Sufficient Conditions Analysis of Coverage Algorithm Constructed Positive Definite Tridiagonal Matrices in WSNs

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Abstract

In this paper, we have studied the coverage algorithm constructed positive definite Tridiagonal Matrices for identifying the positive definite symmetric matrix together with the properties in Wireless sensor networks (WSNs). Firstly, we have summarized the determinations of the positive definite symmetric matrix, respectively, from four different aspects including the definition, the Jordan normal form, the upper-left sub-matrices and the decomposition of the positive definite matrix. At the same time, the objects and precautions applying to the different positive definite matrices are described in detail. Secondly, we study the relevant properties of positive definite matrix, and construct the positive definite matrix by using the non-degenerate matrix on the basis of the determination of positive definite symmetric matrix. Finally, we prove the inequality of *Hadamard* in solving the practical problems by using the positive definite matrix. Furthermore, we get the *Cauchy-Schwarz* inequality satisfied with positive definite matrix [1-16].

Keywords: Symmetric matrix, Positive definite matrix

1 Introduction

The idea of matrix can be traced back to the study of solving linear equations by scholars in the Han Dynasty. The study of matrix, an important basic concept in mathematics, has always been a very hot issue.

Matrix is an important basic concept in mathematics and a main research direction of algebra. Matrix theory has been widely used in geometry, physics, probability theory and optimization theory. Symmetric matrices play an important role in matrix theory because of their special properties [17-29].

Positive definite matrix plays a very important role in the theory of matrix, especially symmetric matrix. Therefore, it is very important to study the judgment and properties of positive definite matrix.

The particularity of positive definite matrices in matrix theory has been discovered long ago. In the 1960s and 1970s, scientists continued to find positive definite matrices in numerical calculation, engineering and other fields of wide application. A large number of judgment methods and properties of positive definite matrices have been discovered constantly, and the study of positive definite matrices is regarded as a breakthrough in the study of matrix theory.

In this paper, we will start with the determination method of positive definite matrix, describe the determination method, application scope and matters needing attention of positive definite matrix from four different perspectives, and then continue to find out the related properties of positive definite matrix and carry out the corresponding properties of the promotion, finally, use the related properties of positive definite matrix to apply it to practical problems.

2 Sequential Master-child Method

Definition 2.2.1 [2]: let $A=(a_{ij})_{nn}$ be a matrix of order n, and the minor

$$A=(a_{ij})_{nn}$$

$$P_i=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1i} \\ a_{21} & a_{22} & \cdots & a_{2i} \\ \vdots & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ii} \end{vmatrix}$$

$$(i=1,2,...,n)$$

It's called the sequential principal minor of matrix.

Theorem 2.2.1 [3]: A symmetric matrix of order n is A positive definite matrix if and only if all the sequential principal minor of A are greater than zero.

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Proof: (necessity)

The positive qualitative decision of symmetric matrix is equivalent to the equal qualitative decision of quadratic form.

Let n-order symmetric matrix A be a positive definite matrix, then for any n-dimensional vector $X = (x_1, x_2, ..., x_n)$, there is a

$$f(X) = X^T \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_i x_j > 0$$

establish.

For every k, $1 \le k \le n$, let $f_k(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}x_i x_j$. Now we only need to prove that f_k is a positive definite quadratic form of k-elements, that is, for any group of real numbers $c_1, \cdots, c_k$ which are not all zero. there is

$$f_k(c_1, c_2, \cdots, c_k) = \sum_{i=1}^{k}\sum_{j=1}^{k} a_{ij}c_i c_j = f(c_1, c_2, 0, \cdots, 0) > 0$$

establish, So $f_k(x_1, x_2, \cdots, x_n)$ is positive definite. Since the determinant of the positive definite matrix is greater than zero, then the determinant of the coefficient matrix of f_k is positive definite

$$\begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \end{vmatrix} > 0$$

Where $k = 1, 2, ..., n$. This proves that all the sequential principal minor of A are greater than zero.

(Sufficiency) Prove it by mathematical induction.

When $n = 1$, $f(x_1) = a_{11}x_1^2$, it is obvious that $f(x_1)$ is positive definite quadratic form from the condition $a_{11} > 0$.

The assumption of sufficiency is true for $n-1$-ary quadratic form. Now let's prove the case of n-ary.

