A Novel Brownian Motion-based Hybrid Whale Optimization Algorithm

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Abstract

The Whale Optimization Algorithm (WOA) has the characteristics of simple implementation and few adjustment parameters, which is remarkable in the optimization algorithm. However, there are shortcomings such as premature convergence, slow convergence in the later period, and low search accuracy. For these shortcomings, a novel Brownian motion-based hybrid whale optimization algorithm (HWOA) is proposed. The search strategy in the Harris hawk optimization algorithm (HHO) is adopted to improve the global search ability of the algorithm, and a soft besiege with progressive rapid dives is introduced to solve the problems of premature convergence and slow convergence. Besides, the Brownian motion model is used to replace WOA. The random parameters in the distance formula are calculated to better simulate the prey's escape during the predation process, and help to jump out of the local optimum. The simulation of 23 benchmark functions shows that compared with the classic and HWOA and metaheuristic, the convergence accuracy and speed have been improved, and the local optimum can be effectively jumped out. At the same time, 10 CEC06-2019 test functions are used to test and analyze it. Compared with WOA, HWOA has better search results, which verifies the superiority of the improved algorithm.

Keywords: Whale optimization algorithm, Harris hawk optimization algorithm, Brownian motion, Metaheuristic, CEC06-2019

1 Introduction

The meta-heuristic algorithm [1] has a great performance in speed in the face of partial optimization problems, and the optimization effect is better than traditional algorithms such as random search [2] and simulated annealing [3], metaheuristics are gradually being used to solve optimization problems. Meta-heuristics are mainly divided into three genres [4]: physics-based, swarm-based and evolutionary-based. The physics-based meta-heuristic algorithms include GSA [5], SDA [6]. The swarm-based meta-heuristic algorithms include PSO [7], ABC [8]. Evolutionary-based meta-heuristic algorithms originate from biological evolution [3] include GA [9] and DE [10] et al.

With the rise of heuristic algorithm, whale optimization algorithm is proposed by S. Mirjalili et al. [11], a scholar at Griffith University in Australia.

WOA got the attention of many scholars once it is announced. For the enhancement of global search capabilities, Xinming Zhang et al. [12] proposed a hybrid WOA with gathering strategies (HWOAG). Applying the random opposition learning strategy to position update, which improves the diversity of the algorithm. Simulations show that the HWOAG algorithm exhibits higher search efficiency. Jiang Li et al. [13] used the cross selection strategy and the chaotic mapping strategy to update the current optimal position of the WOA, and proposed a new chaotic whale optimization algorithm (CWA), which improved the convergence speed, search ability and stability of the algorithm. Mengxing Huang et al. [14] introduced the gravity weight parameters containing dis1 and dis2, the position update equation of the shrinking encircling phase of the WOA to obtain reasonable convergence performance. Sanjoy Chakraborty et al. [15] introduced a selection parameter for balancing the search stage and proposed an enhanced whale optimization algorithm (eWOA). Tests show that eWOA has more advantages in the face of high-dimensional optimization problems and improves the convergence speed.

Although WOA has a simple implementation and adjustment of small parameters, it has shortcomings such as premature convergence, slow late convergence, and low search accuracy [16]. The Harris hawk optimization algorithm is an emerging algorithm [17], with faster search speed and higher search accuracy. Brownian motion is a phenomenon in which suspended particles move randomly caused by collision with other molecules [18]. Using the motion characteristics of Brownian motion can effectively jump out of the local optimum to achieve a better optimization effect. Therefore, this paper proposes a hybrid Harris hawk algorithm and Brownian motion whale optimization algorithm, which aims to improve the search accuracy and convergence speed of WOA, and improve its ability to jump out of the local optimum. The experiments shows that the algorithm have been significantly improved and outperforms other emerging swarm intelligence optimization algorithms.

2 Basic Principles of WOA

There are three stages are proposed by whale optimization algorithm: shrinking encircling, bubble net attacking, and search for prey.

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2.1 Shrinking Encircling

When whales are looking for prey, they are first surrounding the prey. Through continuous communication and keeps getting closer to the prey. Define $\vec{X}^*(t)$ as the optimal position vector of the current whale, other whales approach the optimal position [11].

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D}. \tag{1}$$

$$\vec{D} = \left|\vec{C} \cdot \vec{X}^*(t) - \vec{X}(t)\right|. \tag{2}$$

In the formula, $\vec{X}(t)$ is the current position vector of the whale, t is the number of iterations, $\vec{D}$ is the distance between the current optimal whale and the individual whale, $\cdot$ is the element-by-element multiplication, $\vec{A}$ and $\vec{C}$ are the coefficient vectors [11].

$$\vec{A} = 2\vec{a}(t) \times \vec{r}_1 - \vec{a}(t). \tag{3}$$

$$\vec{C} = 2\vec{r}_2. \tag{4}$$

In the formula, $\vec{r}_1$, $\vec{r}_2$ are random vectors between [0, 1], $\vec{a}(t)$ decreases linearly from 2 to 0, $t_{\max}$ indicating the maximum number of iterations.

2.2 Bubble Net attacking

The bubble net attack stage includes two parts, one is to use formula (2) to shrink encircling, and the other is to use formula (5) for spiral updating position. The update formula is as follows [11]:

$$\vec{X}(t+1) = \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t). \tag{5}$$

$$\vec{D}' = \left|\vec{X}^*(t) - \vec{X}(t)\right|. \tag{6}$$

Where b is the constant that determines the shape of the helical motion, which is 1 according to experience. $l \in [-1,1]$, $\vec{D}'$ representing the absolute value of the distance between the search individual and the current optimal whale.

