# Provably Secure Certificateless Proxy Signature Scheme in the Standard Model 

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#### Abstract

Proxy signature frees the original signer from the heavy signature work. Many certificateless proxy signature (CLPS) schemes have been proposed in the last ten years. The security proofs of most known schemes are given in the random oracle model (ROM). There are only two CLPS schemes with provably security in the standard model (SM). However, in which the size of the system parameter increase linearly with the size of the user's identity information. That increase the storage burden of the key generation center. In this paper, a new CLPS scheme is constructed and the security proofs are showed in SM. The size of system parameters and the master key are constant in the scheme. Requiring only three pairing operations, the new scheme is more efficient and suitable for mobile computing.


Keywords: Certificateless cryptography, Mobile computing, Proxy signature, Pairing, Standard model

## 1. Introduction

With the development of mobile communication technology, mobile services have penetrated into every aspect of people's lives. In 2017, more than 5 billion people were associated with mobile services, and by 2025 independent mobile users will reach 5.9 billion, equivalent to $71 \%$ of the global population. The number of mobile Internet users will increase by 1.75 billion new users by then, reaching a milestone of 5 billion users in 2025. Global mobile data traffic grew strongly, with a compound annual growth rate of $73.46 \%$ in 2010-2017, with smartphone mobile data traffic growing at a compound annual growth rate of $121.69 \%$ from 2010 to 2017.

People are enjoying the convenience of mobile communications while also facing the risk of personal privacy leaks. Due to size constraints, the computing power of personal mobile communication devices is limited. Some previous cryptographic schemes require more computing costs, so they are not suitable for personal mobile communication devices. Therefore, it is meaningful to design secure and efficient cryptographic schemes for mobile computing.

In 1996, Mambo et al. [1] put forward the notion of proxy signatures, which means that when the authorizer (original signer) cannot exercise the signature right for some reason, he can authorize the designated agent to exercise the signature right instead of himself. The original signer designated the
agent to sign instead of himself, and does not need to provide his private key to the agent.

Al-Riyami et al. [2] presented the certificateless public key cryptography (CLPKC). The user's private key is made up of two parts: a partial private key yield by key generation center (KGC) and a secret value chosen by the user himself. It avoids key escrow and certificate management.

### 1.1 Related Work

Li et al. [3] put forward the first certificateless proxy signature (CLPS) scheme. Unfortunately, they did not show the security proofs. Choi et al. [4] and Lu et al. [5] indicated that the scheme [3] is insecure against proxy signature forgery attacks, and proposed an improved scheme, respectively. However, the security proofs of the improved schemes were not shown too. Chen et al. [6] put forward a new security model and constructed a new CLPS scheme with provably security. Xiong et al. [7] presented a CLPS scheme that needs to perform eight pairing operations. Seo et al. [8] and Zhang et al. [9] presented a CLPS scheme and showed the security proofs, respectively. Deng et al. [10] proposed a CLPS scheme that needs to perform two pairing operations. He et al. [11] presented a CLPS scheme from elliptic curve group. Deng et al. [12] proposed a CLPS scheme and showed the security proofs based on RSA problem. These two schemes [11-12] do not use pairing operations. Jin and Wen [13] put forward a certificateless multi-proxy signature (CLMPS) scheme. However, Xu et al. [14] indicated that the scheme has three disadvantages and proposed an improved scheme. Qu et al. [15] presented a CLMPS scheme that does not need to perform pairing operation. The security proofs of the above schemes were shown in ROM. Eslami and Pakniat [16] put forward the first CLPS scheme with provably security in SM. However, they did not provide a concrete security proof. Lu and Li [17] showed that the scheme [16] has some security drawbacks and presented a new scheme that is provably secure in SM. Ming and Wang [18] constructed a CLPS scheme, and gave the security proofs in SM. Yang et al. [19] came up with a new CLPS scheme and claimed that their scheme is provable secure in SM. However, Lin et al. [20] indicated that a Type II adversary can forge a valid signature in the scheme [19].

### 1.2 Motivations and Contributions

[^0]In the last ten years, several concrete CLPS schemes were proposed. There are only two schemes [17-18] with provably security in SM. In the two schemes, the size of the system parameter increases linearly with the size of the user's identity information, and the number of addition operations on the elliptic curve group increases linearly with the size of the user's identity information. Therefore, these two schemes are not suitable for mobile computing scenarios. So it is attractive to design a CLPS scheme for mobile computing that is provable secure in SM and requires a constant number of pairing operations.

In this paper, we constructed a new CLPS scheme with the following features:

- It is secure against Type I/II adversary in SM
- It was constant that the size of system parameters and the size of master secret key.
- It was constant that the number of three kinds of operations (addition, scalar multiplication, and pairing).


### 1.3 Roadmap

The rest of the content are arranged as follows: First, we introduced the bilinear pairing and computation attack algorithm problem in Sec.2. Second, we proposed the system model and a concrete CLPS scheme in Sec. 3 and Sec.4, respectively. Third, we presented the security model and the security proofs of new scheme in Sec. 5 and Sec.6, respectively Next, we made the performance comparisons on several schemes in Sec.7. Lastly, we gave some conclusions in Sec.8.

## 2 Preliminaries

For ease of understanding, we listed the symbols used in the paper in Table 1.

## Bilinear pairing

Let $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$ be a map with the following properties, where $G_{1}$ and $G_{2}$ are an additive group and a multiplicative group, respectively, and their order is $q$

- Bilinearity: $\hat{e}\left(a P_{1}, b P_{2}\right)=\hat{e}\left(P_{1}, P_{2}\right)^{a b}$ for all
- $P_{1}, P_{2} \in G_{1}$ and $a, b \in Z_{q}$.
- Non-degeneracy: There exist $P_{1}, P_{2} \in G_{1}$ such that $\hat{e}\left(P_{1}, P_{2}\right) \neq 1_{G_{2}}$.
- Computability: There is an efficient algorithm to compute $\hat{e}\left(P_{1}, P_{2}\right)$ for all $P_{1}, P_{2} \in G_{1}$.

Definition 1 CAA (Computation attack algorithm) problem[21]. Given a generator $P$ of the group $G_{1}$ and a tuple $(P, x P)$, output a pair $\left(c, \frac{1}{x+c} P\right)$, where $c \in Z_{q}^{*}$.

## 3 System Model

A CLPS scheme consists of four entities: Key generation center (KGC), original signer, proxy signer and verifier.

KGC: It generates system parameters and publishes them to the outside world, generates a partial private key (PPK) according to the user's identity, and sends it to the user through an authenticated channel.

Original signer: $\mathrm{He} /$ she generates a delegation and sends it to the proxy signer.

Proxy signer: $\mathrm{He} /$ she generates a signature on a message on behalf of the original signer according to the delegation and sends it to the verifier.

Verifier: $\mathrm{He} /$ she checks its validity after receiving the signature.