Let $A_1 = \begin{vmatrix} a_{11} & \cdots & a_{1,n-1} \\ \vdots & & \vdots \\ a_{n-1,1} & \cdots & a_{n-1,n-1} \end{vmatrix}, a = \begin{pmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{pmatrix}$

So matrix A can be written in blocks

$$A = \begin{pmatrix} A_1 & \alpha \\ \alpha^T & a_{nn} \end{pmatrix}$$

Since all the sequential principal minor of A are greater than zero, of course, all the sequential principal minor of A_1 are greater than zero. According to the inductive hypothesis, A_1 is a positive definite matrix, that is, there exists an invertible matrix G of order $n-1$ such that

$$G^T A_1 G = E_{n-1}$$

Where E_{n-1} represents the identity matrix of order $n-1$.

Let $C_1 = \begin{pmatrix} G & 0 \\ 0 & 1 \end{pmatrix}$ be

$$C_1^T A C_1 = \begin{pmatrix} G^T & O \\ O & 1 \end{pmatrix}\begin{pmatrix} A_1 & \alpha \\ \alpha^T & a_{nn} \end{pmatrix}\begin{pmatrix} G & O \\ O & 1 \end{pmatrix}\begin{pmatrix} E_{n-1} & G^T\alpha \\ \alpha^T G & a_{nn} \end{pmatrix}$$

Let $C_2 = \begin{pmatrix} E_{n-1} & -G^T\alpha \\ 0 & 1 \end{pmatrix}$ be

$$C_2^T C_1^T A C_1 C_2 = \begin{pmatrix} E_{n-1} & O \\ -\alpha^T G & 1 \end{pmatrix}\begin{pmatrix} E_{n-1} & G^T\alpha \\ \alpha^T G & a_{nn} \end{pmatrix}\begin{pmatrix} E_{n-1} & -G^T\alpha \\ O & 1 \end{pmatrix}$$

$$= \begin{pmatrix} E_{n-1} & O \\ O & a_{nn} - \alpha^T G G^T \alpha \end{pmatrix}$$

Let $C = C_1 C_2, \alpha_{nn} - \alpha^T G G^T \alpha = \alpha$

be $C^T A C = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & a \end{pmatrix}$

Take the determinant of both sides $|C|^2 |A| = a$

From the known conditions, $|A| > 0$, so $A > 0$, obviously

$$\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & a \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \sqrt{a} \end{pmatrix}\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \sqrt{a} \end{pmatrix}$$

That is to say, matrix A is consistent with identity matrix, that is to say, it is proved that A is a positive definite matrix

Example 2.3.2 Symmetric real tridiagonal matrices A

$$A = \begin{pmatrix} a_1 & b_1 & & & \\ b_1 & a_2 & b_2 & & \\ & b_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & b_{n-1} & a_n \end{pmatrix}$$

Where $a_i > 0\,(i = 1, 2, \cdots, n)$ and matrix A is strictly diagonally dominant, then symmetric real tridiagonal matrix A is positive definite.

Proof: since the sequential principal minor of symmetric real tridiagonal matrix A are all greater than zero [4-5]. It can be known from the above theorem 2.2.1, symmetric real tridiagonal matrix A is a positive definite matrix. Therefore, for symmetric real tridiagonal matrix A, it satisfies

$$a_i > |b_i| \prec |b_{i-1}|$$
$$(i = 1, 2, \cdots, n)$$

Where $b_0 = 0$, then a symmetric real tridiagonal matrix A is a positive definite matrix.

This kind of symmetric matrix has a lot of practical applications in computer programming, statistical partial differential equations, engineering science and so on.

2.1 Principal Minor Method

Definition 2.2.2. all the sub formulas with the same row index and column index in the matrix are called principal sub formulas.

Theorem 2.2.2 [6] n symmetric matrix A is a positive definite matrix if and only if all the principal minor of a are greater than zero.

Prove: (sufficiency)

Since the definition of the principal minor shows that all the principal minor of A are greater than zero, including all the sequential principal minor are greater than zero, theorem 2.2.1 shows that the symmetric matrix A of order n is a positive definite matrix. (necessity)

Let $A = (a_{ij})_{n\times n}$ be a positive definite matrix, and its principal minor of order k is

$$|\boldsymbol{A}^{(k)}| = \begin{vmatrix} \boldsymbol{a}_{i_1 i_1} & \cdots & \boldsymbol{a}_{i_1 i_k} \\ \vdots & & \vdots \\ \boldsymbol{a}_{i_k i_1} & \cdots & \boldsymbol{a}_{i_k i_k} \end{vmatrix}$$

For any $y_0 = (b_{i1}, \cdots, b_{ik})^T \neq 0$, let $x_0 = (c_{i1}, \cdots, c_{ik})^T$, where

$$c_i = \begin{cases} b_i & \text{where } i = (i_i, i_2, \cdots, i_k) \\ 0 & \end{cases}$$

other

because $x^T Ax$ is a positive definite quadratic form, there are $x_0^T Ax_0 > 0$ So there is $x_0^T Ax_0 = y_0^T A^{(k)} y_0 > 0$.

