A Deep Learning Based Equalization Scheme for Bandwidth-compressed Non-orthogonal Multicarrier Communication

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Abstract

Spectrally efficient frequency division multiplexing (SEFDM) is a bandwidth-compressed non-orthogonal multicarrier communication scheme, which provides improved spectral efficiency compared to orthogonal frequency division multiplexing (OFDM) system. The loss of orthogonality yields the self-introduced inter-carrier interference (ICI) complicating the equalizer design. In this work, a deep learning (DL) -based SEFDM equalization scheme is proposed to characterize the ICI and to detect the transmitted information bits. The DL-based equalization scheme is trained offline using randomly-generated data and then deployed online. The performance of the equalization scheme is tested by extensive numerical simulations. The results show that the proposed equalization scheme outperforms the linear equalization based equalization scheme, such as zero forcing (ZF), minimum mean squared error (MMSE) and truncated singular value decomposition (TSVD), under additive white Gaussian noise (AWGN) channel in terms of the bit-error rate (BER). Especially for BPSK, the uncoded BER performance approaches the traditional OFDM even for the compression ratio of 0.7, which saves the bandwidth by 30%.

Keywords: Deep neural networks, Equalization scheme, Non-orthogonal signal, Bandwidth-compressed multicarrier

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a successful approach to achieve high-speed data transmission, which is widely adopted multicarrier modulation scheme in wireless broadband communication nowadays. In OFDM, the complex information symbols are modulated onto orthogonal subcarriers, where the frequency separation of subcarriers equals to the OFDM symbol rate. To improve the transmission rate and spectral efficiency in OFDM, higher order modulation is required at the cost of higher signal-to-noise-ratio (SNR). Further, the transmission is more sensitive to the channel impairments such as the multi-path propagation or the nonlinear effects.

In 1975, Mazo's work [1] presented that the data symbols could be transmitted at a rate 25% faster than the Nyquist rate while maintaining optimum performance [2]. Such spectral efficiency improvement method (Faster-than-Nyquist signaling (FTNS)) was later extended to both time and frequency domains by Rusek et.al [3], and a time-frequency compressed single carrier FTNS (TFC-SC-FTNS) scheme to improve the spectral efficiency via two dimensions simultaneously proposed by S. Wen et al. [4]. In 2003, a new non-orthogonal multi-carrier scheme, termed spectrally efficient frequency division multiplexing (SEFDM), was proposed to improve the spectral efficiency by reducing the spacing between the subcarriers [5]. As an underlying technique, SEFDM is required in future communication networks to address the coming challenges in Internet of Vehicle (IoV) and Smart City Applications [6-7]. However, due to the violation of Nyquist principle, such systems suffer from the self-introduced inter-carrier interference (ICI), which complicates the equalization scheme design. The maximum-likelihood (ML) detection achieves the optimum system performance, but facing an NP-hard problem due to the exponentially growing complexity [8], and it was shown not computationally feasible [5]. On the other hand, with much reduced complexity, the classical linear equalization methods, including zero forcing (ZF), minimum mean squared error (MMSE) [9], truncated singular value decomposition (TSVD) [10], were utilized to deal with the ICI-spoiled symbols, but failing to provide competitive BER performance for moderate bandwidth savings or number of

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subcarriers. A detailed view of the history of SEFDM could be found in [11].

1.1 Related Work

Deep learning (DL) and deep neural networks (DNNs) are efficient tools to model complex problems. DL has shown its great potentials in the areas of computer vision, natural language processing, speech recognition and so on [12]. Also, DL is a promising approach for such communication systems, in which the implementation complexity is extremely high, or the mathematical models of some communication modules (e.g., channel model) are hard to build up. H. Ye et. al employed DL to channel estimation and symbol detection in OFDM system [13], showing advantages over the conventional methods, e.g., MMSE. It is established in [14], that the transmitted symbols can be recovered from the inter-symbol interference (ISI) - corrupted received signal by DL method. [15] gives an overview on DL and a comprehensive overview on the application of deep learning for the physical layer can be found in [16]. Inspired by the conclusion that DNN can learn complex interference features using back propagation mechanism. We choose DNN as the network architecture to implement the equlization scheme because it connects all the input neurons and can consider the effects from both adjacent neurons and non-adjacent neurons [17].

1.2 Contribution

The contribution of this paper can be summarized as follows:

We proposed a DL-based equalization scheme, combining the equalizer and symbol demapper to handle the ICI and correlated noise in SEFDM system.