The probability $P\left(P \in [0,1]\right)$ is introduced to determine the update method performed at this stage, when deciding to execute the two parts of WOA, and the expression is as follows [11]:

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & P < 0.5 \\ \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) & P \geq 0.5 \end{cases}. \tag{7}$$

2.3 Search for Prey

In the optimization process of WOA, when the parameter $P < 0.5$ enters the shrinking encircling stage, the value of $\left|\vec{A}\right|$ determines whether the whale performs the search for prey phase or the prey encirclement phase. When $\left|\vec{A}\right| < 1$, the whale uses Equation (1) to execute the shrinking and encircling strategy. When $\left|\vec{A}\right| \geq 1$, the whale searches randomly within a certain range [11].

$$\vec{X}(t+1) = \vec{X}_{rand}(t) - \vec{A} \cdot \vec{D}. \tag{8}$$

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_{rand}(t) - \vec{X}(t)\right|. \tag{9}$$

Where $\vec{X}_{rand}(t)$ is the randomly selected whale position vector in the current group.

3 Brownian Motion-based Hybrid Whale Optimization Algorithm

The WOA is effective in low-dimensional single-modal optimization problems, but it is unsuitable for high-dimensional and multi-modified issues [19]. At the same time, the random selection method is adopted, which leads to large randomness in the early search period [19]. To improve the search accuracy, convergence speed and ability to jump out of the local optimum of WOA, the search strategy in HHO and the soft besiege with progressive rapid dives are integrated into WOA. When dealing with problems with local optima, Brownian motion is introduced to simulate the random swimming of the prey in the whale algorithm, so that the algorithm can avoid premature convergence.

3.1 Introducing the Harris Hawk Strategy

In the search stage, the Harris hawk optimization algorithm uses two different mechanisms based on the location of other search populations and prey, and chooses a random location to inhabit. It can expand the predation area and increase the randomness of the algorithm [20]. To obtain a better search effect, Shangbin Jiao et al. [21] added a nonlinear weight $\vec{w}(t)$ in the HHO search process, which improved the early global search ability. So, the Harris hawk search strategy with nonlinear weights is introduced into the WOA search-predation stage, and its expression is as follows [21]:

$$\vec{X}(t+1) = \begin{cases} \vec{w}(t) \times \vec{X}_{rand}(t) - r_1 \left|\vec{X}_{rand}(t) - 2r_2\vec{X}(t)\right| & q \geq 0.5 \\ \vec{w}(t) \times (\vec{X}^*(t) - \vec{X}_m(t)) - r_3(lb + r_4(ub - lb)) & q < 0.5 \end{cases}. \tag{10}$$

$$\vec{w}(t) = 0.2\cos\left(\frac{\pi}{2} \cdot \left(1 - \frac{t}{t_{\max}}\right)\right). \tag{11}$$

$$\vec{X}_m(t) = \frac{1}{N}\sum_{i=1}^{N} \vec{X}(t). \tag{12}$$

In the formula, $\vec{X}_m(t)$ is the average position vector of the current population, r_1, r_2, r_3, r_4 is a random number between [0, 1], ub and lb are the upper and lower bounds of the population, respectively, and N is the number of search populations.

When the WOA is in the bubble net attack stage, it is necessary to improve and optimize the spiral updating position and shrinking encirclement since the two strategies of spiral updating position and shrinking encirclement are slow to converge in the late search stage and are easy to enter the local optimum [22]. The strategy of HHO algorithm has two functions: first, it contains the Levy flight strategy, which can make the prey have more escape possibilities. Second, use optimal individual to guide the position update to improve the algorithm convergence effect. Therefore, adding the soft besiege with progressive rapid dives strategy in the HHO algorithm to the spiral updating position and shrinking encircling strategy of WOA not only can promote the algorithm's ability to escape from local optima but also speed up the algorithm's convergence speed.

$$\vec{Y}(t+1) = \vec{X}^*(t) - E\left|J\vec{X}^*(t) - \vec{X}(t)\right|. \quad \textbf{(13)}$$

$$\vec{Z}(t+1) = \vec{X}^*(t) - E\left|J\vec{X}^*(t) - \vec{X}(t)\right| + \vec{S} \times LF(Dim). \quad \textbf{(14)}$$

Where E is the escape energy, and the calculation formula is as formula (16); LF is the Levy flight function [17]. Dim is the dimension; $\vec{S}$ is a random vector of Dim dimension.

$$\begin{cases} LF(x) = 0.01 \times \dfrac{l \times m}{|\mu|^{\frac{1}{\beta}}} \\ \sigma = \left(\dfrac{\Gamma(1+\beta) \times \sin(\pi\beta/2)}{\Gamma((1+\beta)/2) \times \beta \times 2^{\frac{\beta-1}{2}}} \right)^{\frac{1}{\beta}} \end{cases}. \quad \textbf{(15)}$$

$$E = 2E_0(1 - \frac{t}{t_{\max}}). \quad \textbf{(16)}$$

Among them, l and m are uniformly distributed between [0,1]; β is 1.5; $E_0 \in [-1,1]$. The update strategy for this link is as follows:

$$\vec{X}(t) = \begin{cases} \vec{Y}(t), & F(\vec{Y}(t)) < F(\vec{X}(t)) \\ \vec{Z}(t), & F(\vec{Y}(t)) < F(\vec{X}(t)) \\ \vec{X}(t), & otherwise \end{cases}. \quad \textbf{(17)}$$

3.2 Introduce Brownian Motion Strategy

In the shrinking encircling phase (2), the random value generated by the random parameter $\vec{C}$ is too small, which causes the individual to fall into the problem of being unable to escape from local optima for a long time, which will have a particular impact on the later optimization. The randomization strategy of Brownian motion allows taking larger steps in the search process, which can avoid the algorithm stalling at local optima [18]. After applying the Brownian motion strategy to the distance formula of WOA, the whale can take a larger step factor and prevent algorithms from prematurely maturing.