Because of the arbitrariness of y_0, we know that $y_0^T A^{(k)} y_0$ is a positive definite quadratic form, so $|A^{(k)}| > 0$.

2.2 Rodan's Standard form Method

For a square matrix A of order n over any complex field, there exists a unique Jordan canonical form. The diagonal elements of each Jordan block are the eigenvalues of the square matrix A. For a special square matrix, positive definite matrix, its eigenvalues are more special.

2.2.1 Some Equivalent Conditions and Mutual Derivation of Jordan Canonical Form of Positive Definite Matrix

Due to the particularity of Jordan canonical form of positive definite matrix, the following three propositions are equivalent [6]:

(1) Square matrix A is a positive definite matrix

(2) There is an orthogonal matrix H of order n such that

$$H^T AH = diag(\lambda_1, \lambda_2, ..., \lambda_n) \quad \textbf{(1)}$$

Where λ_i is the eigenvalue of A and $\lambda_1 > 0$, $i = 1, 2, ..., n$.

(3) There is an invertible square matrix S of order n such that

$$S^T AS = E_n \ (E_n \text{ is the identity matrix of order } n)$$

Prove: (1) $\Rightarrow$ (1)

Considering the relationship between the real symmetric matrix and the symmetric transformation, we only need to prove that the symmetric transformation σ has a standard orthogonal basis made of n eigenvectors. Now we make a mathematical induction for the dimension n of space.

When $k = 1$, there is a conclusion.

Assuming $k = n - 1$, the conclusion holds. It is proved that $k = n$ also holds. For n-dimensional Euclidean space R^n, the linear transformation σ has an eigenvector α_1 whose eigenvalue is a real number λ_1. The transformation unites α_1 and uses α_1 to represent it. The orthogonal complement of $L(\alpha_1)$ is set to V_1, and V_1 is an invariant subspace of σ, whose dimension is $n-1$. $\sigma | V_1$ clearly satisfies the property of a symmetric transformation

$$(\sigma(\alpha), \beta) = (\alpha, (\beta)) \, \forall \alpha, \beta \in R^n$$

That is to say, $\sigma | V_1$ is still a symmetric transformation.

According to the inductive hypothesis, $\sigma | V_1$ has $n-1$ eigenvectors $\alpha_1, \alpha_2, \cdots, \alpha_n$ as the standard orthogonal basis of V_1, so that $\alpha_1, \alpha_2, \cdots, \alpha_n$ are the standard orthogonal basis of R^n, then there are n eigenvectors $\alpha_1, \alpha_2, \cdots, \alpha_n$ as the standard orthogonal basis of σ.

(2) $\Rightarrow$ (3)

By (2) known

$$H^T AH = diag(\lambda_1, \lambda_2, ..., \lambda_n) = \begin{pmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{pmatrix} \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{pmatrix}$$

Let $S = H \begin{pmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{pmatrix}$, and

$$|S| = |H| \begin{pmatrix} \sqrt{\lambda_1} & & \\ & \ddots & \\ & & \sqrt{\lambda_n} \end{pmatrix} \neq 0$$

is an invertible matrix, then

$S^T AS = E_n$ (E_n is the identity matrix of order *n*)

$(3) \Rightarrow (1)$

Let $F(X) = X^T$ make a reversible linear transformation,
let $X = SY$,
then $f(X) = X^T AX = s(SY) = Y^T S^T ASY = Y^T E_n Y$

Let $g(Y) = Y^T E_n y$, since the positive inertia index of real quadratic form $g(Y)$ is *n*, then real quadratic form $g(Y)$ is positive definite quadratic form, that is to say, real quadratic form $f(X)$ is real quadratic form, that is, square matrix *A* is positive definite matrix.

From the variant form of (1) in the above equivalent proposition

$$A = Hdiag(\lambda_1, \lambda_2, ..., \lambda_n)H^T$$

It is called the spectral decomposition of positive definite matrices *A*.

2.2.2 Rodan Canonical Form Method Applicable Objects and Matters Needing Attention

By using the Rodan canonical form method to determine the positive qualitality of the symmetric matrix, the eigenvalue problem of the symmetric matrix can be determined firstly, When all the eigenvalues are positive, the symmetric matrix can be determined as positive definite matrix, then the orthogonal matrix *H* is determined by the eigenvalues. Finally, we get the Rordan canonical form of the symmetric matrix *A*.