The proposed equalization scheme can learn the features from raw data automatically instead of manual extraction. With the a priori knowledge of communication and DL, such as the output range of the activation function, the DNN structure has been tailored, obtaining four types of DNN based equalization scheme for further performance improvement.

·We have carried out comprehensive evaluations to verify and analyze the proposed DL-based equalization scheme for bandwidth-compressed multicarrier Communication. The simulation results show that our equalization scheme achieves excellent BER performance, compared with the linear equalization schemes, and very close to that of the BPSK-modulated OFDM without ICI on the additive white Gaussian noise (AWGN) channel.

The rest of this paper is organized as follows. Sec. 2 introduces the SEFDM system and describes the linear equalization based equalization scheme. The DL principles are briefly introduced in Sec. 3, followed by developing the DL equalization scheme. In Section 4, numerical results are presented to verify the proposed scheme. Finally, some conclusions are drawn in Section 5.

2 Traditional Equalization Scheme for Bandwidth-compressed Multicarrier Communication

2.1 The Principle of SEFDM Signaling

SEFDM is a bandwidth compressed non-orthogonal multicarrier communication technique that can be viewed as a subcarrier spacing compressed version of OFDM. In an OFDM system with N subchannels, the high-speed serial data stream is divided into N low-speed parallel streams, which modulated on a group of mutual orthogonal subcarriers. The above modulation behavior can be performed by inverse discrete Fourier transform (IDFT). Therefore, the OFDM signal can be expressed as

$$x_{OFDM}\left(t\right)=\frac{1}{\sqrt{T}}\sum_{l=-\infty}^{+\infty}\sum_{n=0}^{N-1}S_{l,n}\exp\left(\frac{j2\pi n\left(t-lT\right)}{T}\right), \quad \textbf{(1)}$$

where $S_{l,n}$ (being l the time index and n the carrier index) are the transmitted symbols and the subcarrier spacing is $\Delta f=1/T$. As to SEFDM, the spacing is compressed to α/T, where α is called the bandwidth compression factor (BCF) and $0<\alpha\leq1$, Consequently, the bandwidth saving from SEFDM is $\left(1-\alpha\right)\times100\%$. Then, the transmitted signal (1) is changed as

$$x_{SEFDM}\left(t\right)=\frac{1}{\sqrt{T}}\sum_{l=-\infty}^{+\infty}\sum_{n=0}^{N-1}S_{l,n}\exp\left(\frac{j2\pi n\alpha\left(t-lT\right)}{T}\right). \quad \textbf{(2)}$$

Without loss of generality, the first time slot is considered, i.e., $l=0$ and $0\leq t\leq T$. Meanwhile, it is convenient to deal with the discrete-time SEFDM signal obtained by sampling the continuous-time signal (2) at times $k\frac{T}{Q}$, where $Q=\rho N$ and ρ is the oversampling factor, typically equals to 1. Then the Q-point sequence $\left\{x\left[k\right]\right\}$ is given by

$$x\left[k\right]=\frac{1}{\sqrt{Q}}\sum_{n=0}^{Q-1}S_n\exp\left(\frac{j2\pi n\alpha k}{Q}\right) \quad \textbf{(3)}$$

where $0\leq k\leq Q-1$ and $1/\sqrt{Q}$ is the scaling factor for normalization. In matrix notation, (3) is written as

$$\boldsymbol{X}=\boldsymbol{\Phi S} \quad \textbf{(4)}$$

where $\boldsymbol{S}$ is the symbol vector of N samples, and $\boldsymbol{X}$ is the discrete-time SEFDM signal of Q samples. $\boldsymbol{\Phi}$ is a $Q \times N$ matrix, i.e.

$$\boldsymbol{\Phi} = \frac{1}{\sqrt{Q}} \cdot \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1\times1} & \omega^{1\times2} & \cdots & \omega^{1\times(N-1)} \\ 1 & \omega^{2\times1} & \omega^{2\times2} & \cdots & \omega^{2\times(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{(Q-1)\times1} & \omega^{(Q-1)\times2} & \cdots & \omega^{(Q-1)\times(N-1)} \end{pmatrix} \quad (5)$$

where, $\omega = \exp(j2\pi\alpha/Q)$.