A CLPS scheme contains the following eight algorithms:

- Setup: Inputs a security parameter $v$, KGC generates the system parameters ( params ) and master secret key (msk).
- PPK-Extract: Inputs an identity $I D_{i} \in\{0,1\}^{*}$, PKG generates a partial private key.
- SV-Set: Inputs an identity $I D_{i} \in\{0,1\}^{*}$, the user sets his own secret value.
- UPK-Generate: Inputs an identity $I D_{i} \in\{0,1\}^{*}$, the users generates a user public key.
- Delegate: Inputs a tuple $\left(m_{w}, t_{o}, D_{o}\right)$, the original signer generates a delegation.
- Delegation-Verify: Inputs a tuple ( $m_{w}, \delta$ ), the proxy signer checks whether the delegation is valid.
- Proxy-Sign: Inputs a tuple ( $m, m_{w}, \delta, t_{p}, D_{p}$ ), the proxy signer generates a signature.
- PS-Verify: Inputs a tuple ( $m, m_{w}, \delta, \sigma$ ), the receiver checks whether the signature is valid.

Table 1. Notations

| Symbol | Meaning |
| :---: | :---: |
| $F_{p}$ | A prime finite field |
| $q$ | A prime number |
| $Z_{q}^{*}$ | A set making up of positive integers less than $q$ |
| $G_{1}$ | An additive group with prime order $q$ |
| $G_{2}$ | An multiplicative group with prime order $q$ |
| $\hat{e}$ | A bilinear pairing, where $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$ |
| $x$ | The master secret key of system |
| $P$ | A generator of the group $G_{1}$ |
| $P_{p u b}$ | The public key of system, where $P_{p u b}=x P$ |
| $H_{1}, H_{2}, H_{3}$ | Three secure hash functions |
| $I D_{\text {o }}$ | The identity of the original signer |
| $I D_{p}$ | The identity of the proxy signer |
| $I D_{i}$ | The identity of the $i^{\text {th }}$ user |
| $D_{i}=\left(R_{i}, d_{i}\right)$ | The partial private key of the $i^{\text {th }}$ user, where $R_{i}=r_{i} P$ |