Introduce the Brownian motion strategy into the WOA to get the step factor vector $\vec{L}(t)$

$$\vec{L}(t) = f(\vec{X}(t)) \cdot (\vec{X}^*(t) - f(\vec{X}(t)) \cdot \vec{X}(t)). \quad \textbf{(18)}$$

In formula (18), $f(x)$ represents the mathematical model of Brownian motion, $f_B(x) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-x^2}{2})$. Applying to equations (2) and (6) of WOA, we can get

$$\vec{D} = \left|\vec{L}(t) \cdot \vec{X}^*(t) - \vec{X}(t)\right|. \quad \textbf{(19)}$$

$$\vec{D}' = \left|\vec{L}(t) \cdot \vec{X}^*(t) - \vec{X}(t)\right|. \quad \textbf{(20)}$$

3.3 HWOA Algorithm Flow

Step 1: In the initialization stage, set the relevant parameter variables.

Step 2: Get the fitness of individual, obtain the optimal individual.

Step 3: Update the control parameters in the algorithm.

Step 4: When $P < 0.5$, if $\left|\vec{A}\right| < 1$, use formula (1) and formula (19) containing Brownian motion contraction to update the individual position, and then go to step 6; if $\left|\vec{A}\right| \geq 1$, use Harris hawk search strategy, use formula (10) to update the individual location.

Step 5: When $P \geq 0.5$, use the spiral updating position strategy with Brownian motion to update the individual position through equations (5) and (20), and then go to step 6.

Step 6: Execute HHO exploitation strategy using formula (17).

Step 7: Calculate and record the fitness, and then determine whether t reaches the maximum value. If it is not satisfied, go back to step 3, if it is satisfied, end and output the fitness value.

The HWOA flowchart is shown in Figure 1.

4 Simulation Comparison and Analysis

To demonstrate that the improvement of HWOA is feasible and effective, 23 benchmark functions [20] are used for testing. The description of the benchmark function is as follows. Table 1 is the single-peak test function (F1-F7) in the benchmark function, Table 2 is the multi-dimensional multi-peak test function (F8-F13), and Table 3 is the fixeddimension multi-peak test function (F14-F23). 30 is the dimension of F1-F13, the dimension of F14, F16, F17, and F18 is 2, the dimension of F15, F21, F22, and F23 is 4, the dimension of F19 is 3, and the dimension of F20 is 6. The comparison algorithm is the swarm intelligence optimization algorithm proposed in recent years, including WOA [11], HHO [17], GWO [23], BOA [24], SOA [25], and hybrid firefly–whale optimization algorithm [26] (FA-WOA).

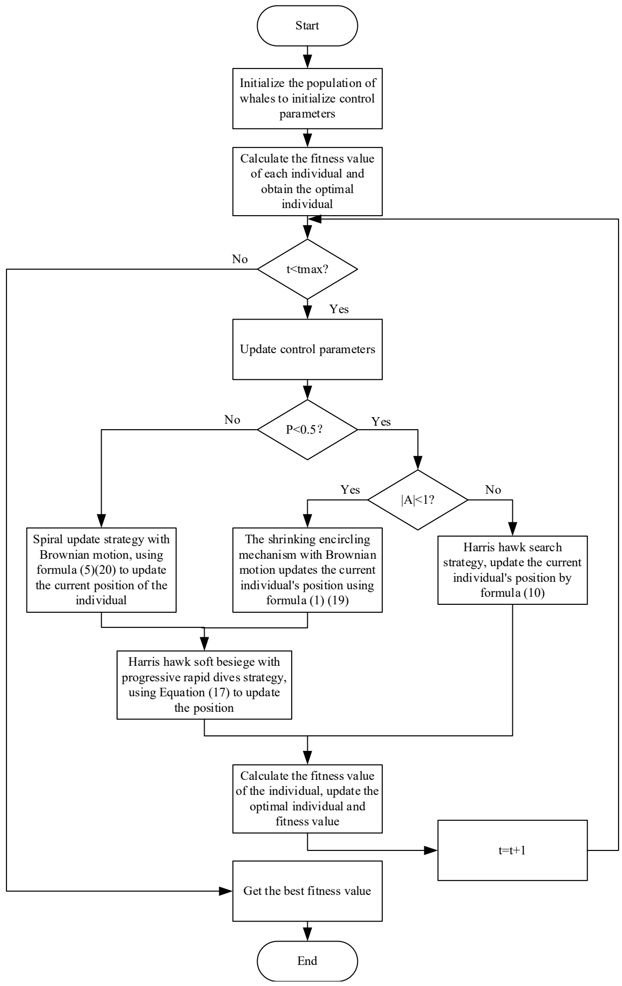

Figure 1. HWOA algorithm flowchart

Table 1. Unimodal benchmark functions

Functions	Function name	Range	F_{min}
$F_1(x)=\sum_{i=1}^{\dim} x_i^2$	Sphere	[-100.100]	0
$F_2(x)=\sum_{i=1}^{\dim}\lvert x_i\rvert+\prod_{i=1}^{\dim}\lvert x_i\rvert$	Schwefel 222	[-10,10]	0
$F_3(x)=\sum_{i=1}^{\dim}\left(\sum_{j=1}^{i} x_j\right)^2$	Schwefel 1.2	[-100,100]	0
$F_4(x)=\max_i\left\{\lvert x_i\rvert,1\le i\le \dim\right\}$	Schwefel 2.21	[-100,100]	0
$F_5(x)=\sum_{i=1}^{\dim-1}\left[100\left(x_{i+1}-x_i^2\right)+\left(x_i-1\right)^2\right]$	Rosenbrock	[-30,30]	0
$F_6(x)=\sum_{i=1}^{\dim}\left(\lvert x_i+0.5\rvert\right)^2$	Step	[-100,100]	0
$F_7(x)=\sum_{i=1}^{\dim} ix_i^4+\text{random}[0,1]$	Quartic	[-1.28,1.28]	0

