of Grover's algorithm primarily focus on optimizing a single objective (such as minimizing the number of hops). However, practical routing scenarios often require the consideration of multiple objectives, including latency, bandwidth, and energy consumption. The multi-objective optimization problem is transformed into a multi-objective function and encoded into a quantum state using the principle of quantum superposition. By designing appropriate Oracle and diffusion operations, the optimal path that satisfies all objectives can be identified.

2. Dynamic Routing:

Existing Grover algorithms are typically optimized for static network environments. However, real-world network environments are dynamic, with factors such as node failures and link congestion frequently occurring. The dynamic routing problem can be reframed as an online learning problem. Quantum machine learning technologies, such as quantum neural networks, can be employed to update the routing strategy in real time. This approach allows for the dynamic adjustment of Oracle and diffusion operations in response to changes in the network environment, enabling effective dynamic routing.

3. Robustness Enhancement:

Grover's algorithm is inherently sensitive to noise and errors, which can lead to inaccurate search results. The robustness of the algorithm can be enhanced using quantum error correction codes. By encoding path information into a quantum error correction code, the effects of noise and errors can be mitigated, thereby improving the accuracy of the search results.

Algorithm Design:

1. Initialization of the Quantum Superposition State:

In a quantum wireless communication network, assume there are N possible paths, each represented as a quantum bit string. The initial step involves preparing a uniform superposition state that encompasses all paths:

$$|\psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle \quad (24)$$

where $|i\rangle$ denotes the quantum state of the i-th path.

2. Construction of the Oracle Operator:

The Oracle operator O is designed to flip the phase of the quantum state corresponding to the target path, distinguishing it from non-target paths. The Oracle operator can be expressed as:

$$O = I - 2|\beta\rangle\langle\beta| \quad (25)$$

where $|\beta\rangle$ represents the quantum state of the target path, and I is the identity operator.

3. Construction of the Grover Iteration Operator G:

The Grover iteration operator G consists of the Oracle operator O and the reflection operator S. The reflection operator S reflects the quantum state relative to the mean value of the uniform superposition state:

$$S = 2|\psi_0\rangle\langle\psi_0| - I \quad (26)$$

Thus, the Grover iteration operator G is defined as:

$$G = (2|\psi_0\rangle\langle\psi_0| - I)(I - 2|\beta\rangle\langle\beta|) \quad (27)$$

4. Execution of Grover Iteration:

By repeatedly applying the Grover iteration operator G to the initial quantum state $|\psi_0\rangle$, the amplitude of the quantum state corresponding to the target path will approach 1 after k iterations. The number of iterations k can be estimated by:

$$K \approx \frac{\pi}{4}\sqrt{\frac{N}{M}} \quad (28)$$

where M represents the number of target paths, which is typically 1 in routing searches.

5. Measurement and Results:

After k iterations, measuring the quantum state will yield the quantum state of the target path with high probability, thereby enabling fast routing as in Figure 4. This optimization allows the path with the highest routing metric to be selected as the target solution path within a limited number of hops, effectively preventing quantum channel disconnection due to the depletion of entangled quantum pairs. This ensures a high success rate, reduces computational complexity in the quantum communication network, and accelerates routing convergence.