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    \(t_{i} \quad\) The secret value of the \(i^{t h}\) user, where \(T_{i}=t_{i} P\)
\(P K_{i}=\left(R_{i}, T_{i}\right) \quad\) The public key of the \(i^{\text {th }}\) user
\(m_{w} \quad\left\{I D_{o}, P K_{o}, I D_{p}, P K_{p}\right\} \subseteq m_{w}\)
\(\delta \quad\) A delegation
\(\sigma \quad\) A proxy signature
```


## 4 Our Scheme

We proposed a new CLPS scheme as follows.

- Setup: Inputs a security parameter $v$, KGC performs following steps.

1. Selects a bilinear pairing $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$, as that defined in sec. 2 .
2. Picks a generator $P$ of $G_{1}$, computes $E=\hat{e}(P, P)$.
3. Selects three secure hash functions $H_{1}, H_{2}, H_{3}:\{0,1\}^{*} \rightarrow Z_{q}^{*}$.
4. Chooses $x \in Z_{q}^{*}$, computes $P_{p u b}=x P$, sets $m s k=\{x\}$
5. Publishes the system parameters params $=\left\{G_{1}, G_{2}, q, \hat{e}, P, P_{p u b}, E, H_{1}, H_{2}, H_{3}\right\}$.
-PPK-Extract: For an identity $I D_{i} \in\{0,1\}^{*}$, KGC randomly picks $r_{i} \in Z_{q}^{*} \quad$, computes $\quad R_{i}=r_{i} P$, $k_{i}=H_{1}\left(I D_{i}, R_{i}\right)$ and $d_{i}=r_{i}+k_{i} x \bmod q$, then forwards $D_{i}=\left(R_{i}, d_{i}\right)$ to the user via an authenticated channel.

The user computes $k_{i}=H_{1}\left(I D_{i}, R_{i}\right)$, and checks whether $d_{i} P=R_{i}+k_{i} P_{p u b}$. Accepts the partial private key if and only if the equation holds.

- SV-Set: The user $I D_{i}$ randomly chooses $t_{i} \in Z_{q}^{*}$.
-UPK-Generate: The user $I D_{i}$ computes $T_{i}=t_{i} P$, and sets $P K_{i}=\left(T_{i}, R_{i}\right)$
-Delegate: The original signer $I D_{o} / P K_{o}$ performs the following steps to generate a delegation.

1. Computes $h=H_{2}\left(m_{w}, I D_{o}, P K_{o}\right)$.
2. Computes $Y=\frac{1}{d_{o}+h t_{o}} P$.
3. Outputs $\delta=Y$ as the delegation.

Where $m_{w}$ includes $I D_{o} / P K_{o}, I D_{p} / P K_{p}$, and the delegation duration and so on.
-Delegation-Verify: To verify a delegation ( $m_{w}, \delta=Y$ ), the verifier does as follows.
1.Computes
$k_{o}=H_{1}\left(I D_{o}, R_{o}\right), \quad h=H_{2}\left(m_{w}, I D_{o}, P K_{o}\right)$.
2. Checks whether
$\hat{e}\left(Y, R_{o}+k_{o} P_{p u b}+h T_{o}\right)=E$.
Accepts the delegation if and only if the equation holds.

- Proxy-Sign: The proxy signer $I D_{p} / P K_{p}$ performs the following steps to generate a signature.
1.Computes $l=H_{3}\left(m, m_{w}, Y, I D_{o}, P K_{o}, I D_{p}, P K_{p}\right)$

2. Computes $Z=\frac{1}{d_{p}+l t_{p}} P$.
3. Outputs $\sigma=(Y, Z)$ as the signature.

- PS-Verify: To verify a signature ( $m, m_{w}, \sigma=(Y, Z)$ ), the verifier does as follow:
1.Computes

$$
\begin{aligned}
& k_{o}=H_{1}\left(I D_{o}, R_{o}\right), k_{p}=H_{1}\left(I D_{p}, R_{p}\right), \\
& h=H_{2}\left(m_{w}, I D_{o}, P K_{o}\right), \\
& l=H_{3}\left(m, m_{w}, Y, I D_{o}, P K_{o}, I D_{p}, P K_{p}\right) . \\
& \text { 2. Checks whether } \\
& \hat{e}\left(Y, R_{o}+k_{o} P_{p u b}+h T_{o}\right)=E, \\
& \hat{e}\left(Z, R_{p}+k_{p} P_{p u b}+l T_{p}\right)=E .
\end{aligned}
$$

Accepts the signature if and only if both of equalities hold.

## 5 Security Model

The security requirements of a CLPS scheme are presented as follows.

Definition 3. In the following two games, if the adversary's advantage is negligible, then the CLPS scheme is unforgeable (UNF-CLPS)

Game I. Challenger $\mathfrak{C}$ plays this game with a
Type I adversary $\mathcal{A}_{1}$.
Initialization. $\mathfrak{C}$ gets the msk and params by running the Setup algorithm, then forwards params to $\mathcal{A}_{1}$ and keeps msk as secret.

Query. $\mathcal{A}_{1}$ issues various queries as follows.

- UPK-Query: $\mathcal{A}_{1}$ inputs an identity $I D_{i}$, $\mathfrak{C}$ returns a public key $P K_{i}$.
-UPK-Replacement: $\mathcal{A}_{1}$ submits a tuple $\left(P K_{i}^{\prime}, ~ I D_{i}\right)$, $\mathfrak{C}$ replaces $P K_{i}$ with $P K_{i}^{\prime}$.
- PPK-Query: $\mathcal{A}_{1}$ submits an identity $I D_{i}, \mathfrak{C}$ returns a partial private key $D_{i} \cdot \mathcal{A}_{1}$ cannot do it if the value $R_{i}$ has been replaced
- SV-Query: $\mathcal{A}_{1}$ submits an identity $I D_{i}, \mathfrak{C}$ returns a secret value $t_{i} \cdot \mathcal{A}_{1}$ can not do it if the value $T_{i}$ has been replaced
-Delegation-Query: $\mathcal{A}_{1}$ submits a warrant $m_{w}, \mathfrak{C}$ returns a delegation $\delta$.
-PS-Query: $\mathcal{A}_{1}$ submits a tuple $\left(m, m_{w}, \delta\right), \mathfrak{C}$ returns a proxy signature $\sigma$.

Forge. $\mathcal{A}_{1}$ outputs a tuple $\left(m_{w}^{*}, \delta^{*}\right)$ or $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$. The adversary wins if one of following cases holds.
-Case 1: The final output is $\left(m_{w}^{*}, \delta^{*}\right)$ and it satisfies the following requirements.

1. Verify $\left(m_{w}^{*}, \delta^{*}\right)=1$.
2. $\delta^{*}$ is not obtained by Delegation-Query.
3. $\mathcal{A}_{1}$ did not make PPK-Query for the original signer $I D_{o}$.
-Case 2: The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and it satisfies the following requirements.
4. Verify $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)=1$.
5. $\sigma^{*}$ is not obtained by PS-Query..
6. $\mathcal{A}_{1}$ did not make Delegation-Query for the warrant $m_{w}^{*}$.
7. $\mathcal{A}_{1}$ did not make PPK-Query for the original signer $I D_{o}$.
-Case 3: The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and it satisfies the following requirements.
8. Verify $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)=1$.
9. $\sigma^{*}$ is not obtained by PS-Query.
10. $\mathcal{A}_{1}$ did not make PPK-Query for the proxy signer $I D_{p}$.

The advantage of $\mathcal{A}_{1}$ is defined as: $A d v_{A_{1}}^{U N F-C L S}=\operatorname{Pr}\left[\mathcal{A}_{1}\right.$ wins]