Table 2. Multidimensional and multimodal benchmark functions

Functions	Function name	Range	F_{min}
$F_8(x)=\sum_{i=1}^{\dim}-x_i\sin(\sqrt{\lvert xi\rvert})$	Schwefel 2.26	[-500,500]	-12569.5
$F_9(x)=\sum_{i=1}^{\dim}\left[x_i^2-10\cos\left(2\pi x_i\right)+10\right]$	Rastrigin	[-5.12,5.12]	0
$F_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{\dim}d\sum_{i=1}^{\dim}x_i^2}\right)-$ $\exp\left(\frac{1}{\dim}\sum_{i=1}^{\dim}\cos\left(2\pi x_i\right)\right)+20+e$	Ackley	[-32,32]	0
$F_{11}(x)=\frac{1}{4000}\sum_{i=1}^{\dim}x_i^2-\prod_{i=1}^{\dim}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	Griewank	[-600,600]	0
$F_{12}(x)=\frac{\pi}{\dim}10\sin^2\left(\pi y_1\right)+\sum_{i=1}^{\dim-1}\left(y_i-1\right)^2\left[1+10\sin^2\left(\pi y_{i+1}\right)\right]$ $+\left(y_{\dim}-1\right)^2+\sum_{i=1}^{\dim}U\text{ fun}\left(x_i,10,100,4\right)$	Penalized1	[-50,50]	0
$F_{13}(x)=0.1\begin{Bmatrix}\sin^2\left(3\pi x_1\right)+\sum_{i=1}^{\dim}\left(x_i-1\right)^2\left[1+\sin^2\left(3\pi x_{i+1}+1\right)\right]+\\ \left(x_{\dim}-1\right)^2\left[1+\sin^2\left(2\pi x_{dim}\right)\right]\end{Bmatrix}$ $+\sum_{i=1}^{\dim}U\text{ fun}\left(x_i,5,100,4\right)$	Penalized2	[-50,50]	0

Table 3. Fixed-dimensional multimodal benchmark functions

Functions	Function name	Range	F_{min}
$F_{14}(x)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\Sigma_{i=1}^{2}\left(x_i-a_{ij}\right)^6}\right)^{-1}$	Foxholes	[-65.5360, 65.5360]	1
$F_{15}(x)=\sum_{k=1}^{11}\left[a_k-\frac{x_1\left(b_k^2+b_kx_2\right)}{b_k^2+b_kx_3+x_4}\right]^2$	Kowalik	[-5,5]	0.00030
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^{\ 2}+4x_2^4$	Six Hump Camel Bcak	[-5,5]	-1.0316
$F_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	Branin	[-5,0 10,15]	0.398
$F_{18}(x)=\left[1+\left(x_1+x_2+1\right)^2\left(19-14x_1+3x_1^{\ 2}-14x_2+6x_1x_2+3x_2^2\right)\right]\times$ $[30+\left(2x_1-3x_2\right)^2\times\left(18-32x_1+12x_1^{\ 2}+48x_2-36x_1x_2+27x_2^{\ 2}\right)\Big]$	GoldStein Price	[-5,5]	3
$F_{19}(x)=-\sum_{i=1}^{4}cH_i\exp\left(-\sum_{j=1}^{3}aH_{ij}\left(x_j-pH_{ij}\right)^2\right)$	Hartman3	[0,1]	-3.86
$F_{20}(x)=-\sum_{i=1}^{4}cH_i\exp\left(-\sum_{j=1}^{6}aH_{ij}\left(x_j-pH_{ij}\right)^2\right)$	Hartman6	[0,1]	-10.1532
$F_{21}(x)=-\sum_{i=1}^{5}\left[\left(x-a_i\right)\left(x-a_i\right)^T+c_i\right]^{-1}$	Shekel5	[0,10]	-10.1532
$F_{22}(x)=-\sum_{i=1}^{7}\left[\left(x-a_i\right)\left(x-a_i\right)^T+c_i\right]^{-1}$	Shekel7	[0,10]	-10.4028
$F_{23}(x)=-\sum_{i=1}^{10}\left[\left(x-a_i\right)\left(x-a_i\right)^T+c_i\right]^{-1}$	Shekel10	[0,10]	-10.5363

4.1 Comparison with Hybrid Whale Optimization Algorithm

FA-WOA and HWOA are improved algorithms formed by hybridization and fusion of other swarm intelligence algorithms and WOA. Test experiments were performed using the parameters in Ref. [26], the max iterations t_{max} =500, the population number N=100. Under the conditions of independent repetition of the experiment, the data in the Table 4 is repeated 30 times and recorded after averaging.

The convergence performance of HWOA is better than that of WOA and FA-WOA. For the multidimensional multimodal test function $F_8 \sim F_{13}$, its ability to escape from local optima is also improved, which proves that for the multidimensional multimodal test function and unimodal test functions the HWOA algorithm has good advantages. After comparison, HWOA has preferable search results to FA-WOA in $F_1 \sim F_{13}$ and some fixed-dimensional multimodal test functions.

