A Chaotic Discriminant Algorithm for Arrival Traffic Flow Time Series Based on Improved Alternative Data Method

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Abstract

Chaos discrimination is a prerequisite for the application of chaos theory modeling. Since the average orbital period of an air traffic flow system is long, it is difficult to obtain time series with a small time scale and many data points, so the Small-Data Method is often adopted to quantitatively calculate the chaotic characteristic quantity. However, when using the power spectrum method, it is found that the Small-Data Method is prone to false judgments when the data volume is small. To reduce spurious judgments, we apply a chaos discrimination algorithm based on an Improved Alternative Data Method combined with the Small-Data Method for air traffic flow and analyze it by example. The algorithm was experimentally demonstrated to correct the false judgment results of the Small-Data Method. In particular, when the data volume is only 150, the discrimination accuracy of the improved algorithm is as high as 80%, which is 26% higher than the discrimination accuracy of the Small-Data Method. Moreover, the improved algorithm has better discriminative performance than the Small-Data Method under the same data volume condition, which is suitable for the chaotic discriminative analysis of the arrival traffic flow time series.

Keywords: Small-Data Method, Improved Alternative Data Method, Chaos discrimination, Air traffic flow

1 Introduction

In the research on traffic flow, scholars have found that although the current air traffic flow density and flow rate are much smaller than the ground traffic flow, there are similarities between air traffic and ground traffic, both of which have uncertain characteristics. In the time dimension, chaotic characteristics also exist in the air traffic flow sequence [1]. Therefore, various algorithms based on chaos theory have been gradually introduced into the field of air traffic flow, which have significant application prospects in traffic flow characterization, traffic flow forecast, and traffic flow control. The application of chaos theory to these aspects involves chaos discrimination problems. In air traffic flow modeling and analysis, if the time series are blindly applied to chaos without chaos discrimination, it may be difficult to meet the requirements and the conclusions obtained may be unconvincing. Therefore, accurate chaos identification of air traffic flow is a prerequisite for using chaos theory to solve air traffic flow modeling, which is one of the key scientific issues that urgently requires a solution.

The measured chaotic time series not only have chaotic signals similar to random noise generated within the system, but also mix with noisy signals due to external factors, making it particularly difficult to distinguish chaotic time series from noisy ones. Therefore, it is necessary to select a suitable chaos identification method to accurately identify the chaos of the time series to ensure the feasibility of using chaos theory methods for time series analysis.

Numerical analysis methods for chaos discrimination can be divided into qualitative and quantitative discrimination methods. The qualitative discriminant method is to visually determine the existence of chaos by observing whether the characteristics of the signal sequence exhibited in the conversion to time domain or frequency domain curves are consistent with chaotic motion. It is mainly represented by the phase trajectory diagram method, stroboscopic method, Poincare cross-section method and power spectrum method. T. Nagatani achieved effective control of complex motions of trams by the phase diagram method based on probing the relationship between tram stopping time and speed to the dynamic response of retrieval schedules [2]. X. L. Meng et al. proposed an improved model to identify and analyze chaos for the chaotic phenomenon of railroad traffic flow with the Poincare method to draw section phase diagrams to further verify the accuracy of the results and provide information support for railroad scheduling [3]. V. Socha et al. proposed the use of power spectrum analysis to study controller heart rate variability and use curve fluctuation parameters to accurately assess the workload of air traffic controllers [4]. Such methods are simple and easy to understand by means of experiments and can provide support for visual verification of non-time-varying system results, but it is difficult to achieve fast and dynamic accurate discrimination in real-time discrimination scenarios.

The quantitative discrimination method is more objective in discriminating chaos by calculating the characteristic parameters that can quantitatively describe the characteristics of chaos. The characteristic parameters include Maximum

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Lyapunov Exponent [5], K-entropy [6] and the correlation dimension [7], etc. B. Krese used the 0-1 chaos test method for chaos discrimination in order to analyze the differences in chaotic dynamics between motorway and ring road traffic flows, and proved that both types of traffic flow are in chaotic state but differ significantly in the degree of chaos by means of the Lyapunov exponent [8]. H. X. Yu researched the effect of nonlinearity of flight accident time series with different time scales on the identification of chaotic characteristics by Lyapunov exponential spectrum to determine that the time series has chaotic characteristics. Yu also analyzed the influence of different time scales on the accuracy of the prediction model built, which provided a new way for accurate prediction analysis of flight accidents [9]. C. M. Pablo used BDS and Lyapunov to examine the non-linear dynamic properties of liquid bulk cargo in transport in order to accurately predict the volume of liquid cargo in Spanish seaports and to rationalize the allocation of cargo loads to port and shipping respectively, which provides theoretical support for maritime traffic modeling and forecasting [10].

Among them, Lyapunov exponents are applied most widely in chaos discrimination and are suitable for short-time online processing. The current main methods to obtain the maximum Lyapunov exponent are shown in Table 1.

Table 1. Main methods for maximum Lyapunov exponent

	References	Conclusion
Jacobian method	[11-12]	Suitable for noisy data with approximately linear small vectors in the cut space.
Wolf method	[13-14]	Suitable for noise-free high-dimensional sequences, with long computation time and the required samples need to be more than 8000.
P-norm method	[11, 15]	It has a strong filtering effect on noisy signals, but requires a large sample size.
Small-data method	[16]	Smaller computational effort, more adaptable to phase space reconstruction parameters, smaller sample size required.