Game II. Challenger $\mathfrak{C}$ plays this game with a Type II adversary $\mathcal{A}_{2}$

Initialization. $\mathfrak{C}$ gets the msk and params by running the Setup algorithm, then forwards them to $\mathcal{A}_{2}$.

Query. $\mathcal{A}_{2}$ issues various queries as those in Game I.
Forge. $\mathcal{A}_{2}$ outputs a tuple $\left(m_{w}^{*}, \delta^{*}\right)$ or $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$. The adversary wins if one of following cases holds.
-Case 1: The final output is $\left(m_{w}^{*}, \delta^{*}\right)$ and it satisfies the following requirements.

1. Verify $\left(m_{w}^{*}, \delta^{*}\right)=1$.
2. $\delta^{*}$ is not obtained by Delegate-Query.
3. $\mathcal{A}_{2}$ did not make SV-Query for the original signer $I D_{o}$.
4. $\mathcal{A}_{2}$ did not make UPK-Replacement for the original signer $I D_{o}$.
-Case 2: The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and it satisfies the following requirements.
5. Verify $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)=1$.
6. $\sigma^{*}$ is not obtained by PS-Query..
7. $\mathcal{A}_{2}$ did not make Delegation-Query for the warrant $m_{w}^{*}$.
8. $\mathcal{A}_{2}$ did not make SV-Query for the original signer $I D_{o}$.
9. $\mathcal{A}_{2}$ did not make UPK-Replacement for the original signer $I D_{o}$.
-Case 3: The final output is ( $\left.m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and it satisfies the following requirements.
10. Verify $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)=1$.
11. The value $\sigma^{*}$ is not obtained through PS-Query.
12. $\mathcal{A}_{2}$ did not make SV-Query for the proxy signer $I D_{p}$.
13. $\mathcal{A}_{2}$ did not make UPK-Replacement for the proxy signer $I D_{p}$.

The advantage of $\mathcal{A}_{2}$ is defined as: $A d v_{A_{2}}^{U N F-C L S}=\operatorname{Pr}\left[\mathcal{A}_{2}\right.$ wins]

Remark: To forge a delegation $\left(m_{w}^{*}, \delta^{*}\right), \mathcal{A}_{1}$ can make SV-Query for the original signer $I D_{o}^{*}$ or even replace the value $T_{o}^{*}$. However, he can not get the value $d_{o}^{*}$. On the other hand, $\mathcal{A}_{2}$ can make PPK-Query for the original signer $I D_{o}^{*}$. However, he can not get the value $t_{o}^{*}$.

To forge a proxy signature $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right), \mathcal{A}_{1}$ can make SV-Query for the proxy signer $I D_{p}^{*}$ or even replace the value $T_{p}^{*}$.However, he can not get the value $d_{p}^{*}$. On the other hand, $\mathcal{A}_{2}$ can make PPK-Query for the proxy signer $I D_{p}^{*}$. However, he can not get the value $t_{p}^{*}$.

## 6 Security of Scheme

We gave the security proofs in SM. In the following proofs, the adversary can directly calculate the hash function instead of querying the challenger

### 6.1 Correctness of Delegation

$$
\begin{aligned}
& \hat{e}\left(Y, R_{o}+k_{o} P_{p u b}+h T_{o}\right) \\
& =\hat{e}\left(\frac{1}{d_{o}+h t_{o}} P, r_{o} P+k_{o} x P+h t_{o} P\right) \\
& =\hat{e}\left(\frac{1}{r_{o}+k_{o} x+h t_{o}} P,\left(r_{o}+k_{o} x+h t_{o}\right) P\right)=\hat{e}(P, P) \\
& =E
\end{aligned}
$$

### 6.2 Correctness of Proxy Signature

$$
\begin{aligned}
& \hat{e}\left(Z, R_{p}+k_{p} P_{p u b}+l T_{p}\right) \\
& =\hat{e}\left(\frac{1}{d_{p}+l t_{p}} P, r_{p} P+k_{p} x P+l t_{p} P\right) \\
& =\hat{e}\left(\frac{1}{r_{p}+k_{p} x+l t_{p}} P,\left(r_{p}+k_{p} x+l t_{p}\right) P\right) \\
& =\hat{e}(P, P) \\
& =E
\end{aligned}
$$

Theorem 1. If the $C A A$ problem is hard, the scheme is unforgeable against an adversary $\mathcal{A}_{1}$ in SM.

Proof. Suppose that the challenger $\mathfrak{C}$ want to solve an instance of the $C A A$ problem $(P, a P)$, he does as follows.

Initialization. $\mathfrak{C}$ runs the Setup algorithm with a parameter $v$, then gives $\mathcal{A}_{1}$ the
params $=\left\{G_{1}, G_{2}, q, \hat{e}, P, P_{p u b}=x P, E, H_{1}, H_{2}, H_{3}\right\}$
Queries. $\mathcal{A}_{1}$ will first perform UPK-query for each identity.

- UPK-Query: $\mathfrak{C}$ maintains a list $L_{U}$ of tuple $\left(I D_{i}, t_{i}, r_{i}\right)$.When $\mathcal{A}_{1}$ inputs an identity $I D_{i}, \mathfrak{C}$ does as follows:

1. For $i=j$, picks at random $t_{j} \in Z_{q}^{*}$, sets $I D_{j}=I D^{\diamond}$, returns $P K_{j}=P K^{\diamond}=\left(t_{j} P, a P\right)$, then stores the tuple $\left(I D_{j}, t_{j}, \diamond\right)$ in the list $L_{U}$
2. For $i \neq j$, randomly picks $t_{i}, r_{i} \in Z_{q}^{*}$ and returns $P K_{i}=\left(t_{i} P, r_{i} P\right)$, then stores the tuple $\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$.

- UPK-Replacement: $\mathfrak{C}$ maintains a list $L_{R}$ of tuple $\left(I D_{i}, P K_{i}, P K_{i}^{\prime}\right)$. When $\mathcal{A}_{1}$ submits a tuple $\left(I D_{i}\right.$, $P K_{i}^{\prime}$ ), $\mathbb{C}$ replaces $P K_{i}$ with $P K_{i}^{\prime}$ and adds $\left(I D_{i}, P K_{i}, P K_{i}^{\prime}\right)$ to the list $L_{R}$.
- PPK-Query: $\mathfrak{C}$ maintains a list $L_{D}$ of tuple $\left(I D_{i}, D_{i}\right)$. When $\mathcal{A}_{1}$ submits an identity $I D_{i}$. If $I D_{i}=I D^{\diamond}$, $\mathfrak{C}$ fails. Otherwise, $\mathfrak{C}$ finds $\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$, gives the $D_{i}$ by running PPK-Extract algorithm and adds $\left(I D_{i}, D_{i}\right)$ to the list $L_{D}$.
- SV-Query: When $\mathcal{A}_{1}$ submits an identity $I D_{i}, \mathfrak{C}$ finds the tuple $\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$, and responds with $t_{i}$.
- Delegation-Query: When $\mathcal{A}_{1}$ submits a warrant $m_{w}$, $\mathfrak{C}$ generates a delegation as follows:

1. $I D_{o} \neq I D^{\diamond}$ and $I D_{o} \notin L_{R}, \mathfrak{C}$ outputs a delegation $\delta$ by running Delegate algorithm.