Table 4. Comparing the optimization accuracy of different algorithms

Function	Algorithm	Mean	Best	Worst	Std
F_1	WOA	4.641E-97	2.270E-103	5.150E-96	1.168E-96
	FA-WOA	4.374E-95	3.409E-95	3.797E-77	3.817E-78
	HWOA	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
F_2	WOA	2.122E-57	5.782E-64	5.668E-56	1.032E-56
	FA-WOA	4.259E-54	9.691E-63	1.173E-52	1.786E-53
	HWOA	**2.140E-249**	**5.982E-276**	**6.421E-248**	**0.000E+00**
F_3	WOA	1.439E+04	4.562E+03	2.594E+04	5.636E+03
	FA-WOA	7.938E+03	2.758E+01	2.575E+04	5.734E+03
	HWOA	**6.166E-23**	**1.265E-69**	**1.007E-21**	**2.151E-22**
F_4	WOA	1.483E+01	1.567E-09	7.364E+01	1.906E+01
	FA-WOA	2.438E-02	2.130E-05	1.089E-01	2.869E-02
	HWOA	**1.849E-49**	**4.249E-140**	**5.546E-48**	**1.013E-48**
F_5	WOA	2.674E+01	2.615E+01	2.729E+01	3.024E-01
	FA-WOA	2.786E+01	2.672E+01	2.884E+01	6.264E-01
	HWOA	**1.068E-03**	**8.993E-10**	**1.009E-02**	**2.236E-03**
F_6	WOA	4.400E-03	1.900E-03	1.500E-03	8.100E-03
	FA-WOA	2.037E-04	4.667E-05	4.810E-04	**8.965E-05**
	HWOA	**4.565E-05**	**1.320E-07**	**3.273E-04**	9.03E-05
F_7	WOA	8.580E-04	5.753E-05	4.269E-03	1.037E-03
	FA-WOA	3.298E-03	9.311E-06	2.530E-02	4.527E-03
	HWOA	**3.687E-05**	**3.338E-06**	**1.178E-04**	**3.489E-05**
F_8	WOA	-1.160E+04	-1.257E+04	-8.375E+03	1.293E+03
	FA-WOA	-1.068E+04	-1.257E+04	-7.447E+03	1.499E+03
	HWOA	**-1.257E+04**	**-1.257E+04**	**-1.257E+04**	3.532E-01
F_9	WOA	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	FA-WOA	1.137E-15	0.000E+00	5.684E-14	7.99E-15
	HWOA	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
F_{10}	WOA	3.908E-15	0.000E+00	7.105E-15	2.696E-15
	FA-WOA	4.086E-15	8.882E-16	7.994E-15	2.286E-15
	HWOA	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
F_{11}	WOA	2.228E-03	0.000E+00	3.456E-02	8.483E-03
	FA-WOA	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	HWOA	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
F_{12}	WOA	2.861E-03	1.173E-04	4.401E-02	8.212E-03
	FA-WOA	6.461E-05	1.753E-05	2.580E-04	3.675E-05
	HWOA	**7.529E-07**	**1.677E-10**	**1.210E-05**	**2.505E-06**

F_{13}	WOA	3.096E-02	3.762E-03	1.530E-01	3.382E-02
	FA-WOA	6.475E-01	2.228E-04	2.773E+00	8.857E-01
	HWOA	**3.552E-05**	**4.029E-09**	**7.179E-04**	**1.365E-04**
F_{14}	WOA	1.197E+00	5.467E-01	9.980E-01	2.982E+00
	FA-WOA	7.148E+00	9.980E-01	1.267E+01	4.900E+00
	HWOA	**9.980E-01**	**4.779E-12**	**9.980E-01**	**9.980E-01**
F_{15}	WOA	6.652E-04	3.932E-04	3.103E-04	1.600E-03
	FA-WOA	9.054E-04	3.079E-04	1.657E-02	2.229E-03
	HWOA	**3.168E-04**	**1.247E-05**	**3.075E-04**	**3.606E-04**
F_{16}	WOA	-1.032E+00	-1.032E+00	-1.032E+00	1.595E-11
	FA-WOA	1.032E+00	-1.032E+00	-1.032E+00	4.665E-15
	HWOA	**-1.032E+00**	**-1.032E+00**	**-1.032E+00**	**2.241E-09**
F_{17}	WOA	3.979E-01	3.979E-01	3.979E-01	8.225E-08
	FA-WOA	3.979E-01	3.979E-01	3.979E-01	**4.348E-11**
	HWOA	**3.979E-01**	**3.979E-01**	**3.979E-01**	9.382E-08
F_{18}	WOA	3.000E+00	3.000E+00	3.000E+00	9.950E-07
	FA-WOA	3.000E+00	3.000E+00	3.000E+00	**2.028E-15**
	HWOA	**3.000E+00**	**3.000E+00**	**3.000E+00**	5.145E-06
F_{19}	WOA	-3.005E-01	-3.005E-01	-3.005E-01	2.258E-16
	FA-WOA	-3.862E+00	-3.863E+00	-3.855E+00	1.881E-03
	HWOA	**-3.005E-01**	**-3.005E-01**	**-3.005E-01**	**2.258E-16**
F_{20}	WOA	-3.240E+00	-3.322E+00	-3.108E+00	7.023E-02
	FA-WOA	-3.242E+00	-3.322E+00	-2.840E+00	1.025E-01
	HWOA	**-3.255E+00**	**-3.322E+00**	**-3.115E+00**	**6.986E-02**
F_{21}	WOA	-9.303E+00	-1.015E+01	-5.048E+00	1.933E+00
	FA-WOA	-8.387E+00	-1.015E+01	-8.810E-01	2.601E+00
	HWOA	**-1.015E+01**	**-1.015E+01**	**-1.015E+01**	**1.210E-03**
F_{22}	WOA	-9.294E+00	-1.040E+01	-3.724E+00	2.267E+00
	FA-WOA	-8.565E+00	-1.040E+01	-1.837E+00	2.551E+00
	HWOA	**-1.040E+01**	**-1.040E+01**	**-1.040E+01**	**1.702E-03**
F_{23}	WOA	-8.561E+00	-1.054E+01	-3.835E+00	2.865E+00
	FA-WOA	-7.760E+00	-1.053E+01	-1.676E+00	3.253E+00
	HWOA	**-1.053E+01**	**-1.054E+01**	**-1.053E+01**	**1.918E-03**

4.2 Comparison with Hybrid Whale Optimization Algorithm

The convergent image of HWOA, GWO, BOA, HHO, SOA are as follows. The population was 100, the max iterations were 500, and 30 experiments were independently repeated to record data. Due to limited space, only the first four convergence curves of the first two types of test function are given.