The Alternative Data Method is a means to indirectly confirm the presence of chaos in time series by verifying the nonlinear dynamical components in the original data. It can go beyond the limitation of direct calculation of characteristic parameters, and will obtain better rigor when specific algorithms are combined with it. However, the original alternative data method suffers from the large magnitude of the imaginary part of the generated data, which cannot retain the characteristics of the original data well. So, further optimization in generating alternative data is necessary. Therefore, we apply a chaotic discrimination algorithm built on an Improved Alternative Data Method combined with Small-Data Method to overcome the problem that Small-Data Method is prone to false judgments in the air traffic flow time series discrimination on the basis of obtaining the phase space reconstruction parameters and using the maximum Lyapunov exponent as the base feature quantity. Finally, the chaos discrimination accuracy of the Small-Data Method and the improved algorithm is compared and analyzed by using the power spectrum method as the benchmark.

The contribution of this paper is briefed on three points:

1) It was found that using the Small-Data Method to discriminate the chaos of air traffic flow time series online is prone to the problem of spurious determination, and the enhancement of the data volume is still difficult to meet the discriminative accuracy requirements.

2) An Improved Alternative Data Method is used to improve the limitations of the Small-Data Method, and a chaos discrimination algorithm based on the Improved Alternative Data Method combined with the Small-Data Method is applied for air traffic flow.

3) We compare and analyze the chaotic discrimination accuracy of the Small-Data Method and the improved algorithm under different data volume levels, and demonstrate the applicability of the improved algorithm in air traffic flow.

2 Data Pre-processing

Compared with radar data, Automatic Dependent Surveillance –Broadcast (ADS-B) data has a faster update rate, higher navigation accuracy and more comprehensive downlink information, so this paper uses ADS-B data as the data source.

Data pre-processing is the first step of data analysis. Our research object is the terminal area arrival traffic flow, so the raw data needs to be cleaned and the terminal area arrival trajectory needs to be divided from the raw data. There are three main steps as follows:

1) Using the latitude and longitude coordinates of the terminal area boundary to make the terminal area projection plane area using the ray method and combined with the terminal area height range to screen the terminal area traffic flow.

2) Performing validity identification and processing of the trajectory data, removing duplicate data, and removing severely broken trajectory data by the following low-pass filter [17].

$$\hat{P}_1(x,y,h) = P_1(x,y,h). \quad \textbf{(1)}$$

$$\hat{P}_n(x,y,h) = \alpha P_n(x,y,h) + (1-\alpha) P_{n-1}(x,y,h). \quad \textbf{(2)}$$

where $\hat{P}_i(x,y,h)$ is the i^{th} coordinate point of a track, x, y are the distances of point P from the center of the airport in the Cartesian coordinate system respectively, h is the height of point P from the center of the airport in the Cartesian coordinate system, α is the filtering coefficient.

3) The height characteristics of different types of trajectories are used to distinguish the arrival trajectories, flyover trajectories and departure trajectories and to extract the arrival trajectory data.

The data pre-processing process is illustrated in Figure 1.

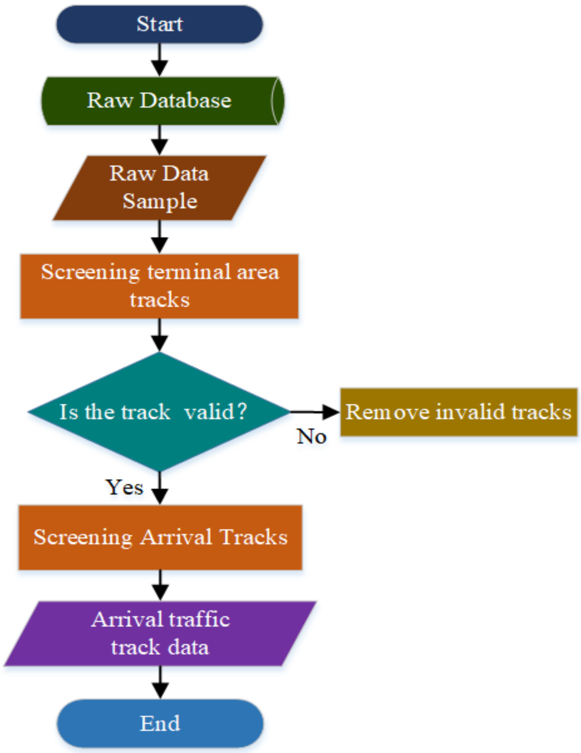


Figure 1. Data pre-processing process

After obtaining the terminal area arrival traffic flow, the flight flow is counted according to the terminal area flight procedure map with the time of flight descent to the final arrival altitude. Referring to statistical scales for the pre-tactical management phase of air traffic flow systems in Europe and the USA, this paper selects 30 min as the statistical time scale to extract arrival traffic flow time series $\{x_1, x_2, x_3, \ldots, x_N\}$ in the terminal area, which provides the basic input data for the identification and analysis of chaotic characteristics.

3 Chaos Discrimination Based on Small-Data Method

3.1 Phase Space Reconstruction

Phase space reconstruction is an important part of the flow time series chaos identification, and the high-dimensional phase space that can highly reproduce the properties of the proto-dynamic system is constructed by applying the delay coordinates to the one-dimensional time series $\{x_1, x_2, x_3, \ldots, x_N\}$ output from the system. According to the embedding theory proposed by Takens [18], the appropriate embedding dimension m and delay time τ can be selected to construct a system with differential homogeneity with the original dynamical system, recover the chaotic attractor of the system, obtain the information of the dynamical system hidden in the individual components, and then excavate the evolution law of the original dynamical system.