2. $I D_{o} \in L_{R}$, then $P K_{o}=\left(t_{o} P, r_{o} P\right)$ has been replaced by $P K_{o}^{\prime}=\left(t_{o}^{\prime} P, r_{o}^{\prime} P\right)$. If $t_{o}^{\prime} \neq t_{o}$ (or $r_{o}^{\prime} \neq r_{o}$ ), $\mathcal{A}_{1}$ must send the value $t_{o}^{\prime}$ (or $r_{o}^{\prime}$ ) to $\mathfrak{C}, \mathfrak{C}$ obtains the value $t_{o}^{\prime} \quad$ (or $d_{o}^{\prime}$ ) by running SV-Set (or PPK-Extraction) algorithm, and finally outputs a delegation $\delta$ by running Delegate algorithm.
3. $I D_{o}=I D^{\diamond}$, then $\mathbb{C}$ fails.

- PS-Query: When $\mathcal{A}_{1}$ submits a tuple $\left(m, m_{w}, \delta\right), \mathfrak{C}$ gerenates a signature as follows:

1. $I D_{p} \neq I D^{\diamond}$ and $I D_{p} \notin L_{R}, \mathfrak{C}$ outputs a proxy signature $\sigma$ by running Proxy Sign algorithm.
2. $I D_{p} \in L_{R}$, then $P K_{p}=\left(t_{p} P, r_{p} P\right)$ has been replaced by $P K_{p}^{\prime}=\left(t_{p}^{\prime} P, r_{p}^{\prime} P\right)$. If $t_{p}^{\prime} \neq t_{p}$ (or $\left.r_{p}^{\prime} \neq r_{p}\right)$, $\mathcal{A}_{1}$ must send the value $t_{p}^{\prime}$ (or $r_{p}^{\prime}$ ) to $\mathfrak{C}, \mathfrak{C}$ then obtains the value $t_{p}^{\prime}$ (or $d_{p}^{\prime}$ ) by running SV-Set (or PPK-Extract) algorithm, and finally outputs a proxy signature $\sigma$ by running Proxy Sign algorithm.
3. $I D_{p}=I D^{\diamond}$, then $\mathbb{C}$ fails.

Forge. $\mathcal{A}_{1}$ outputs a tuple $\left(m_{w}^{*}, \delta^{*}\right)$ or $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$. $\mathfrak{C}$ aborts if the output does not satisfy any of the cases in Game
I. Otherwise, $\mathfrak{C}$ resolves the $C A A$ example as follows:

- Case 1. The final output is $\left(m_{w}^{*}, \delta^{*}\right)$ and fulfills the conditions of Case 1 in Game I.

In fact, $\delta^{*}$ is a signature on the warrant $m_{w}^{*}$, then $\delta^{*}$ $=Y^{*}=\frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P \quad$. If $\quad I D_{o}^{*}=I D^{\diamond} \quad$, then $I D_{o}^{*}=I D_{j}, P K_{o}^{*}=P K_{j}$, namely,
$\left(t_{o}^{*} P, r_{o}^{*} P\right)=\left(t_{j} P, a P\right) . \mathfrak{C}$ finds $t_{j}$ in the list $L_{U}$, computes $\quad h^{*}=H_{2}\left(m_{w}^{*}, I D_{o}^{*}, P K_{o}^{*}\right), \quad k_{o}^{*}=H_{1}\left(I D_{o}^{*}, a P\right)$ and $c=k_{o}^{*} x+h^{*} t_{o}^{*} \quad\left(\right.$ where $\left.t_{o}^{*}=t_{j}\right), \quad$ outputs a solution of the $C A A$ example in the end.

$$
\begin{aligned}
& \left(c, \delta^{*}\right)=\left(c, \frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P\right) \\
& =\left(c, \frac{1}{a+k_{o}^{*} x+h^{*} t_{o}^{*}} P\right)=\left(c, \frac{1}{a+c} P\right) .
\end{aligned}
$$

Probability. Let $q_{U}, q_{R}$ and $q_{D}$ be the number of UPKQuery, UPK-Replacement and PPK-Query , respectively.

Some notations are defined as follows.
$\pi_{1}: \mathcal{A}_{1}$ did not make PPK-Query on $I D_{o}^{*}$, nor did make UPK-Replacement on it.
$\pi_{2}: \mathfrak{C}$ did not fail in Delegation-Query and PS-Query.
$\pi_{3}: I D_{o}^{*}=I D^{\diamond}$.
It is a reasonable assumption that $L_{R} \cap L_{D}=\varnothing$. Hence it is not difficult to obtain the following results:

$$
\begin{aligned}
& \quad \operatorname{Pr}\left[\pi_{1}\right]=\frac{q_{U}-q_{R}-q_{D}}{q_{U}}, \operatorname{Pr}\left[\pi_{2} \mid \pi_{1}\right]=1-\frac{1}{q_{U}} \\
& \operatorname{Pr}\left[\pi_{3} \mid \pi_{1} \wedge \pi_{2}\right]=\frac{1}{q_{U}-q_{R}-q_{D}} \\
& \quad \operatorname{Pr}[\mathbb{C} \text { success }]=\operatorname{Pr}\left[\pi_{1} \wedge \pi_{2} \wedge \pi_{3}\right] \\
& \quad=\operatorname{Pr}\left[\pi_{1}\right] \cdot \operatorname{Pr}\left[\pi_{2} \mid \pi_{1}\right] \cdot \operatorname{Pr}\left[\pi_{3} \mid \pi_{1} \wedge \pi_{2}\right] \\
& \\
& =\frac{q_{U}-q_{R}-q_{D}}{q_{U}} \cdot\left(1-\frac{1}{q_{U}}\right) \cdot \frac{1}{q_{U}-q_{R}-q_{D}} \\
& \quad \approx \frac{1}{q_{U}}
\end{aligned}
$$

Therefore, $\mathfrak{C}$ can resolve the $C A A$ example with the probability $\frac{\varepsilon}{q_{U}}$ if $\mathcal{A}_{1}$ can succeed with the probability $\varepsilon$

- Case 2. The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and fulfills the conditions of Case 2 in Game I.

In fact, $\sigma^{*}=\left(Y^{*}, Z^{*}\right)$ is a proxy signature on the tuple $\left(m^{*}, m_{w}^{*}\right)$, then $Y^{*}=\frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P$
. If $I D_{o}^{*}=I D^{\diamond}$, then $I D_{o}^{*}=I D_{j}, P K_{o}^{*}=P K_{j}$, namely, $\left(t_{o}^{*} P, r_{o}^{*} P\right)=\left(t_{j} P, a P\right) . \mathfrak{C}$ finds $t_{j}$ in the list $L_{U}$, computes $k_{o}^{*}=H_{1}\left(I D_{o}^{*}, a P\right) \quad, \quad h^{*}=H_{2}\left(m_{w}^{*}, I D_{o}^{*}, P K_{o}^{*}\right) \quad$ and
$c=k_{o}^{*} x+h^{*} t_{o}^{*} \quad$ (where $t_{o}^{*}=t_{j}$ ), outputs a solution of the $C A A$ example in the end.

$$
\begin{aligned}
& \left(c, Y^{*}\right)=\left(c, \frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P\right) \\
& =\left(c, \frac{1}{a+k_{o}^{*} x+h^{*} t_{o}^{*}} P\right)=\left(c, \frac{1}{a+c} P\right)
\end{aligned}
$$

Probability. Same as that in Case 1.

- Case 3. The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and fulfills the conditions of Case 3 in Game I.

In fact, $\sigma^{*}=\left(Y^{*}, Z^{*}\right)$ is a proxy signature on the tuple $\left(m^{*}, m_{w}^{*}\right)$, then $Z^{*}=\frac{1}{r_{p}^{*}+k_{p}^{*} x+l^{*} t_{p}^{*}} P$
. If $I D_{p}^{*}=I D^{\diamond}$, then $I D_{p}^{*}=I D_{j}, P K_{p}^{*}=P K_{j}$, namely, $\left(t_{p}^{*} P, r_{p}^{*} P\right)=\left(t_{j} P, a P\right)$. $\mathfrak{C}$ finds $t_{j}$ in the list $L_{U}$, computes $k_{p}^{*}=H_{1}\left(I D_{p}^{*}, a P\right) \quad, \quad l^{*}=H_{3}\left(m^{*}, m_{w}^{*}\right.$, $Y^{*}, I D_{o}^{*}, P K_{o}^{*}, I D_{p}^{*}, P K_{p}^{*}$ ) and $\quad c=k_{p}^{*} x+l^{*} t_{p}^{*} \quad$ (where $t_{p}^{*}=t_{j}$ ), outputs a solution of the CAA example in the end.

$$
\begin{aligned}
& \left(c, Z^{*}\right)=\left(c, \frac{1}{r_{p}^{*}+k_{p}^{*} x+l^{*} t_{p}^{*}} P\right) \\