For Figure 2, the convergence speed of HWOA is preferable to other algorithms. For unimodal test functions, the consequence of F_1, F_2, F_4 following test is outstanding. F_3 is slightly worse than HHO. The reason is that the search strategy of WOA is a ring-shaped encirclement, and F_3 is a function whose optimal value is located at the bottom of the parabolic valley [27]. It could get the valley in a short time, but it's hard to get the lowest point value. The annular encircling search model in the WOA is difficult to reach the bottom of the parabolic valley when searching for the optimal value, so it is hard for HWOA to get the minimum value of the F_3 in a relatively short time. For $F_8 \sim F_{11}$, the convergence speed of HWOA is preferable to comparison algorithms, and the search accuracy is ideal in most functions. In the test function $F_9 \sim F_{11}$, the convergence curve of HHO has an optimum local phenomenon, which affects the convergence speed, while the HWOA algorithm does not have a relatively obvious local optimum phenomenon, so under the same optimization results, the HWOA convergence speed is the fastest. This can prove that HWOA has done well to escape from local optima. Table 5 presents the optimization data of the test function.

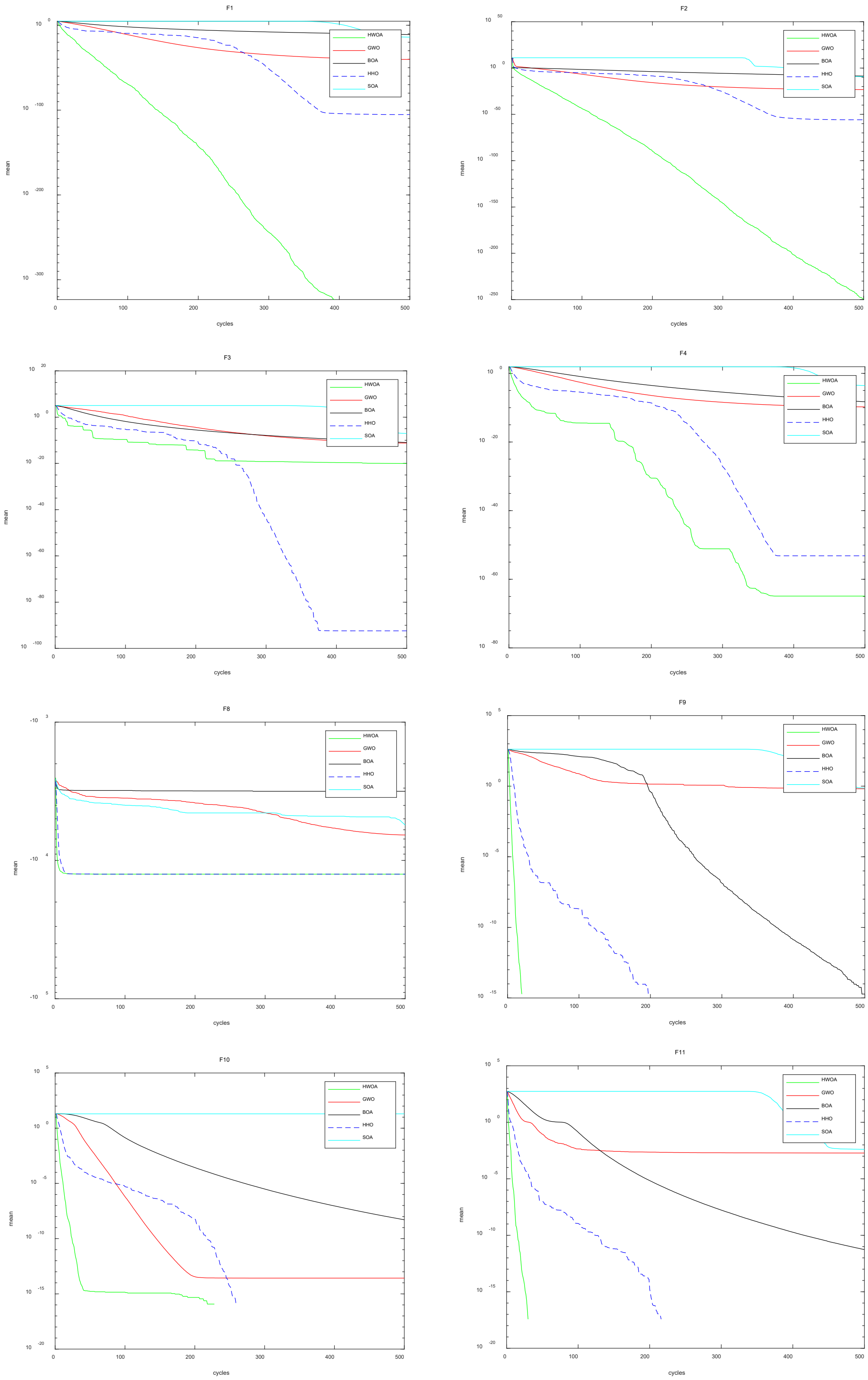

Figure 2. Convergence curves of HWOA and other algorithms

Table 5. Optimization data of test function

Function	Algorithm	Mean	Std	Best	Worst
F_1	HWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
	GWO	5.88E-41	5.83E-41	3.79E-42	2.52E-40
	BOA	1.10E-11	7.33E-13	9.57E-12	1.27E-11
	HHO	5.85E-106	3.01E-105	1.19E-121	1.65E-104
	SOA	1.02E-14	3.51E-14	4.24E-16	1.94E-13
F_2	HWOA	**5.47E-250**	**0.00E+00**	**8.78E-283**	**1.64E-248**
	GWO	5.29E-24	4.47E-24	6.73E-25	1.74E-23
	BOA	2.32E-09	1.33E-09	6.99E-12	4.76E-09
	HHO	1.49E-56	3.79E-56	1.80E-64	1.77E-55
	SOA	5.07E-10	3.12E-10	1.20E-10	1.56E-09
F_3	HWOA	8.90E-21	4.70E-20	1.08E-60	2.58E-19
	GWO	6.20E-12	1.27E-11	8.72E-16	6.19E-11
	BOA	1.10E-11	5.80E-13	9.50E-12	1.17E-11
	HHO	**3.94E-93**	**1.99E-92**	**1.21E-113**	**1.09E-91**
	SOA	9.01E-08	2.12E-07	6.43E-10	1.08E-06
F_4	HWOA	**1.27E-65**	**6.80E-65**	**2.47E-121**	**3.73E-64**
	GWO	1.74E-10	1.51E-10	1.80E-11	6.03E-10
	BOA	5.29E-09	2.96E-10	4.61E-09	5.79E-09
	HHO	6.91E-54	3.48E-53	5.26E-63	1.91E-52
	SOA	2.81E-04	7.80E-04	5.43E-06	4.00E-03
F_8	HWOA	**-1.26E+04**	3.54E-01	**-1.26E+04**	**-1.26E+04**
	GWO	-6.55E+03	6.39E+02	-7.64E+03	-5.21E+03
	BOA	-3.16E+03	3.42E+02	-2.67E+03	-4.14E+03
	HHO	-1.26E+04	**8.30E-02**	-1.26E+04	-1.26E+04
	SOA	-5.55E+03	7.83E+02	-7.43E+03	-4.29E+03
F_9	HWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
	GWO	6.54E-01	1.56E+00	0.00E+00	5.22E+00
	BOA	1.89E-15	1.04E-14	0.00E+00	5.68E-14
	HHO	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SOA	7.24E-01	2.13E+00	0.00E+00	8.33E+00
F_{10}	HWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
	GWO	2.65E-14	3.58E-15	1.78E-14	3.20E-14
	BOA	5.11E-09	2.90E-10	4.58E-09	5.76E-09
	HHO	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SOA	2.00E+01	1.40E-03	2.00E+01	2.00E+01