The time series $\{x_1, x_2, x_3, \ldots, x_N\}$ is spatially reconstructed according to m with τ to obtain $N-(m-1)\tau$ m-dimensional phase space vectors.

$$X_j = [x_j, x_{j+\tau}, \cdots, x_{j+(m-1)\tau}]^T, \tag{3}$$

where $j = 1, 2, \ldots, M'$, $M = N-(m-1)\tau$.

Therefore, on the premise of ensuring accurate calculation results of the phase space reconstruction parameters and reducing the tedious calculations caused by blind human selection, the improved C-C method [19] is used in this paper to determine the phase space reconstruction parameters.

3.2 Small-Data Method to Find the Maximum Lyapunov Exponent

Lyapunov exponent is used to quantitatively describe the attractor orbit dispersion rate, which is generally denoted by $\lambda_{\max}$ in the formula. Due to the chaotic attractor, the chaotic system as a whole tends to be stable, but due to the initial value sensitivity of the system, the local instability of the system leads to a Lyapunov exponent greater than 0. According to the proof presented by C. Grebogi [20], as long as the maximum Lyapunov exponent of the phase space system after time series reconstruction is larger than 0, the system can be judged as chaotic.

Small-Data Method is based on the previously obtained m and τ to find the average slope of the phase space orbital period over time, which has the advantages of small calculation volume and the calculation results are more reliable for small data sets of air traffic flow. Therefore, this paper uses Small-Data Method to calculate the maximum Lyapunov exponent, and the calculation steps are as follows:

1) The extracted time series $X_n = \{x_{t_1}, x_{t_2}, \ldots, x_{t_N}\}$ is subjected to spectral analysis, and the average period P of the series is calculated according to the FFT transform.

2) The improved C-C method is used to simultaneously calculate the phase space reconstruction of m and τ.

3) After phase space reconstruction, the neighboring points $X_{j'}$ of each phase point X_j are found according to the Euclidean distance closest principle, and the transient separation is restricted by the time series averaging period P.

$$d_j(0) = \min\left\|X_j - X_{j'}\right\|, \left|j - j'\right| > P, \tag{4}$$

where $d_j(0)$ is the closest distance to the j^{th} point.

4) Calculate the distance $d_j(i)$ between each phase point and its corresponding neighbor $X_{j'}$ after i discrete steps:

$$d_j(i) = \left\|X_{j+i} - X_{j'+i}\right\| = C_j e^{\lambda_1(i\Delta t)}, \quad i = 1, 2\cdots, \min(M-j, M-j') \tag{5}$$

where Δt is the time series sampling step.

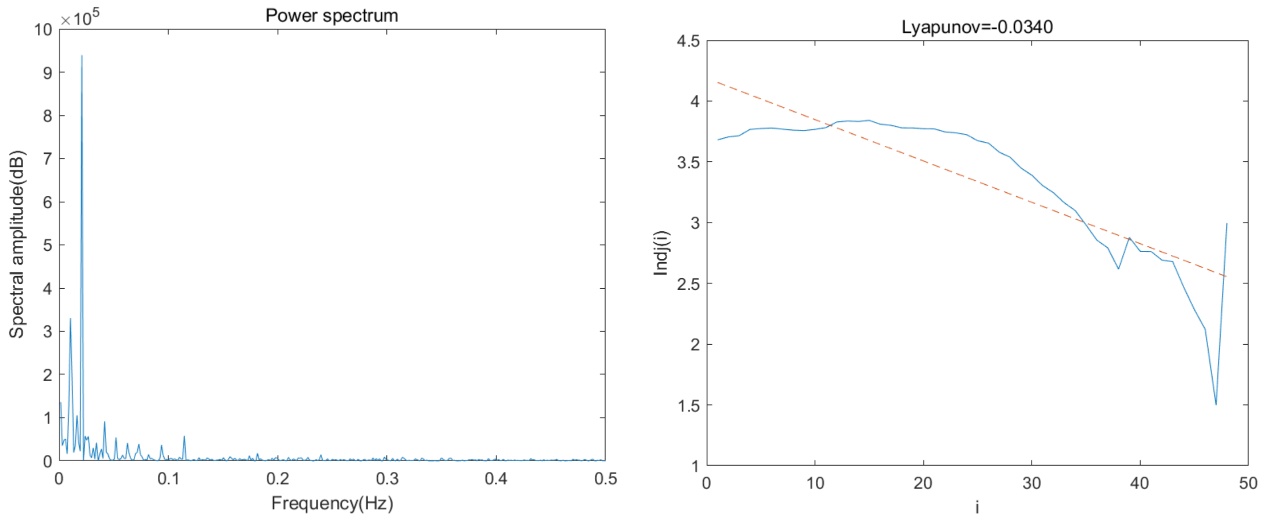
5) Taking logarithms on both sides of Equation (5), for each i, calculate the average of all $\ln(d_j(i))$'s $f(i)$.

$$f(i) = \frac{1}{q\Delta t}\sum_{j=1}^{q}\ln(d_j(i)), \tag{6}$$

where q denotes the number of non-zero $d_j(i)$'s and $f(i)$ is the mean of the cumulative sum of distances $d_j(i)$ to q.