Due to the violation of orthogonality, SEFDM signals cannot be generated directly by conventional IDFT, but can be generated as a sum of multiple IDFT outputs [18]. In our model, the SEFDM signal is transmitted through the AWGN channel. At the equalization scheme, the multiple DFTs is employed to demodulate the received signal [19], yielding the observation **R**,

$$\boldsymbol{R} = \boldsymbol{\Phi}^H \boldsymbol{X} + \boldsymbol{\Phi}^H \boldsymbol{Z} = \boldsymbol{\Phi}^H \boldsymbol{\Phi} \boldsymbol{S} + \boldsymbol{\Phi}^H \boldsymbol{Z} = \boldsymbol{C}\boldsymbol{S} + \boldsymbol{Z}_\omega \quad (6)$$

Where $[\cdot]^H$ denotes the conjugate transpose. $\boldsymbol{Z}_\omega$ is the vector of correlated noise samples. $\boldsymbol{C} = \boldsymbol{\Phi}^H \boldsymbol{\Phi}$ is the cross correlation coefficient matrix. The coefficients in correlation matrix are

$$\begin{aligned} c_{m,n} &= \frac{1}{Q}\sum_{k=0}^{Q-1} \exp\left(\frac{j2\pi\alpha km}{Q}\right) \cdot \exp\left(-\frac{j2\pi\alpha kn}{Q}\right) \\ &= \frac{1}{Q} \times \begin{cases} Q & , m = n \\ \dfrac{1-\exp\left(j2\pi\alpha(m-n)\right)}{1-\exp\left(j2\pi\alpha(m-n)/Q\right)}, & m \neq n \end{cases} \\ &= \begin{cases} 1 & , m = n \\ \dfrac{\exp\left(j\pi\alpha(m-n)\right)}{\exp\left(-j\pi\alpha(m-n)/Q\right)} \cdot \dfrac{\operatorname{sinc}\left(\alpha(m-n)\right)}{\operatorname{sinc}\left(\alpha(m-n)/Q\right)}, & m \neq n \end{cases} \end{aligned} \quad (7)$$

In the case of $\alpha = 1$, then $\boldsymbol{C} = \boldsymbol{I}$, i.e., no ICI introduced where it becomes the OFDM.

2.2 Linear Equalization Based Detection Scheme

Practically, the suboptimal linear equalization was developed to retrieve the transmitted symbols from the ICI-spoiled observation according to some criteria, e.g., ZF and MMSE. Hence, we give an illustration of SEFDM system employing linear equalization methods as depicted in Figure 1.

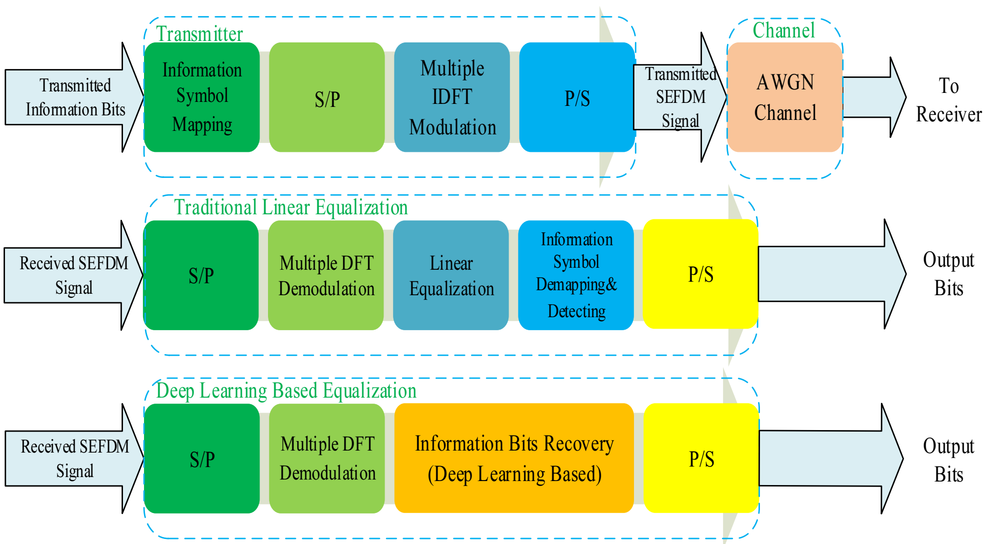

Figure 1. SEFDM system model based on linear equalization and DL-based equalization scheme

The ZF equalization can be expressed as

$$\hat{\boldsymbol{S}}_{ZF} = \boldsymbol{G}_{ZF}\boldsymbol{R} = (\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H\boldsymbol{R}, \quad (8)$$

where $\hat{\boldsymbol{S}}_{ZF}$ denotes the estimated symbol vector. The MMSE equalization is implemented as

$$\hat{\boldsymbol{S}}_{MMSE} = \boldsymbol{G}_{MMSE}\boldsymbol{R} = \boldsymbol{C}^H\left(\boldsymbol{C}\boldsymbol{C}^H + \frac{\sigma^2}{\sigma_S^2}\boldsymbol{I}\right)^{-1}\boldsymbol{R}. \quad (9)$$

where σ^2 and σ_s^2 denote the noise and signal power.