F_{11}	HWOA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
	GWO	1.90E-03	4.50E-03	0.00E+00	1.61E-02
	BOA	5.38E-12	1.38E-12	2.44E-12	7.51E-12
	HHO	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	SOA	4.10E-03	8.80E-03	2.55E-15	2.91E-02

Through data comparison, HWOA can get the minimum value in F_1, F_8, F_9, F_{10}, F_{11}, and the two test functions of F_2, F_4 have a good performance in the optimization accuracy. In F_3, the optimization results of the mean and the worst value. It does not show an absolute advantage over other comparison algorithms, but has different degrees of improvement compared to GWO, BOA, and SOA.

In order to prove that the improvement of HWOA is effective and has no correlation with the two test results of WOA, the Wilcoxon function [28] is used to test the eight test functions of HWOA and WOA in Table 5. The SPSS software test and the result retain three decimal places, and the obtained statistical test value p-value is shown in Table 6.

Table 6. Wilcoxon signed-rank test results

Function	p-value
F_1	0.000
F_2	0.000
F_3	0.000
F_4	0.000
$_8$	0.000
F_9	0.317
F_{10}	0.000
F_{11}	0.180

For Table 6, the p-value of the two functions of F_9, F_{11} is greater than 0.05, which proves that the significance level is low. In the optimization process of F_9, F_{11}, both HWOA and WOA have found the optimal value many times in 30 optimization processes, so the algorithm is not very significant in F_9, F_{11}. Most of the p-value test results of the remaining functions are less than 0.01, which is very

significant, so it can be judged that HWOA and WOA are statistically significant.

4.3 CEC06-2019 Benchmark Function

Using the CEC06-2019 benchmark test function [29], the HWOA algorithm is compared with WOAGWO [30], WOA-BAT [31], FOX [32], CDDO [33], LPB [34], which are both hybrid whale optimization algorithms and the latest proposed swarm intelligence algorithm. During the test and comparison process, the population number N is defined as 30. The $t_{\max} = 500$, and every function is repeated 30 times to obtain the average value. The IEEE CEC06-2019 benchmark fuction is given in Table 7.

For Table 8, HWOA compared with the other six algorithms in the CEC06-2019 benchmark function test, 5 functions have the best optimization effect. The optimization effect of HWOA on the functions of $F_{24}, F_{28}, F_{29}, F_{30}, F_{33}$ is not as good as other comparison algorithms, but the optimization results are still superior and better than WOA. Subsequently, the rank of the performance of each algorithm in the face of different functions was recorded. For Table 9, the average ranking of algorithms at CEC2019 was listed. It can be seen that HWOA has a higher average ranking among the seven algorithms, which can better reflect the excellent optimization effect of HWOA.

According to this result, it can be concluded that the search performance of HWOA is preferable to WOA, WOAGWO, WOA-BAT, FOX, CDDO and LPB in dealing with most problems.