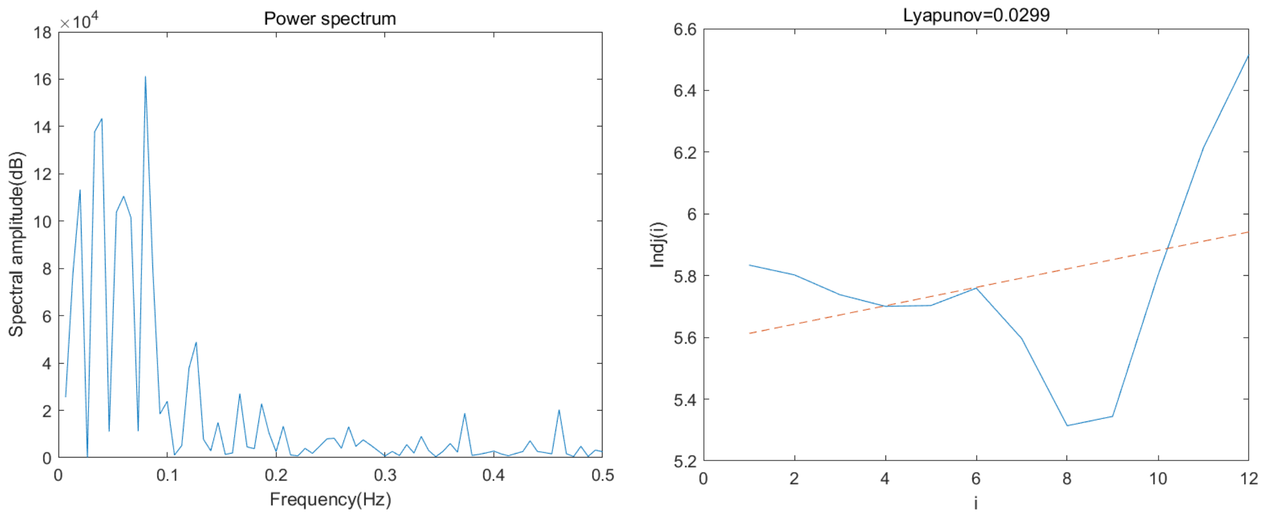
6) Plot $i - f(i)$ is made and the linear part of the curve is fitted linearly using least squares. The slope of the fitted line is λ_{max}.

From the Small-Data Method processing steps, it can be found that when using the least squares method for linear fitting, the linear part of the region needs to be artificially selected through experience, without uniform guideline limits, and also in the context of short time online discrimination, the time series is short and contains noise, which all tend to cause false judgments. As shown in Figure 2 and Figure 3 below, there are two cases of false judgment. As can be seen from Figure 2, the power spectrum curve exhibits features that are not part of a random sequence. There are broad peaks and small spikes of noise, and the amplitude curve is not uniformly distributed in the frequency domain. It can also be clearly distinguished that the power spectrum belongs to a chaotic sequence, but the maximum Lyapunov exponent calculated by the Small-Data Method is negative. The system is judged to be stable and there is no chaotic characteristic. However, it is found from the Lyapunov exponent diagram that the orbital period decreases first with the increase of sampling points i and then increases suddenly and rapidly, and the system state jumps from the stable state under the action of the attractor to the unstable chaotic state with the characteristics of chaotic motion. As shown in Figure 3, the amplitude of the power spectrum of the analyzed data is independent from the power and spreads over the whole frequency domain, which shows the characteristics of a random sequence, but the Lyapunov index result is positive. These indicate that the Lyapunov exponent calculated by the Small-Data Method is a spurious judgment.



(a) Power spectrum at Lyapunov exponent false judgement (b) Lyapunov index in case of false judgements

Figure 2. Small-Data Method false judgment example 1



(a) Power spectrum at Lyapunov exponent false judgment (b) Lyapunov index in case of false judgements

Figure 3. Small-Data Method false judgment example 2

4 Improved Chaos Discrimination Algorithm

The improved chaos discrimination algorithm [21] is based on the combination of the Improved Alternative Data Method and Small-Data Method.

4.1 Improved Alternative Data Method

The algorithmic idea of the Alternative Data Method is to introduce controlled data noise, generate alternative data that retains the linear correlation components of the original data, and then make a null hypothesis on the original data. The decision to accept the null hypothesis is made by calculating the magnitude of the difference in the number of features between the original and alternative data. It is widely used as a hypothesis testing method for testing the non-linear properties of time series. In order to retain the advantages of Small-Data Method and combine the rigor of the Alternative Data Method, this paper applies Improved Alternative Data Method combined with Small-Data Method to discriminate chaos and improve the accuracy of chaos identification.

The steps of Improved Alternative Data Method are as follows:

1) Time series discrete Fourier transform

A discrete Fourier transform is applied to a one-dimensional traffic flow time series $X_n = \{x_{t_1}, x_{t_2}, \ldots, x_{t_N}\}$ of length N.

$$X^{'}(j) = Y\{x_n\} = \sum_{n=1}^{N} x_n e^{[-2\pi i(n-1)(j-1)/N]}, j = 1, 2, \ldots N, \tag{7}$$

where $x_n = x_{t_n}$.

2) Phase randomization

Randomizing the phase of $X^{'}(j)$, the Fourier transform form can be rewritten as: $X^{'}(j)=A(j)e^{\Phi(j)}$, where $\Phi(j)$ is the phase, $A(j)$ is the amplitude. Applying a random phase angle $\theta(j)$ to the phase with a range belonging to $[-\pi, \pi]$ leads to $\tilde{X}^{'}(j)$.

$$\tilde{X}^{'}(j) = X^{'}(j) e^{-i[\Phi(j)+\theta(j)]}. \tag{8}$$

To ensure that the phase of the generated data is skew-symmetrically distributed, the phase angle is restricted according to the index parity of the sequence data. When the index of the sequence data is even,

$$\begin{cases} \theta(1) = \theta(N/2+1) = 0 \\ \theta(k) = -\theta(N-k+2) \end{cases}, k = 2, 3, \ldots, N/2+1;$$

when the index of the sequence data is odd,

$$\begin{cases} \theta(1) = 0 \\ \theta(k) = -\theta(N-k+2) \end{cases}, k = 2, 3, \ldots, (N+1)/2.$$

The algorithm is improved by transforming the algorithm so that the generated data are all real numbers by the difference in the parity of the data index.