With the increase of the subcarrier number and the decrease of α, the correlation matrix $\boldsymbol{C}$ becomes ill-conditioned and singular [20]. To solve this problem, the pseudo inverse of $\boldsymbol{C}$ is used to produce a better quality [21-22]. The method is called the Truncated Singular Value Decomposition (TSVD), which discards the small singular values. First, a SVD of is performed as

$$\boldsymbol{C} = \boldsymbol{U}\Sigma\boldsymbol{V}^H \quad (10)$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrixes, i.e., $\boldsymbol{U}^H\boldsymbol{U}=\boldsymbol{I}$, $\boldsymbol{V}^H\boldsymbol{V}=\boldsymbol{I}$. $\Sigma = diag(\sigma_1,\sigma_2,\cdots,\sigma_N)$ is the diagonal matrix formed by the singular values of $\boldsymbol{C}$. Then the pseudo inverse of matrix $\boldsymbol{C}$ is given by

$$\boldsymbol{C}_\xi^{-1} = \boldsymbol{V}\Sigma_\xi^{-1}\boldsymbol{U}^H \tag{11}$$

Where $\boldsymbol{C}_\xi^{-1}$ indicates the pseudo inverse, and $\Sigma_\xi^{-1} = diag\left(\frac{1}{\sigma_1},\frac{1}{\sigma_2},\cdots,\frac{1}{\sigma_\xi},0,..,0\right)$ with ξ, being the truncation index. The optimal truncation index is given by [7]:

$$\xi = \lceil \alpha N \rceil + 1. \tag{12}$$

Where $\lceil \cdot \rceil$ denotes the celling operation. The TSVD method ignores the small singular values and forces their reciprocals to zero when computing Σ^{-1}. In this way, the TSVD equalization method avoids amplifying the input noise and, preventing the further degradation of the system performance. The estimated received symbols can be expressed as

$$\hat{\boldsymbol{S}}_{TSVD} = \boldsymbol{G}_{TSVD}\boldsymbol{R} = \boldsymbol{C}^{-1}\boldsymbol{R} \tag{13}$$

3 DL-based Equalization Scheme for Bandwidth-compressed Multicarrier Communication

In this section, we develop a new equalizer and demapper joint design based on DL for the SEFDM system. The system block is shown in Figure 1. After training, the DNN is utilized to retrieve the transmitted information bits.

3.1 Deep Learning Basics

DNNs are neural networks that have multiple layers of different perceptrons. The structure of DNN is shown in Figure 2.

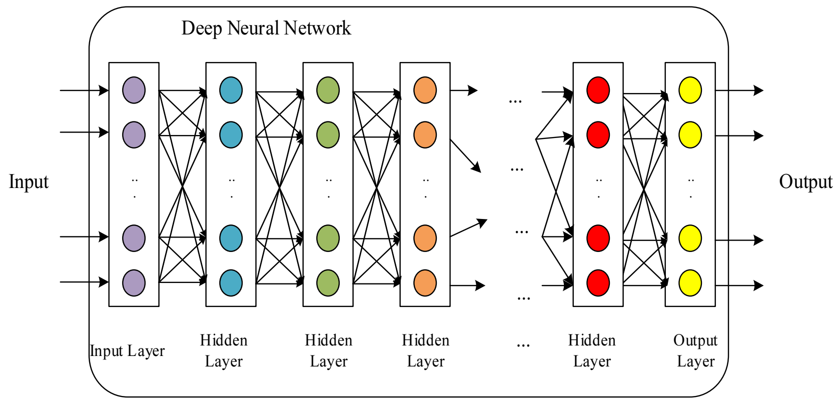

Figure 2. The DNN model

Each layer of the network consists of multiple neurons, with a nonlinear function alled activation function computing the weighted sum of the preceding layer. The activation function generally includes the Sigmoid function, the Relu function, or the tanh function, which are defined as $f_{Sigmoid}(a)=\frac{1}{1+e^{-a}}$,