& =\left(c, \frac{1}{a+k_{p}^{*} x+l^{*} t_{p}^{*}} P\right)=\left(c, \frac{1}{a+c} P\right)
\end{aligned}
$$

Probability. Same as that in Case 1.
Theorem 2. If the $C A A$ problem is tricky, the scheme is unforgeable against an adversary $\mathcal{A}_{2}$ in SM.

Proof. Suppose that the challenger $\mathfrak{C}$ want to solve an instance of the $C A A$ problem $(P, a P)$, he does as follows.

Initialization. $\mathfrak{C}$ runs the Setup algorithm with a parameter $v$, then gives $\mathcal{A}_{2}$ the

$$
\text { params }=\left\{G_{1}, G_{2}, q, \hat{e}, P, P_{p u b}=x P, E, H_{1}, H_{2}, H_{3}\right\}
$$

$m s k=\{x\}$
Queries. $\mathcal{A}_{2}$ will first perform UPK-query for each identity..

- UPK-Query: $\mathfrak{C}$ maintains a list $L_{U}$ of tuple $\left(I D_{i}, t_{i}, r_{i}\right)$.When $\mathcal{A}_{2}$ inputs an identity $I D_{i}, \mathfrak{C}$ does as follows:

1. For $i=j$, selects at random $r_{j} \in Z_{q}^{*}$, sets $I D_{j}=I D^{\circ}$, returns $P K_{j}=P K^{\diamond}=\left(a P, r_{j} P\right)$, then stores the tuple $\left(I D_{j}, \diamond, r_{j}\right)$ in the list $L_{U}$
2. For $i \neq j$, selects at random $t_{i}, r_{i} \in Z_{q}^{*}$ and returns $P K_{i}=\left(t_{i} P, r_{i} P\right)$, then stores the tuple $\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$ 。

- UPK-Replacement: Same as that in the Theorem 1.
- PPK-Query: When $\mathcal{A}_{2}$ submits an identity $I D_{i}$, $\mathfrak{C}$ finds
$\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$, gives the $D_{i}$ by running the PPK-Extract algorithm.
- SV-Query: $\mathfrak{C}$ maintains a list $L_{E}$ of tuple $\left(I D_{i}, t_{i}\right)$. When $\mathcal{A}_{2}$ submits an identity $I D_{i}$, If $I D_{i}=I D^{\diamond}, \mathfrak{C}$ fails. Otherwise, $\mathfrak{C}$ finds $\left(I D_{i}, t_{i}, r_{i}\right)$ in the list $L_{U}$, responds with $t_{i}$, then adds $\left(I D_{i}, D_{i}\right)$ to the list $L_{E}$.
- Delegation-Query: Same as that in the Theorem 1.
- PS -Query: Same as that in the Theorem 1.

Forge. $\mathcal{A}_{2}$ outputs a tuple $\left(m_{w}^{*}, \delta^{*}\right)$ or $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$. $\mathfrak{C}$ aborts if the output does not satisfy any of the cases in Game II. Otherwise, $\mathfrak{C}$ resolves the $C A A$ example as follows:

- Case 1. The final output is $\left(m_{w}^{*}, \delta^{*}\right)$ and fulfills the conditions of Case 1 in Game II.

In fact, $\delta^{*}$ is a signature on the warrant $m_{w}^{*}$, then $\delta^{*}$ $=Y^{*}=\frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P$. If $I D_{o}^{*}=I D^{\diamond}$, then $I D_{o}^{*}=I D_{j}$, $P K_{o}^{*}=P K_{j}$ namely, $\left(t_{o}^{*} P, r_{o}^{*} P\right)=\left(a P, r_{j} P\right) . \mathfrak{C}$ finds $r_{j}$ in the list $L_{U}$, computes $\quad h^{*}=H_{2}\left(m_{w}^{*}, I D_{o}^{*}, P K_{o}^{*}\right), \quad k_{o}^{*}=$ $H_{1}\left(I D_{o}^{*}, a P\right)$ and $c=h^{*-1}\left(r_{o}^{*}+k_{o}^{*} x\right) \quad\left(\right.$ where $\left.\quad r_{o}^{*}=r_{j}\right)$, outputs a solution of the $C A A$ example in the end.

$$
\begin{aligned}
& \left(c, h^{*} \delta^{*}\right)=\left(c, \frac{h^{*}}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P\right) \\
& =\left(c, \frac{h^{*}}{r_{o}^{*}+k_{o}^{*} x+h^{*} a} P\right)=\left(c, \frac{1}{h^{*-1}\left(r_{o}^{*}+k^{*} x\right)+a} P\right) \\
= & \left(c, \frac{1}{c+a} P\right)
\end{aligned}
$$

Probability. Let $q_{U}, q_{R}$ and $q_{E}$ be the number of UPKQuery, UPK-Replacement and SV-Query, respectively. Some notations are defined as follows.
$\pi_{1}: \mathcal{A}_{2}$ did not make SV-Query on $I D_{o}^{*}$, nor did make UPK-Replacement on it.
$\pi_{2}: \mathfrak{C}$ did not fail during Delegation-Query and PS-Query.
$\pi_{3}: \quad I D_{o}^{*}=I D^{\diamond}$.
It is a reasonable assumption that $L_{R} \cap L_{E}=\varnothing$. Hence it is not difficult to obtain the following results:

$$
\begin{aligned}
& \operatorname{Pr}\left[\pi_{1}\right]=\frac{q_{U}-q_{R}-q_{E}}{q_{U}}, \operatorname{Pr}\left[\pi_{2} \mid \pi_{1}\right]=1-\frac{1}{q_{U}} \\
& \operatorname{Pr}\left[\pi_{3} \mid \pi_{1} \wedge \pi_{2}\right]=\frac{1}{q_{U}-q_{R}-q_{E}}
\end{aligned}
$$

$\operatorname{Pr}\left[\begin{array}{ll}\mathbb{C} & \text { success }\end{array}\right]=\operatorname{Pr}\left[\pi_{1} \wedge \pi_{2} \wedge \pi_{3}\right]$
$=\operatorname{Pr}\left[\pi_{1}\right] \cdot \operatorname{Pr}\left[\pi_{2} \mid \pi_{1}\right] \cdot \operatorname{Pr}\left[\pi_{3} \mid \pi_{1} \wedge \pi_{2}\right]$
$=\frac{q_{U}-q_{R}-q_{E}}{q_{U}} \cdot\left(1-\frac{1}{q_{U}}\right) \cdot \frac{1}{q_{U}-q_{R}-q_{E}}$
$\approx \frac{1}{q_{U}}$
Therefore, $\mathfrak{C}$ can resolve the $C A A$ example with the
probability $\varepsilon / q_{U}$ if $\mathcal{A}_{2}$ can succeed with the probability $\varepsilon$.

- Case 2. The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and fulfills the conditions of Case 2 in Game II.

In fact, $\sigma^{*}=\left(Y^{*}, Z^{*}\right)$ is a proxy signature on the tuple $\left(m^{*}, m_{w}^{*}\right)$, then $Y^{*}=\frac{1}{r_{o}^{*}+k_{o}^{*} x+h^{*} t_{o}^{*}} P$

If $I D_{o}^{*}=I D^{\diamond}$, then $I D_{o}^{*}=I D_{j}, P K_{o}^{*}=P K_{j}$, namely
, $\left(t_{o}^{*} P, r_{o}^{*} P\right)=\left(a P, r_{j} P\right)$. $\mathfrak{C}$ finds $r_{j}$ in the list $L_{U}$, computes $\quad h^{*}=H_{2}\left(m_{w}^{*}, I D_{o}^{*}, P K_{o}^{*}\right), \quad k_{o}^{*}=H_{1}\left(I D_{o}^{*}, a P\right)$ and $\quad c=h^{*-1}\left(r_{o}^{*}+k_{o}^{*} x\right)\left(\right.$ where $\left.r_{o}^{*}=r_{j}\right)$, outputs a solution of the $C A A$ example in the end.

$$
\begin{aligned}
& \left(c, h^{*} Y^{*}\right)=\left(c, \frac{h^{*}}{r_{o}^{*}+k_{o}^{*} x+h^{*} a} P\right) \\
& =\left(c, \frac{1}{h^{*-1}\left(r_{o}^{*}+k_{o}^{*} x\right)+a} P\right)=\left(c, \frac{1}{a+c} P\right)
\end{aligned}
$$

Probability. Same as that in Case 1.