3) Find the inverse Fourier transform of the sequence $\tilde{X}^{'}(j)$ to obtain the alternative data $\hat{X}(n)$.

$$\hat{X}(n) = Y^{-1}\{\tilde{X}^{'}(j)\} = \frac{1}{N}\sum_{n=1}^{N} \tilde{X}^{'}(j) e^{[2\pi i(n-1)(j-1)/N]}, \quad j = 1, 2, \ldots N \tag{9}$$

4.2 Improved Chaos Discriminant Algorithm Process

By combining the Improved Alternative Data Method with Small-Data Method, the level of difference of the maximum Lyapunov exponent between the alternative data and the measured data is calculated and expressed as a combined eigenvalue significant level S, which in turn discriminates against the chaotic nature of the time series. The algorithm process is as follows:

1) Formulate the null hypothesis H_0: the original time series is generated by a linear stochastic process.

2) Following Improved Alternative Data Method, multiple sets of alternative data are generated that retain the spectral characteristics of the original time series and satisfy the proposed null hypothesis.

3) Using Small-Data Method to find index $\lambda_{\max}$ of the original time series and the maximum Lyapunov index $\lambda_{\max}$ of the alternative data, respectively.

4) Calculate the maximum Lyapunov index standard deviation σ_λ and the mean $\bar{\lambda}^{t}_{\max}$ for multiple sets of alternative data corresponding to the original data and further calculate the significance level S.

$$S = \frac{\left|\lambda^{y}_{\max} - \bar{\lambda}^{t}_{\max}\right|}{\sigma_\lambda}. \tag{10}$$

The significance level $\alpha = 0.95$ is taken and $S = 1.96$ is used as the decision boundary. If the significance level $S > 1.96$, then there is a 95% confidence level that the original data is chaotic, and if the significance level $S < 1.96$, then there is a 95% confidence level that the original data is linearly random.

5 Experiments

For verifying the degree of improvement of the improved chaos discrimination algorithm on the false discrimination problem and the robustness of the algorithm, the Small-Data Method and the improved chaos discrimination algorithm were used to perform chaos discrimination on data with different sample sizes, and the results of the power spectrum method were used as a benchmark for verification.

In this paper, a traffic time series with a time scale of 30 minutes and a length of 1,487 is extracted based on the arrival traffic flow at O'Hare International Airport from August 1 to August 31, 2019, as shown in Figure 4.

The power spectrum of the series is shown in Figure 5, and the power spectrum is a continuous broad spectrum with small spikes of noise, and the maximum Lyapunov exponent of the series is 0.03. All the results obtained suggested that the series was a chaotic time series.