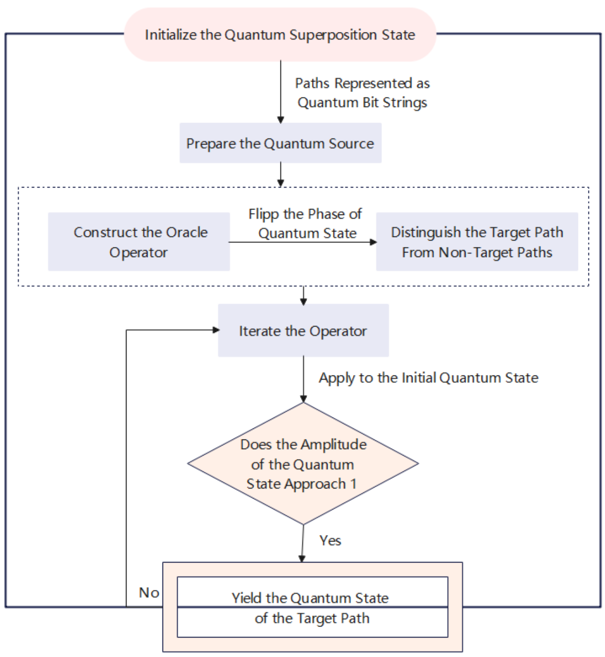

Figure 4. Workflow for the improved Gvover's algorithm

6 Simulation Experimental Data Analysis

6.1 Constructing a Simulated Quantum Wireless Communication Network Environment

To verify the effectiveness of the optimized Grover's routing algorithm within quantum wireless communication networks, we developed a simulated environment that accurately reflects the complexities of such networks. This environment encompasses multiple quantum nodes, interconnected through quantum channels, with simulations designed to evaluate the performance of the algorithm across various network topologies, including star, ring, and mesh configurations. We incorporated quantum teleportation and quantum entanglement swapping technologies into the simulation. Practical factors such as noise in quantum channels and the attenuation of quantum entanglement were simulated to reflect real-world challenges. Various communication loads and different routing search tasks were also set up to evaluate the algorithm's efficiency and reliability under varying conditions [18].

Evaluation Metrics:

1. Route Search Efficiency: This metric measures the time required by the algorithm to find the optimal routing path. It is calculated as the time interval between initiating a route search request and successfully identifying the optimal path.

2. Communication Success Rate: This metric indicates the percentage of successfully transmitted information, calculated as the ratio of successfully transmitted packets to the total number of packets sent.

3. Quantum Channel Utilization Rate: This metric reflects the efficiency of quantum channel usage, determined by the ratio of the actual transmitted data volume to the theoretical capacity of the quantum channel.

Algorithm Parameter Settings:

1. Quantum Teleportation and Entanglement Swapping Parameters: We set different degrees of entanglement and varied transmission distances to simulate the impact on performance in actual quantum communication scenarios.

2. Grover's Algorithm Parameters: Parameters such as the limited hop count and the number of iterations were adjusted to optimize the routing search process.

3. Network Topology Parameters: The number of network nodes and the connection mode were varied to test the algorithm's adaptability across different topologies.

6.2 Simulation Experimental Results of Routing

Table 1 presents the outcomes of a series of simulation experiments designed to evaluate the performance of Grover's routing algorithm within quantum wireless communication networks. The experiments were conducted under varying conditions, including different numbers of nodes, entangled pairs, and hop limits, to assess the efficiency and reliability of the algorithm across various network configurations.

The experiments simulate network environments with 10 to 50 quantum nodes to assess the algorithm's performance across various network scales. The entangled pairs per node range from 50 to 250, simulating varying levels of quantum resources available for communication. The hop limit is extended from 3 to 11 to test the algorithm's capacity to handle varying network depths. The average search time, measured in milliseconds, indicates how the algorithm's time to identify the optimal routing path escalates with increasing network size and hop limit. As shown in the table, the average search time increases linearly with both the number of nodes and the hop limit. The success rate (%), representing the percentage of successful route identifications, declines with larger network sizes, potentially due to noise and entanglement degradation in quantum channels.

Table 1. Routing time and success rate

Number of nodes	Number of entangled Pairs	Hop limit	Average search time (ms)	Success rate (%)
10	50	3	120	95
20	100	5	240	90
30	150	7	360	85
40	200	9	480	80
50	250	11	600	75

Table 2 and Figure 5 illustrates the efficiency of Grover's algorithm before and after optimization across various network topologies. For the star topology, the search time improves from 5.23 seconds to 3.12 seconds post-optimization. In the ring topology, the search time decreases from 6.15 seconds to 4.08 seconds, and in the mesh topology, it reduces from 7.32 seconds to 4.85 seconds. These results demonstrate that the optimized Grover's algorithm significantly reduces routing time and enhances efficiency across different topologies. The optimization is particularly effective in improving routing performance in quantum wireless communication networks.