$f_{Relu}(a)=\max(0,a)$ and $f_{\tanh}(a)=\tanh(a)=\frac{e^a-e^{-a}}{e^a+e^{-a}}$, respectively. Hence, the output of the DNN can be expressed as:

$$\begin{aligned}
\boldsymbol{Z}^{[1]} &= f^{[1]}\left(\boldsymbol{W}^{[1]}x+b^{[1]}\right)\\
\boldsymbol{Z}^{[2]} &= f^{[2]}\left(\boldsymbol{W}^{[2]}\boldsymbol{Z}^{[1]}+b^{[2]}\right)\\
&\dots\\
\boldsymbol{y}=\boldsymbol{Z}^{[L\text{-}1]} &= f^{[L\text{-}1]}\left(\boldsymbol{W}^{[L\text{-}1]}\boldsymbol{Z}^{[L-2]}+b^{[L\text{-}1]}\right).
\end{aligned} \tag{14}$$

where superscript $[l]$ denotes the quantity associated with the l-th layer; x and $\boldsymbol{y}$ are respectively the input and the output of the DNN; $\boldsymbol{Z}^{[l]}$ is the output of the l-th layer, and $f^{[l]}$ is the activation function. $\boldsymbol{W}^{[l]}$ and $b^{[l]}$ are respectively the weight matrix and bias vector. The DNN can be formulated as an abstract function:

$$y=\boldsymbol{Z}^{[L-1]} = f(x;\theta) = f^{[L-1]}\left(f^{[L-2]}\left(\cdots f^{[1]}(x)\right)\right) \tag{15}$$

where θ denotes the set of parameters in the neural network, which needs to be optimized before the online deployment.

3.2 DNN Based SEFDM Equalization Scheme Design

The observation $\boldsymbol{R}$ is The DNN based SEFDM equalization scheme architecture is also illustrated in

Figure 1. The observation $\boldsymbol{R}$ is As we know, the source bits are regularly assumed to be Bernoulli distributed, with values of 0 or 1. The transmitter modulates the information bits in proper forms to fit the channel. The purpose of the equalization scheme is to find a proper architecture to retrieve the information bit sequence. That is to say, the equalization scheme needs to recover the information bits from the observation while there exists different kinds of distortions, from the transmitter, the channel, and even the equalization scheme itself. The final output of the equalization scheme is the estimated bits, also with values of 0 or 1. Therefore, in communication system, the output of the DNN requires predicting the value of a binary variable. Sigmoid units are suitable for Bernoulli output distributions and classification problems with two classes can be cast in this form [9]. In the study, we adopt the Sigmoid function in the output layer of our developed DNN.

From (15), we design an abstract function, of which the input is from the received symbols and the output estimates are the information bits. Then we give the expression of the function:

$$\hat{\boldsymbol{X}} = f\left(\boldsymbol{R};\theta\right) \quad \textbf{(16)}$$

where $\hat{\boldsymbol{X}}$ is an estimate of the transmitted bit vector $\boldsymbol{X}$. From the definition of Sigmoid in Sec. III, we know that the Sigmoid returns values restricted to the open interval $(0,1)$. A simple design is thereby to train the model to minimize the difference between the output of the DNN, and the information bits from the source. In the designed DNN-based detector, hence, we use the mean squared error to portray the difference,

$$\mathcal{L}(\theta) = \left\|\hat{\boldsymbol{X}} - \boldsymbol{X}\right\|_2^2 \quad \textbf{(17)}$$

We perform the system training in a supervised manner by using a set of labeled data. The DNN parameter vector θ is iteratively updated via RMSPropOptimizer aimed at minimizing the loss $\mathcal{L}(\theta)$ over a mini-batch from the training set, given by (15). After the training, the output of the DNN is a vector of real numbers restricted in $(0,1)$ that are close to the original information bits. In the stage of the online deployment, it is reasonable to use a comparator gate with threshold of 0.5, to force output to 0 or 1, as the output bits. The received complex information symbols are divided into the real and imaginary parts, and then sent into the DNN. A detailed illustration of Information Bits Recover part in Figure 1 is given in Figure 3.