- Case 3. The final output is $\left(m^{*}, m_{w}^{*}, \sigma^{*}\right)$ and fulfills the conditions of Case 3 in Game II.

In fact, $\sigma^{*}=\left(Y^{*}, Z^{*}\right)$ is a proxy signature on the tuple $\left(m^{*}, m_{w}^{*}\right)$, then $Z^{*}=\frac{1}{r_{p}^{*}+k_{p}^{*} x+l^{*} t_{p}^{*}} P$. If $I D_{p}^{*}=I D^{\diamond}$, then $I D_{p}^{*}=I D_{j}, \quad P K_{p}^{*}=P K_{j}$, namely, $\left(t_{p}^{*} P, r_{p}^{*} P\right)=\left(a P, r_{j} P\right) . \mathfrak{C}$ finds $r_{j}$ in the list $L_{U}$, computes $k_{p}^{*}=H_{1}\left(I D_{p}^{*}, a P\right)$, $l^{*}=H_{3}\left(m^{*}, m_{w}^{*}, Y^{*}, I D_{o}^{*}, P K_{o}^{*}, I D_{p}^{*}, P K_{p}^{*}\right) \quad$ and $c=l^{*-1}\left(r_{p}^{*}+k_{p}^{*} x\right)$ (where $\left.r_{p}^{*}=r_{j}\right)$, outputs a solution of the $C A A$ example in the end.

$$
\begin{aligned}
& \left(c, l^{*} Z^{*}\right)=\left(c, \frac{l^{*}}{r_{p}^{*}+k_{p}^{*} x+l^{*} a} P\right) \\
& =\left(c, \frac{1}{l^{*-1}\left(r_{p}^{*}+k_{p}^{*} x\right)+a} P\right)=\left(c, \frac{1}{a+c} P\right)
\end{aligned}
$$

Probability. Same as that in Case 1.

## 7 Efficiency and Comparison

We compared the performance of the new scheme with the other two CLPS schemes. We listed the symbols in Table 2 that need to be used in this section.

Table 2. Notations

| $B_{P}$ | $S_{G_{1}}$ | $A_{G_{1}}$ | $M_{G_{2}}$ | $E_{G_{2}}$ | $H$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 5.427 | 2.165 | 0.013 | 0.001 | 0.339 | 0.007 |

We used third-party data to analyze the performance of the three CLPS schemes. The running times on basic cryptographic operations are listed in Table 3. In order to
accomplish 1024-bit RSA security level, the hardware/software parameters used in the experiments [22] are as follows: cryptographic library (MIRACL) and a computer (Dell with an $55-4460 \mathrm{~S} 2.90 \mathrm{GHz}$ processor, 4 G bytes memory and the Window 8 operating system), a Tate pairing $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$ and $G_{1}$ with order $q$ is an additive group over a super singular curve $E / E_{p}: y^{2}=x^{3}+1$, where $p$ and $q$ are 512-bits prime number and 160 -bits prime number, respectively.

We used a simple, intuitive method to evaluate the computational efficiency of the three schemes, as shown in Figure 1. Without loss of generality, it is assumed that the size of the identity information of a user is 60 bits, and the size of the output of a hash function (SHA1) is 160 bits. Namely, $\left|B_{I D}\right|=n=60$ (bits) and $\left|B_{m}\right|=\left|B_{w}\right|=u=160$ (bits). Further,

$$
\left|\Delta_{I D}\right|=\frac{n}{2}=30 \quad \text { (bits) and } \quad\left|\Delta_{m}\right|=\left|\Delta_{w}\right|=\frac{u}{2}=80
$$

(bits).

In Delegation, the scheme [17] requires 3 scale multiplication operations in $G_{1}, 2$ addition operations in $G_{1}$ and 1 hash function opretions. In Delegation-Verify, it requires 4 pairing operations, 1 scale multiplication operations in $G_{1}, \frac{n+1}{2}$ addition operations in $G_{1}, 2$ multiplication operations in $G_{2}$ and 2 hash function opretions. In Proxy-Sign, it requires 6 scale multiplication operations in $G_{1}, 6$ addition operations in $G_{1}$ and 2 hash function opretions. In PS-Verify, it requires 5 pairing operations, $n+2$ addition operations in $G_{1}$, and 2 scale multiplication operations in $G_{1}$, 5 multiplication operation in $G_{2}$ and 2 hash function opretions. So the resulting computation time is $5.427 \times 9+2.165 \times 12+$

$$
0.013 \times\left(\frac{3 n}{2}+11\right)+0.001 \times 7+0.007 \times 6=0.013
$$

$\times \frac{3 n}{2}+75.015$. When $n=60$, the computation time is $0.013 \times \frac{3 \times 60}{2}+75.015=76.185 \mathrm{~ms}$.

Table 3. Operation time (in milliseconds)

| Symbol | Meaning |
| :---: | :--- |
| $B_{I D}$ | A bit string of the identity information of a <br> user, where $\left\|B_{I D}\right\|=n$ |
| $B[i]_{I D}$ | The ith bit of the identity information of a user <br> $\Delta_{I D}$ |
|  | A set of indices $i$ such that $B[i]_{I D}=1$, <br> namely $\Delta_{I D}=\left\{i: B[i]_{I D}=1\right\}$ |
| $B_{w}$ | The output bit string of a warrant hush <br> function, where $\left\|B_{w}\right\|=u$ |
| $B[i]_{w}$ | The ith bit of a warrant |
| $\Delta_{w}$ | A set of indices $i$ such that $B[i]_{w}=1$, <br> namely $\Delta_{w}=\left\{i: B[i]_{w}=1\right\}$ |
| $B_{m}$ | The output bit string of a message hush <br> function, where $\left\|B_{m}\right\|=u$ |

$B[i]_{m} \quad$ The $\mathrm{i}^{\text {th }}$ bit of a message
$\Delta_{m} \quad$ A set of indices $i$ such that $B[i]_{m}=1$, namely $\Delta_{m}=\left\{i: B[i]_{m}=1\right\}$
$B_{P} \quad$ A pairing operation
$S_{G_{1}} \quad$ A scale multiplication operation in $G_{1}$
$A_{G_{1}} \quad$ A addition operation in $G_{1}$
$M_{G_{2}} \quad$ A multiplication operation in $G_{2}$
$E_{G_{2}} \quad$ An exponentiation operation in $G_{2}$
$H \quad$ A hash function operation
$\left|G_{1}\right| \quad$ An element in $G_{1}$
$\left|G_{2}\right| \quad$ An element in $G_{2}$
$\left|Z_{q}^{*}\right| \quad$ An element in $Z_{q}^{*}$

In Delegation, the scheme [18] requires 4 scale multiplication operations in $G_{1}$ and $\frac{u}{2}+1$ addition operations in $G_{1}$ and 1 hash function opretions. In DelegationVerify, it requires 3 pairing operations, $\frac{n+u}{2}$ addition operations in $G_{1}, 3$ multiplication operations in $G_{2}$ and 1 hash function opretions. In Proxy-Sign, it requires 10 scale multiplication operations in $G_{1}, n+u+8$ addition
operations in $G_{1}$ and 4 hash function opretions. In PSVerify, it requires 5 pairing operations, $n+u$ addition operations in $G_{1}$, 5 multiplication operation in $G_{2}$ and 4 hash function opretions. So the resulting computation time is $5.427 \times 8+2.165 \times 14+0.013 \times\left(\frac{5}{2} n+3 u+9\right)$
$+0.001 \times 8+0.007 \times 10=0.013 \times\left(\frac{5}{2} n+3 u+9\right)+73.804 \mathrm{~ms}$. When $n=60, u=160$, the computation time is