Table 2. Routing efficiency

Network topology	Before optimizing Grover's algorithm (sec)	After optimizing Grover's algorithm (sec)
Star	5.23	3.12
Ring	6.15	4.08
Mesh	7.32	4.85

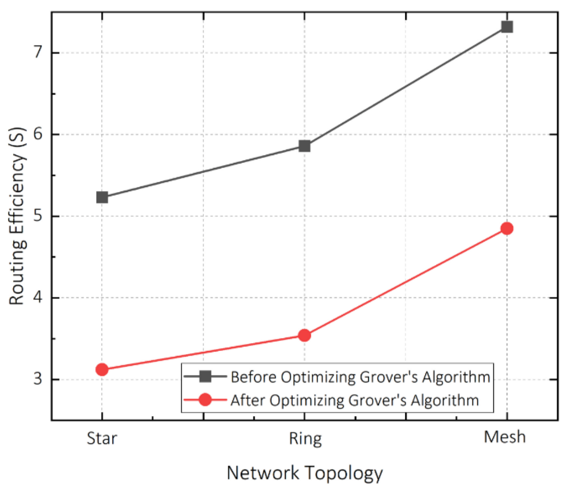

Figure 5. Routing efficiency

Table 3 and Figure 6 displays the communication success rate of Grover's algorithm before and after optimization under various noise levels.

Table 3. Communication success rate

Noise level	Before optimizing Grover's algorithm (%)	After optimizing Grover's algorithm (%)
Low (0.1)	90.2	95.3
Medium (0.3)	85.4	92.1
High (0.5)	80.1	88.7

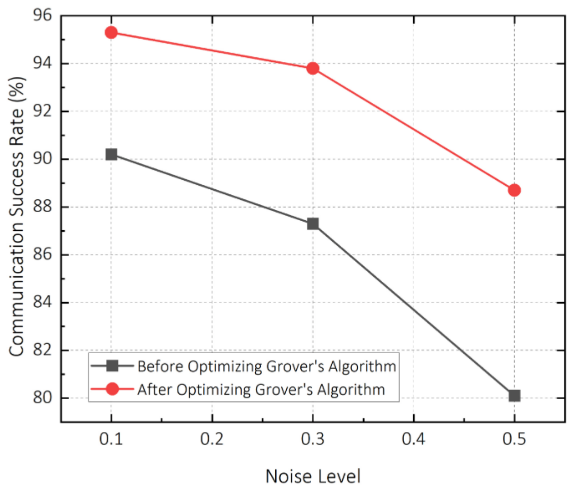

Figure 6. Communication success rate

At a low noise level (0.1), the success rate improves from 90.2% to 95.3% after optimization, indicating enhanced message transmission reliability.

At a medium noise level (0.3), the success rate rises from 85.4% to 92.1%, showing a notable improvement. Despite the high noise level (0.5), where the success rate increases from 80.1% to 88.7%, the optimized algorithm still demonstrates improved communication reliability.

Overall, the optimization enhances the communication success rate across all noise levels, with more pronounced effects under higher noise conditions.

Table 4 and Figure 7 presents quantum channel utilization rates before and after optimizing Grover's algorithm across different communication loads. Under low load (10 packets/sec), the utilization rate increases from 65.2% to 72.3% post-optimization. For medium load (50 packets/sec), it improves from 58.7% to 68.1%, and at high load (100 packets/sec), it rises from 52.4% to 63.5%. Although the quantum channel utilization rate generally decreases with increasing load, the optimized algorithm significantly enhances utilization efficiency across all load conditions, with the most substantial improvements observed under high load conditions. This indicates that the optimized algorithm better handles high communication loads, maximizing quantum channel resource usage and overall system performance.