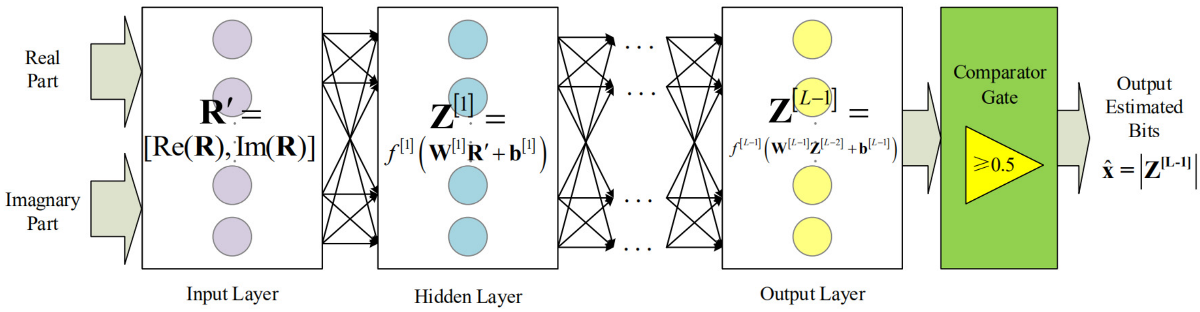

Figure 3. The implementation of the block of Information Bits Recovery

The number of layers and the number of neurons in each layer influence the system performance significantly. The DNN design relates to the SEFDM system configuration, such as the compression factor, the modulation constellation, the prototype filter. We present two kinds of DNNs which have different numbers of neurons. One is the *narrower* one there contains less neurons, and a *wider* one with more neurons.

Another aspect that affects the performance is the activation function. The activation function introduces the non-linearity to make DNNs possible to handle complex problems. Different activation functions have different effects on the DNN based equalization scheme because of the different output range. Since the information symbols are in normalized constellations, whose range in value is usually in $[-1,1]$. According to the definition in Sec. III-A of the Relu and tanh function, the output range of Relu is $[0,+\infty)$. As for tanh, the range is $(-1,+1)$. The two kinds of activation functions are designed in the DNN-based equalization scheme to investigate the system error-rate performance.

4 Simulation Results

In this section, simulation results are presented to verify the developed DL-based SEFDM detection scheme. The number of subcarriers of the SEFDM system is chosen as 64, and the oversampling factor ρ is 1. In the simulation, we assume perfect synchronization and perfect knowledge of the noise variance for the MMSE detectors. In the simulation platform, we consider a few typical values of BCF α, including 0.9, 0.8, and 0.7. We use BPSK and QPSK as the modulation formats and *no forward error correction (FEC) codes* are considered in our system. Without exhaustive hyperparameters (i.e., the number of layers and the number of neurons in each layer) optimization, the DNNs are manually optimized to achieve an optimal performance. All the DNNs we considered contain 5 layers. The numbers of neurons in the layers of the *narrower* DNN are respectively 128, 512, 256, 128, 64(for BPSK)/128 (for QPSK), labeled as "DNN-n" in the BER plots. For the *wider* one, the numbers are 128,2048, 1024, 512, 64 (for BPSK)/128 (for QPSK), then labeled as "DNN-w". The output layer is fixed using the Sigmoid function, while the other layers adopt the same activation function, specifically, we only consider Relu and tanh, refered to as "relu DNN" and "tanh DNN" in the plots. The performance of the equalization scheme based on four different DNNs, i.e., "relu DNN-n", "relu DNN-w", "tanh DNN-n" and "tanh DNN-w", is examined, respectively. The optimizer for DNN we utilized is the RMSProp with learning rate decay. The training SNR is set to 2 dB SNR per bit (Eb/N0). The networks are trained with randomly-generated data for a known α, and then deployed online to assess the system performance. The models are trained with 50000 batches of data, and each training batch contains 1000 SEFDM symbols.

Figure 4 to Figure 6 demonstrate the BER performance of the BPSK-SEFDM system with different values of α. From the BER plots, it follows that the DL-based scheme outperforms the conventional linear equalization schemes, and achieves a bandwidth saving by up to 30%. In the high bandwidth compressing case when $\alpha = 0.7$ DL-based equalization scheme could improve the performance by at least 3 dB at BER of 10^{-2}, and an obvious improvement in higher Eb/N0 range, which means that the characteristics of the ICI and the correlated noise can be learned by the DNN in the training stage. Moreover, the combination of equalizer and demapper could help improve the performance. Meanwhile the BER plots marked as "tanh DNN-n" are very close to the performance reference of OFDM system without ICI, which shows a significant performance improvement, even no FEC codes used in the system. The results also show that the network with tanh activation function has a better performance than Relu, which improves the BER performance by approximately 0.5 dB at BER of 10^{-4} for all BCFs, since the output range of the tanh is more suitable for the equalization scheme output. With the same activation function, DNN-n outperforms DNN-w, resulting from a fact that there are more weights in DNN-w to be trained in the learning stage.