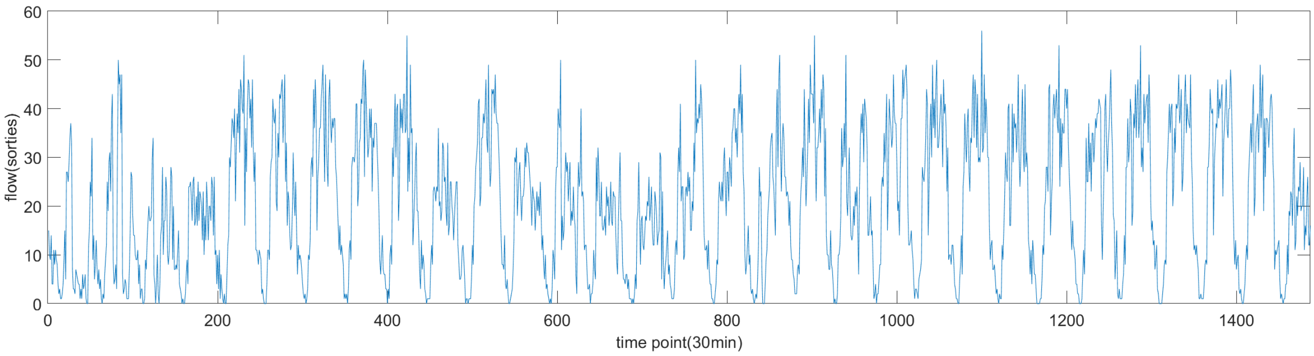


Figure 4. Arrival traffic flow time series in terminal area

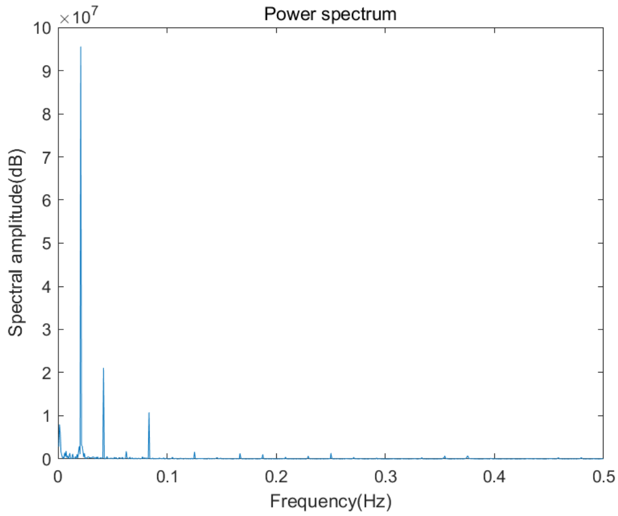


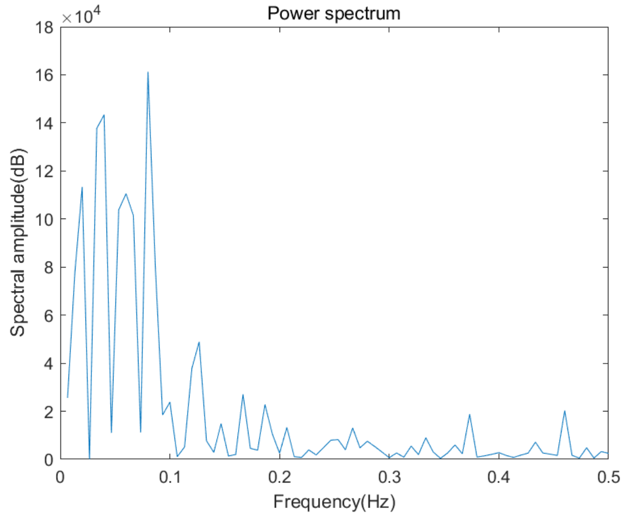
Figure 5. Power spectrum curve of arrival traffic flow time series

To assess the impact of different sample sizes on the accuracy of the Small-Data Method and the improved discrimination algorithm, sub-series with sample sizes of 150, 300 and 672 were selected from the extracted long time series for comparison experiments based on the average period of the series. To get the most out of the data information and to increase the number of experimental groups, the sample data were obtained by sliding the window from front to back and then from back to front, as the number of data could not be divided. Considering the reasonableness of the experimental iteration accuracy and algorithm operation efficiency, a set of measured traffic flow data and five sets of alternative data are used as one experiment for experimental analysis.

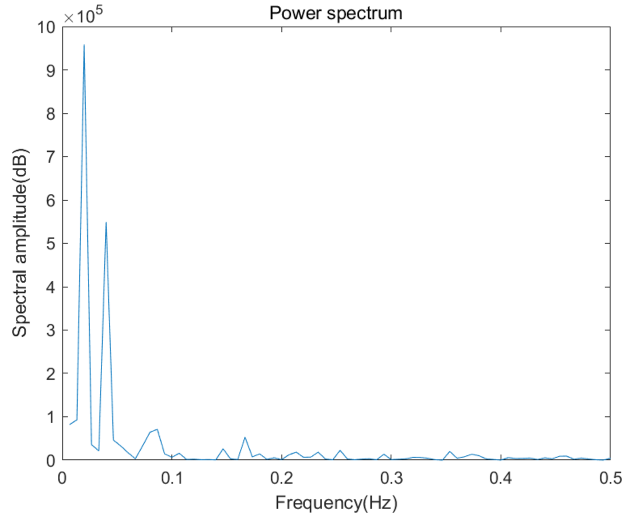
To simplify the space, only the power spectrum curves of the sub-series with sequence sample points indexed 1-150, 901-1050, 301-600, 588-887, 673-1344, 816-1487 are given here, as detailed in Figure 6. Then, the chaotic features of each sub-series were identified by applying the Small-Data Method and the improved chaos discrimination algorithm, respectively. The calculated parameter values and discriminant results for the above six sub-series are shown in Table 2.

Based on the power spectrum curves of the first five sample groups and the experimental results of the improved discrimination algorithm in Table 2, it can be seen that the improved chaos discrimination algorithm can correct the false discrimination of the Small-Data Method and make the chaotic discrimination results more accurate when the false discrimination occurs in Small-Data Method.

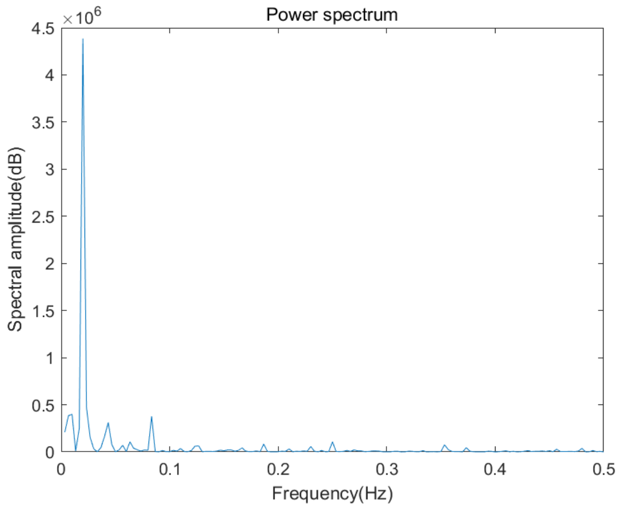
For further verification of the accuracy of the improved algorithm under different data volumes, 50 sets of experiments were conducted for each sample data volume, and the accuracy of the Small-Data Method and the improved chaos discrimination algorithm under each sample data volume are shown in Table 3. From the table it is clear that the accuracy of both discrimination algorithms was significantly improved as the data volume increased and more comprehensive data features were extracted. However, the accuracy of the Small-Data Method is only 54% when the data volume is 150, which cannot meet the requirement of online time series discrimination accuracy. The improved chaos discrimination algorithm has an accuracy of 80% and a short running time under the same data volume conditions. With the increase of data volume, the accuracy of the algorithm also has a significant improvement, which can meet the requirements of fast and accurate online dynamic discrimination of small sample size time series.