$0.013 \times\left(\frac{5 \times 60}{2}+3 \times 160+9\right)+73.804=82.111 \mathrm{~ms}$.
In Delegation, new scheme requires 1 scale function opretions. In Delegation-Verify, it requires 1 pairing operations, 2 scale multiplication operations in $G_{1}, 2$ addition operations in $G_{1}$ and 1 hash function opretions. In Proxy-Sign, it requires 1 scale multiplication operations in $G_{1}$ and 1 hash function opretions. In PS-Verify, it requires 2 pairing operations, 4 scale multiplication operations in $G_{1}$, 4 addition operations in $G_{1}$ and 4 hash function opretions. So the resulting computation time is $5.427 \times 3+2.165 \times 8+0.013 \times 6+0.007 \times 8=33.735 \mathrm{~m}$.

We listed the computation costs for the three CLPS schemes in Table 4.

Table 4. Comparison of three CLPS schemes

| Scheme | Lu [17] | Ming [18] | Our scheme |
| :---: | :---: | :---: | :---: |
| Delegate | $3 S_{G_{1}}+2 A_{G_{1}}+H$ | $4 S_{G_{1}}+\left(\frac{u}{2}+1\right) A_{G_{1}}+H$ | $S_{G_{1}}+H$ |
| Delegation- <br> Verify | $\begin{aligned} & 4 B_{P}+\left(\frac{n}{2}+1\right) A_{G_{1}} \\ + & S_{G_{1}}+2 M_{G_{2}}+H \end{aligned}$ | $\begin{gathered} 3 B_{P}+\left(\frac{n+u}{2}\right) A_{G_{1}} \\ +3 M_{G_{2}}+H \end{gathered}$ | $\begin{array}{r} B_{P}+2 S_{G_{1}} \\ +2 A_{G_{1}}+2 H \end{array}$ |
| Time $(n=60, u=160)$ | $\begin{gathered} 0.013 \times \frac{n}{2}+30.423 \\ (30.813) \\ \hline \end{gathered}$ | $\begin{gathered} 0.013 \times\left(\frac{n}{2}+u\right)+24.971 \\ (27.441) \\ \hline \end{gathered}$ | 11.969 |
| Proxy-Sign | $6 S_{G_{1}}+6 A_{G_{1}}+2 H$ | $10 S_{G_{1}}+(n+u+8) A_{G_{1}}+4 H$ | $S_{G_{1}}+H$ |
| PS-Verify | $\begin{array}{r} 5 B_{P}+(n+2) A_{G_{1}} \\ +2 S_{G_{1}}+5 M_{G_{2}}+2 H \end{array}$ | $\begin{aligned} & 5 B_{P}+(n+u) A_{G_{1}} \\ & +5 M_{G_{2}}+4 H \end{aligned}$ | $\begin{gathered} 2 B_{P}+4 S_{G_{1}}+4 A_{G_{1}} \\ +4 H \end{gathered}$ |
| Time ( $n=60, u=160$ ) | $\begin{gathered} 0.013 n+44.592 \\ (45.372) \\ \hline \end{gathered}$ | $\begin{gathered} 0.013 \times(2 n+2 u)+48.95 \\ (54.670) \\ \hline \end{gathered}$ | 21.766 |
| Total Time $(n=60, u=160)$ | $\begin{gathered} 0.013 \times \frac{3 n}{2}+75.015 \\ (76.185) \\ \hline \end{gathered}$ | $\begin{gathered} 0.013 \times\left(\frac{5 n}{2}+3 u\right)+73.921 \\ (82.111) \end{gathered}$ | 33.735 |
| Size of Params ( $n=60, u=160$ ) | $\begin{array}{r} (n+8)\left\|G_{1}\right\| \\ \text { (4352 bytes) } \end{array}$ | $\begin{aligned} & (n+2 u+5)\left\|G_{1}\right\| \\ & \text { (24640 bytes) } \end{aligned}$ | $\begin{aligned} & 2\left\|G_{1}\right\|+\left\|G_{2}\right\| \\ & \text { (192 bytes) } \end{aligned}$ |
| Size of MSK | $\left\|G_{1}\right\| \quad(64$ bytes $)$ | $\left\|G_{1}\right\| \quad(64$ bytes) | $\left\|Z_{q}^{*}\right\|(20$ bytes $)$ |
| Size of Signature | $5\left\|G_{1}\right\| \quad(320$ bytes $)$ | $5\left\|G_{1}\right\| \quad(320$ bytes $)$ | $2\left\|G_{1}\right\|$ (128bytes) |



Figure 1. Computation cost
Follow on, we evaluated the size of system parameters, the size of master key and the size of signature, as shown in Figure 2. In the scheme [17], the system parameters contain $n+8$ points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $[(60+8) \times 512] / 8=4352$ bytes. The master secret key contains one point over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $512 / 8=64$ bytes. The signature contains five points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $(5 \times 512) / 8=320$ bytes. In the scheme [18], the system parameters contain $n+2 u+5$ points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1 \quad$ thus the size is $[(60+320+5) \times 512] / 8=24640$ bytes. The master secret key contains one point over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $512 / 8=64$ bytes. The signature contains five points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $(5 \times 512) / 8=320$ bytes. In our scheme, the system parameters contain three points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $[3 \times 512] / 8=192$ bytes. The master secret key contains one point over $Z_{q}^{*}$, thus the size is $160 / 8=20$ bytes. The signature contains two points over an elliptic curve $E / E_{p}: y^{2}=x^{3}+1$, thus the size is $(2 \times 512) / 8=128$ bytes.


With the continuous advancement of network technology, electronic signatures are widely used in various scenarios, such as e-commerce, electronic voting, and remote access
control. The original signer may be inconvenient to generate a signature in certain situations (for example sickness, imprisonment, etc.), and he/she can authorize a trustworthy person to exercise the right to sign. In the new sheme, the proxy signer generates a new signature with his own private key and delegation generated by the original signer, then sends it to the receiver. Verifying the signature, the receiver can determine whether the signature and delegation are valid. The new scheme requires only three pairing operations and enjoys lower computation cost.

## 8 Conclusion

It was in ROM that most known CLPS schemes is proved to be secure. There are only two CLPS schemes with provably secure in SM. In ROM, the hash function value obtained by the adversary is provided by the challenger, rather than by a real function computation. A cryptography scheme that has been proven to be secure in ROM is not necessarily secure in real-world applications. In this paper, we constructed a new CLPS scheme and showed the security proofs in SM. In the scheme, it was constant that the size of system parameters and
the size of master secret key, it was constant that the number of three kinds of operations (addition, scalar multiplication and pairing). It was shown that the proposed scheme is more efficient and suitable for mobile computing.

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