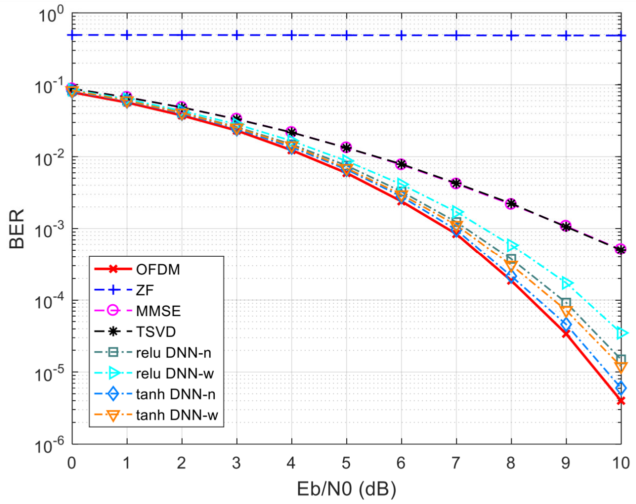

Figure 4. BER versus Eb/N0, $\alpha = 0.9$, BPSK modulation

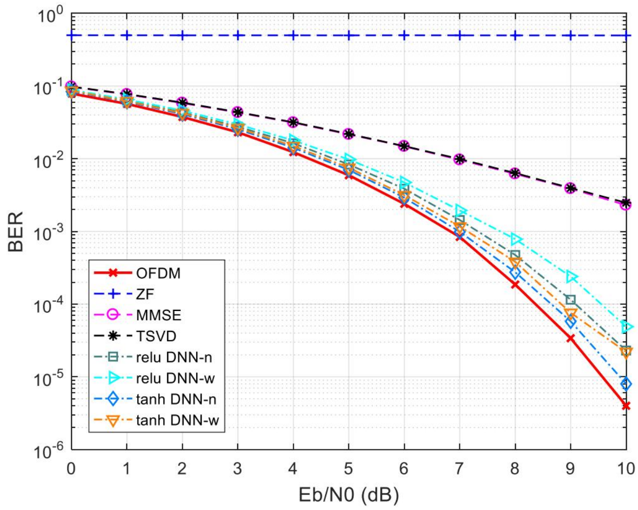

Figure 5. BER versus Eb/N0, $\alpha = 0.8$, BPSK modulation

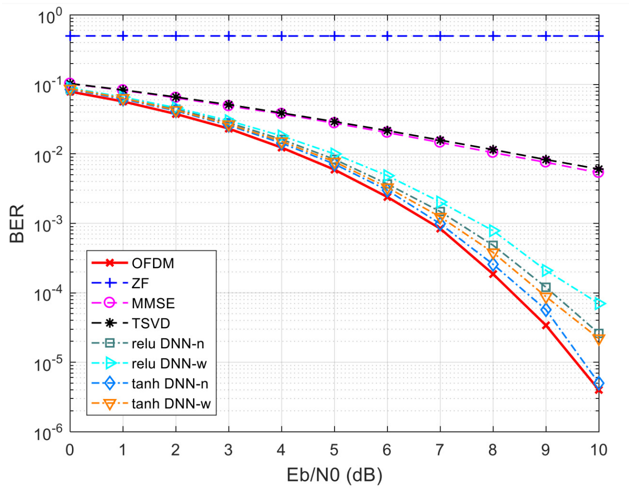

Figure 6. BER versus Eb/N0, $\alpha = 0.7$, BPSK modulation

The BER performance, for QPSK modulation, is depicted in Figure 7 to Figure 9. Compared with the BPSK signaling schemes, the performance of proposed scheme degrades significantly as α decreases Nevertheless, there still shows some improvements in BER, compared with the linear equalization based schemes. From Figure 7, the performance is also improved by 3 dB at BER of 10^{-2} with the network "tanh DNN-w" when $\alpha = 0.9$. However, other networks fail to improve the performance in this case. As for higher bandwidth compression when $\alpha = 0.8$ and $\alpha = 0.7$ the performance improvement degraded significantly, even though there still shows some improvements in high SNR ranges, as depicted in Figure 8 and Figure 9. For QPSK, DNN-w network shows better performance, which means the QPSK is more complex and needs more parameters to characterize. In general, the tanh activation function also brings some performance gain.