Table 7. IEEE CEC06-2019 benchmark function

Function	Function name	$F_{\min}$	Dim	Range
F_{24}	Storn's Chebyshev Polynomial Fitting Problem	1	9	[-8192,8192]
F_{25}	Inverse Hilbert Matrix Problem	1	16	[-16.384,16.384]
F_{26}	Lennard–Jones Minimum Energy Cluster	1	18	[-4,4]
F_{27}	Rastrigin's Function	1	10	[-100,100]
F_{28}	Griewank's Function	1	10	[-100,100]
F_{29}	Weierstrass Function	1	10	[-100,100]
F_{30}	Modified Schwefel's Function	1	10	[-100,100]
F_{31}	Expanded Schaffer's F6 Function	1	10	[-100,100]
F_{32}	Happy Cat Function	1	10	[-100,100]
F_{33}	Ackley's Function	1	10	[-100,100]

Table 8. CEC06-2019 benchmark function test results

Function	Algorithm	Ave	Std	Rank
	HWOA	5.04E+04	6.73E+03	3
	WOA	3.03E+10	3.80E+10	6
	WOAGWO	4.76E+04	5.19E+03	2
F_{24}	WOA-BAT	7.60E+07	4.16E+08	5
	FOX	**2.58E+04**	2.25E+01	1
	CDDO	5.69E+05	4.62E+05	4
	LPB	6.17E+10	5.47E+10	7
	HWOA	**1.73E+01**	3.60E-03	1
	WOA	1.74E+01	8.10E-03	2
	WOAGWO	1.83E+01	4.72E-04	5
25	WOA-BAT	1.75E+01	1.21E-01	3
	FOX	1.83E+01	4.25E-04	6
	CDDO	1.81E+01	4.58E-01	4
	LPB	3.34E+01	1.89E+01	7
	HWOA	**1.27E+01**	6.62E-09	1
	WOA	1.27E+01	2.64E-06	2
	WOAGWO	1.37E+01	1.83E-05	4
F_{26}	WOA-BAT	1.27E+01	9.53E-04	6
	FOX	1.37E+01	7.11E-15	7
	CDDO	1.27E+01	2.37E-05	5
	LPB	1.27E+01	2.77E-07	3
	HWOA	**1.24E+02**	5.42E+01	1
	WOA	3.36E+02	1.27E+02	3
	WOAGWO	2.54E+02	5.39E+02	2
F_{27}	WOA-BAT	2.12E+03	1.01E+03	6
	FOX	1.06E+03	8.35E+02	5
	CDDO	3.07E+03	8.79E+02	7
	LPB	8.21E+02	3.67E+01	4
	HWOA	1.66E+00	2.62E-01	2
	WOA	1.88E+00	4.81E-01	3
	WOAGWO	2.43E+00	2.62E-01	5
F_{28}	WOA-BAT	2.44E+00	6.67E-01	6
	FOX	5.31E+00	8.00E-01	7
	CDDO	2.41E+00	1.14E-01	4
	LPB	**1.25E+00**	1.25E-01	1
	HWOA	8.28E+00	1.05E+00	3
	WOA	9.27E+00	1.15E+00	4
	WOAGWO	1.14E+01	1.64E+00	6
F_{29}	WOA-BAT	1.11E+01	1.55E+00	5
	FOX	**5.03E+00**	1.50E+00	1
	CDDO	1.15E+01	9.61E-01	7
	LPB	6.18E+00	8.60E-01	2
	HWOA	3.11E+02	1.45E+02	3
	WOA	4.77E+02	2.63E+02	4
	WOAGWO	5.88E+02	3.49E+02	5
F_{30}	WOA-BAT	6.06E+02	3.90E+02	6
	FOX	3.07E+02	1.42E+02	2
	CDDO	1.03E+03	1.73E+02	7
	LPB	**2.63E+02**	1.57E+02	1

F_{31}	HWOA	**4.96E+00**	8.12E-01	1
	WOA	5.80E+00	5.49E-01	6
	WOAGWO	5.59E+00	1.02E+00	4
	WOA-BAT	5.72E+00	7.18E-01	5
	FOX	5.46E+00	3.82E-01	3
	CDDO	6.74E+00	3.83E-01	7
	LPB	5.53E+00	5.58E-01	2
F_{32}	HWOA	**2.74E+00**	2.44E-01	1
	WOA	5.29E+00	2.54E+00	4
	WOAGWO	5.67E+00	8.81E-01	5
	WOA-BAT	2.28E+01	4.92E+01	6
	FOX	3.79E+00	4.41E-01	3
	CDDO	2.43E+02	8.25E+01	7
	LPB	3.20E+00	3.30E-01	2
F_{33}	HWOA	2.02E+01	8.39E-02	2
	WOA	2.03E+01	1.08E-01	3
	WOAGWO	2.16E+01	9.22E-02	7
	WOA-BAT	2.12E+01	2.26E-01	6
	FOX	2.10E+01	1.11E-02	5
	CDDO	2.06E+01	1.18E-01	4
	LPB	**2.01E+01**	3.70E-02	1

Table 9. Average ranking of algorithms

Algorithm	Mean rank
HWOA	**1.8**
WOA	3.7
WOAGWO	4.5
WOA-BAT	5.4
FOX	4
CDDO	5.6
LPB	3

5 Conclusion

This paper proposes a HWOA to deal with the disadvantages of WOA. Firstly, the Harris hawk search strategy and HHO search strategy are introduced into WOA, improving the convergence accuracy and speed. Secondly, Brownian motion is added to the whale individual update process to improve the ability to escape from local optima. To demonstrate the effectiveness of the improvement, 23 function tests are used to show that HWOA is better than FA-WOA, which is also a hybrid algorithm, and its data outperforms other algorithms in most tests. In addition, the CEC06-2019 function is used to test HWOA. By comparing with the other six algorithms, it is concluded that HWOA performs well in 5 of the 10 test functions and has different degrees of improvement compared to WOA.

HWOA needs to be further studied in future research: 1) When dealing with the single extreme point problem, the convergence speed still needs to be improved. 2)For the problem of multiple extreme points, the search accuracy is still insufficient. It is necessary to seek a better way to improve the search accuracy while quickly jumping out of the local optimum. 3) High-dimensional problems are not tested in this study, and it needs to handle projects with high-dimensional.

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