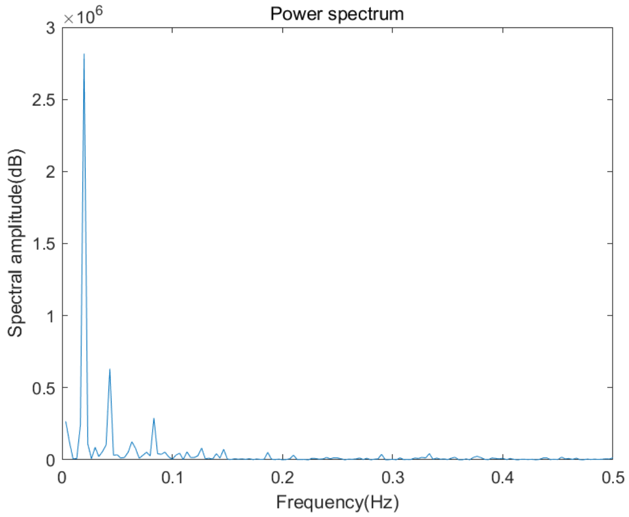
(a) 1-150 series power spectrum



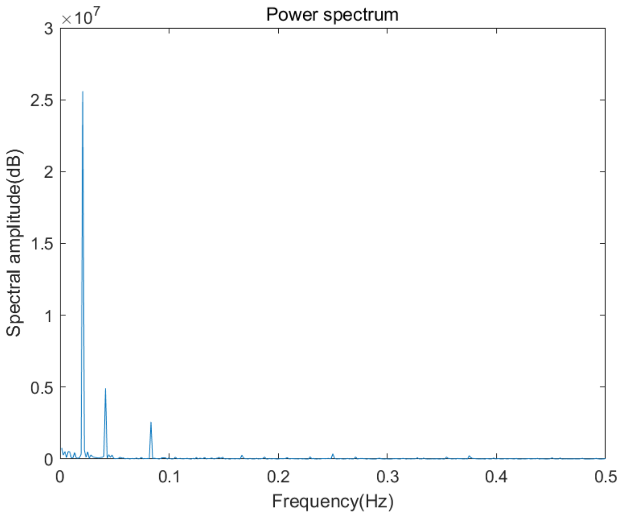
(b) 901-1050 series power spectrum



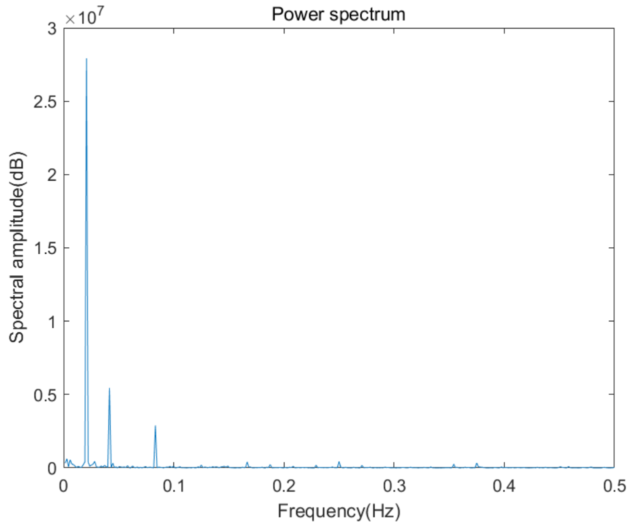
(c) 301-600 series power spectrum



(d) 588-887 series power spectrum



(e) 673-1344 series power spectrum



(f) 816-1487 series power spectrum

Figure 6. Power spectrum of sub-series with different data volume

Table 2. Examples of chaotic decision results of sub time series with different data volumes

Sample Point Index	$\lambda^{y}_{\max}$	$\bar{\lambda}^{t}_{\max}$	Alternative data deviation σ_2	S
1-150	0.0299	0.0376	0.0113	0.6816
901-1050	-0.0297	-0.0038	0.0047	5.5439
301-600	-0.0107	0.0072	0.0049	3.6750
588-887	-0.0154	-0.0034	0.0031	3.9088
673-1344	-0.0005	0.0116	0.0035	3.4209
816-1487	0.0208	0.0070	0.0012	11.6509

Table 3. Algorithm accuracy under different data volumes %

Algorithms	Accuracy with 150 data volume	Accuracy with 300 data volume	Accuracy with 672 data volume
Small-Data Method	54%	74%	78%
Improved Chaos Discriminant Algorithm	80%	88%	92%

6 Conclusion

In purpose of accurately identifying the chaotic characteristics of the arrival traffic flow time series in the terminal area, this paper firstly pre-processes the original data, and on the basis of phase space reconstruction, finds that using the Small-Data Method to calculate the maximum Lyapunov exponent is prone to the problem of false judgments.

To compensate for the shortcomings of the Small-Data Method, the Improved Alternative Data Method is employed to improve the tightness and noise immunity of the discrimination algorithm and enhance the accuracy of the discrimination. Through experimental analysis, even when the data volume is small, the improved algorithm still has high discrimination accuracy, and the algorithm discrimination accuracy is less sensitive to the data volume than the Small-Data Method.

In summary, the improved algorithm is suitable for the identification of chaotic arrival traffic flows with short traffic time series and is difficult to obtain, providing a theoretical basis and reference for air traffic chaos identification.

However, this paper mainly addresses the problem of false judgments of Small-data Method in air traffic flow, with traffic flow operation scenarios as arrivals, but no further applicability verification has been carried out for various operation scenarios in the airspace. The future chaos discrimination method must be developed towards machine learning, with strong self-adaptability [22] through artificial intelligence algorithms for learning and training, and dynamic adjustment of algorithm model parameters according to different operation scenarios or higher dimensional space [23].

Acknowledgements

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References

[1] L. Yang, S. W. Yin, M. H. Hu, K. Han, H. H. Zhang, Empirical exploration of air traffic and human dynamics in terminal airspaces, *Transportation Research Part C: Emerging Technologies*, Vol. 84, pp. 219-244, November, 2017.