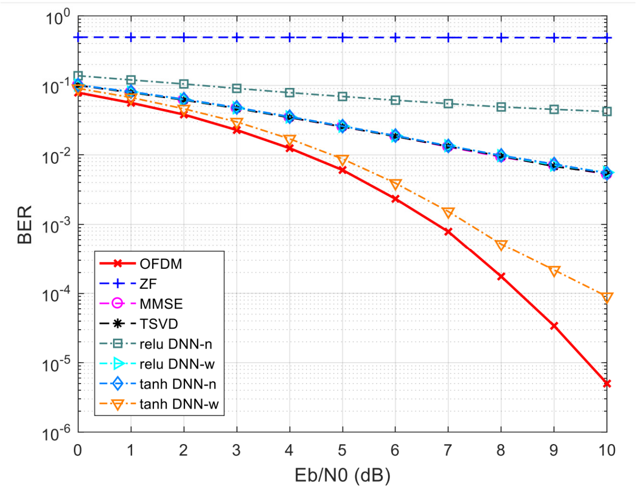

Figure 7. BER versus Eb/N0, $\alpha = 0.9$, QPSK modulation

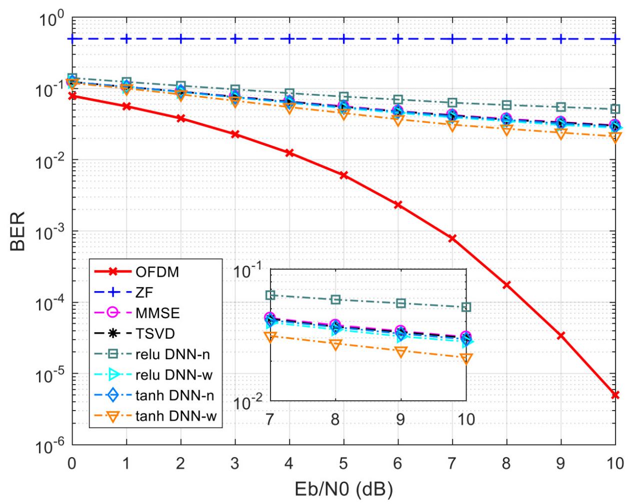

Figure 8. BER versus Eb/N0, $\alpha = 0.8$, QPSK modulation

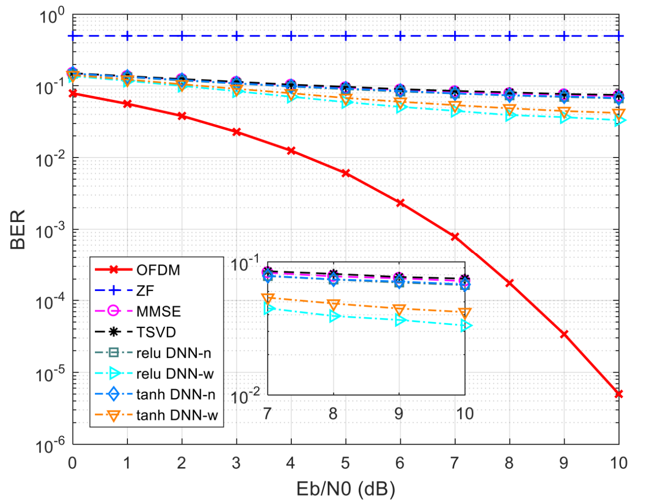

Figure 9. BER versus Eb/N0, $\alpha = 0.7$, QPSK modulation

5 Conclusion

In this paper,we propose a novel DL-based equalization scheme for SEFDM non-orthogonal multicarrier communication system, which outperforms the conventional equalization schemes. The neural network is trained over randomly-generated data and learns the interference features of SEFDM system automatically. Numerical results demonstrate the scheme can effectively improve the BER performance. The simulation results show that the DL method has advantages when solving ill-conditioned problems. However, the proposed scheme works on condition that the number of subcarriers is fixed. When we want to change the number of subcarriers, we have to train another NN, which inspires us to do more indepth research in the future, on the network design of the DL-based SEFDM equalization scheme.

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