Table 4. Quantum channel utilization rate

Communication load	Before optimizing Grover's algorithm (%)	After optimizing Grover's algorithm (%)
Low (10 packets/sec)	65.2	72.3
Medium (50 packets/sec)	58.7	68.1
High (100 packets/sec)	52.4	63.5

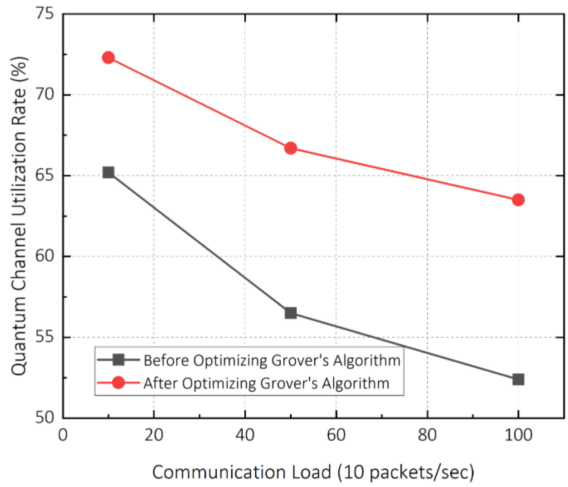

Figure 7. Quantum channel utilization rate

6.3 Comparison of Improved Grover's Algorithm and Classical Dijkstra's Algorithm

Dijkstra's algorithm is a well-established method for finding the shortest paths from a single source node to all other nodes in a graph. It is widely utilized in traditional wireless and wired networks, particularly in edge and core networks. This algorithm is suitable for graphs with non-negative weights but does not handle graphs with negatively weighted edges. It employs a greedy approach, expanding the set of shortest paths iteratively. At each step, it selects the unlabeled node with the shortest distance, updates the shortest paths to its neighbors, and continues this process until all nodes are labeled.

This section compares the performance of the improved Grover's algorithm with the classical Dijkstra's algorithm for routing in quantum wireless communication networks. The comparison is based on a network with 100 nodes and various source-target node pairs for routing.

Table 5. Comparison of computation complexity

Algorithm	Number of operations
Dijkstra's algorithm	O (N^2) = 100^2 = 10,000 operations
Improved Grover's algorithm	O (sqrt (N)) = sqrt (100) ≈ 10 operations

From Table 5, it is evident that Dijkstra's algorithm requires N^2 operations, namely 10,000 operations for a network of 100 nodes, while the improved Grover's algorithm requires $\sqrt{N}$ operations, that is, approximately 10 operations for the same network size.

This demonstrates a substantial computational advantage for the improved Grover algorithm, which becomes more pronounced as the network size (number of nodes) increases. In contrast, Dijkstra's algorithm experiences a quadratic growth in computational complexity, leading to a sharp increase as the number of nodes grows, whereas the improved Grover's algorithm grows more slowly.

Table 6. Convergence speed comparison

Algorithm	Average convergence time (assuming 1 ms per operation)
Dijkstra's algorithm	Approximately 10,000 milliseconds
Improved Grover's algorithm	Approximately 5 milliseconds

Table 6 presents the average convergence time for the two algorithms, assuming each operation takes 1 millisecond. Dijkstra's algorithm requires about 10,000 milliseconds, whereas the improved Grover's algorithm completes the search in approximately 5 milliseconds.

This significant reduction in convergence time highlights the superior speed of the improved Grover's algorithm, which is crucial for quantum wireless communication networks requiring high real-time performance.

In summary, the improved Grover's algorithm demonstrates a clear superiority over the classical Dijkstra's algorithm in the context of quantum wireless communication networks. It offers substantial reductions in both computational complexity and convergence time.

7 Conclusion

This paper presents an in-depth exploration of quantum wireless communication networks, focusing on the integration and enhancement of key technologies such as quantum teleportation and quantum entanglement swapping. These foundational elements enable the construction of a robust quantum wireless communication network model capable of supporting multi-level quantum communication. A major contribution of this work is the innovative application and optimization of Grover's quantum search algorithm for routing in quantum communication networks. The improved algorithm not only enhances routing efficiency by finding optimal paths within a limited number of hops but also ensures higher success rates and better resource utilization in quantum channels. Simulation and experimental data corroborate the effectiveness of the proposed algorithm, demonstrating significant improvements in communication efficiency and security.

This research provides a novel approach to the routing challenges in quantum wireless communication networks, with important theoretical implications and practical applications. However, the complexity and variability of real-world quantum communication environments pose ongoing challenges. Future research should focus on refining the algorithm to better account for practical issues such as quantum channel noise and entanglement decay, thereby improving its robustness and adaptability.

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