[2] T. Nagatani, Dynamical model for retrieval of tram schedule, *Physica A: Statistical Mechanics and its Applications*, Vol. 377, No. 2, pp. 661-671, April, 2007.

[3] X. L. Meng, W. L. Xiang, L. M. Jia, J. Xu, Train flow chaos analysis based on an improved cellular automata model, *Chaos, Solitons & Fractals*, Vol. 81. pp. 43-51, December, 2015.

[4] V. Socha, L. Hanakova, V. Valenta, L. Socha, R. Abela, S. Kusmirek, T. Pilmannova, J. Tecl, Workload assessment of air traffic controllers, *Transportation Research Procedia*, Vol. 51, pp. 243-251, 2020.

[5] Y. Liu, J. Z. Zhang, Predicting Traffic Flow in Local Area Networks by the Largest Lyapunov Exponent, *Entropy*, Vol. 18, No. 1, Article No. 32, January, 2016.

[6] L. D. Fang, Q. Q. Zeng, Y. Faraj, N. Zhao, Z. H. Wei, X. T. Li, Analysis of chaos characteristics of gas-liquid two-phase flow noise, *Flow Measurement and Instrumentation*, Vol. 65, pp. 98-109, March, 2019.

[7] R. A. Hoshi, C. M. Pastre, L. C. M. Vanderlei, M. F. Godoy, Poincare plot indexes of heart rate variability: Relationships with other nonlinear variables, *Autonomic Neuroscience- Basic & Clinical*, Vol. 177, No. 2, pp. 271-274, October, 2013.

[8] B. Krese, E. Covekar, Analysis of traffic dynamics on a ring road-based transportation network by means of 0–1 test for chaos and Lyapunov spectrum, *Transportation Research Part C*, Vol. 36, pp. 27-34, November, 2013.

[9] H. X. Yu, X. Li, On the chaos analysis and prediction of aircraft accidents based on multi-timescales, *Physica A: Statistical Mechanics and its Applications*, Vol. 534, Article No. 120828, November, 2019.

[10] P. Coto-Millan, L. Inglada-Pérez, Testing for nonlinearity and chaos in liquid bulk shipping,

Transportation Research Procedia, Vol. 48, pp. 1605-1614, 2020.

[11] G. Barna, I. Tsuda, A new method for computing Lyapunov exponents, *Physics Letters A*, Vol. 175, No. 6, pp. 421-427, April, 1993.

[12] D. Martini, D. Angeli, G. Innocenti, A. Tesi, Ruling Out Positive Lyapunov Exponents by Using the Jacobian's Second Additive Compound Matrix, *IEEE Control Systems Letters*, Vol. 6, pp. 2924-2928, June, 2022.

[13] A. Wolf, J. B. Swift, H. L. Swinney, J. A. Vastano, Determining Lyapunov exponents from a time series, *Physica D: Nonlinear Phenomena*, Vol. 16, No. 3, pp. 285-317, July, 1985.

[14] P. Alberti-Villalobos, S. Molina-Adasme, P. Maldonado-Cárdenas, High performance implementation of Wolf's method, *AIP Conference Proceedings*, Vol. 1978, No. 1, Article No. 470047, July, 2018.

[15] O. Gorshkov, H. Ombao, Multi-Chaotic Analysis of Inter-Beat (R-R) Intervals in Cardiac Signals for Discrimination between Normal and Pathological Classes, *Entropy*, Vol. 23, No. 1, Article No. 112, January, 2021.

[16] S. Zhou, Y. Feng, W. Y. Wu, W. H. Wang, A novel method based on the fuzzy C-means clustering to calculate the maximal Lyapunov exponent from small data, *Acta Physica Sinica*, Vol. 65, No. 2, Article No. 020502, January, 2016.

[17] C. Frazier, K. M. Kockelman, Chaos theory and transportation systems - Instructive example, *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1897, No. 1, pp. 9-17, January, 2004.

[18] F. Takens, Detecting strange attractors in turbulence, In: D. Rand, L. S. Young (Eds.), *Dynamical Systems and Turbulence, Warwick 1980. Lecture Notes in Mathematics*, Vol. 898, Berlin Springer Verlag, 1981, pp. 366-381.

[19] Z. Y. Zhang, A. Zhang, C. Sun, S. D. Xiang, S. M. Li, Data-Driven Analysis of the Chaotic Characteristics of Air Traffic Flow, *Journal of Advanced Transportation*, Vol. 2020, Article No. 8830731, September, 2020.

[20] J. Awrejcewicz, L. Dzyubak, C. Grebogi, A direct numerical method for quantifying regular and chaotic orbits, *Chaos, Solitons & Fractals*, Vol. 19, No. 3, pp. 503-507, February, 2004.

[21] Y. Lu, *The Study on the Real-time Identification of Chaos in Traffic Flow*, Ph. D. Thesis, Tianjin University, Tianjin, China, 2007.

[22] S. Liu, S. C. Huang, X. Y. Xu, J. Lloret, K. Muhammad, Efficient Visual Tracking Based on Fuzzy Inference for Intelligent Transportation Systems, *IEEE Transactions on Intelligent Transportation Systems*, pp. 1-12, January, 2023.

[23] S. Liu, S. C. Huang, S. Wang, K. Muhammad, P. Bellavista, J. D. Ser, Visual Tracking in Complex Scenes: A Location Fusion Mechanism Based on the Combination of Multiple Visual Cognition Flows, *Information Fusion*, pp. 1-26, February, 2023.

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