If the standard form method is used to determine the positive definiteness of symmetric matrix, there is no general algebraic solution and formula for finding the roots of $n(n \geq 5)$ degree equation. Therefore, this method should be used with caution when the order of symmetric matrix *A* is $n \geq 5$.

2.3 Special Decomposition Method of Positive Definite Matrix

2.3.1 Real full Rank Square Matrix Decomposition of Positive Definite Matrix

For the real full rank square matrix decomposition of positive definite matrix, that is to say, the positive definite matrix is expressed as the product form of invertible matrix. According to the proof process of Jordan canonical form method 2.3 of positive definite matrix, the order symmetric matrix is positive definite matrix if and only if there is an invertible matrix, such that

$$A = B^T B$$

It is a real full rank square matrix decomposition form of positive definite matrix.

Another special decomposition of orthogonal matrices is introduced——before the trigonometric invertible matrix factorization on the positive line, let's first prove the lemma.

Lemma 1 [7] Any real invertible matrix can be uniquely decomposed into the product of an orthogonal matrix and a trigonometric matrix on a positive line.

Prove: let matrix $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ is an invertible matrix, then its *n* column vectors are denoted as $\alpha_1, \alpha_2, \cdots, \alpha_n$, because $|A| \neq 0$, so $\alpha_1, \alpha_2, \cdots, \alpha_n$, are linearly independent, so $\alpha_1, \alpha_2, \cdots, \alpha_n$, are a group of bases of R^n.

By using Schmidt orthogonalization process, from $\alpha_1, \alpha_2, \cdots, \alpha_n$, get an orthonormal basis $\beta_1, \beta_2, \cdots, \beta_n$, which satisfies

$$\begin{aligned} &\beta_1 = \alpha_1 \\ &\beta_2 = \alpha_2 - (\alpha_2, \eta_1)\eta_1 \\ &\vdots \\ &\beta n = \alpha_n - (\alpha_n, \eta_1)\eta_1 - \cdots - (\alpha_n, \eta_{n-1})\eta_{n-1} \end{aligned}$$

Among them $\eta_i = \frac{\beta_i}{|\beta_i|}$ $(i = 1, 2, \cdots, n)$. Let $\beta_i = |\beta_i| \eta_i$ $(i = 1, 2, \cdots, n)$ to the left of each equation, transference arrangement can be obtained

$$\begin{cases} \alpha_1 = t_{11}\eta_1 \\ \alpha_2 = t_{12}\eta_1 + t_{22}\eta_2 \\ \vdots \\ \alpha_n = t_{1n}\eta_1 + t_{2n}\eta_2 + \cdots + t_{nn}\eta_n \end{cases}$$

Among them $t_{ii} = |\beta_i| > 0$ $(i = 1, 2, \cdots, n)$, then

$$A = (\alpha_1, \alpha_2, \cdots, \alpha_n)$$
$$= (\eta_1, \eta_2, \cdots, \eta_n)\begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ & t_{22} & \cdots & t_{2n} \\ & & \ddots & \vdots \\ & & & t_{nn} \end{pmatrix}$$

Let $T = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ & t_{22} & \cdots & t_{2n} \\ & & \ddots & \vdots \\ & & & t_{nn} \end{pmatrix}$, then T is a triangle matrix on the plus line. η_i is n dimensional column vectors, matrix Q is composed of $\eta_1, \eta_2, \cdots, \eta_n$ columns, that is

$$Q = (\eta_1, \eta_2, \cdots, \eta_n)$$

Because $\eta_1, \eta_2, \cdots, \eta_n$ are orthonormal bases, so Q is an orthogonal matrix, and $A = QT$. If there are $Q_1, T_1, A = Q_1 T_1$ is another decomposition of A, then

$$Q_1, T_1 = QT$$
$$Q_1^{-1} Q = T_1 T^{-1}$$

Because Q_1, Q are orthogonal matrices, then $Q_1^{-1} Q$ is also orthogonal matrix, so $T_1 T^{-1}$ is also orthogonal matrix. On the other hand, $T_1 T^{-1}$ is an upper triangular matrix, so the main diagonal elements of $T_1 T^{-1}$ are diagonal matrices of 1 or - 1, while the main diagonal elements of T, T_1 are positive, so

$$T_1 T^{-1} = E$$

that is $\quad T = T_1$

thus $\quad Q_1 = Q$

Theorem 2.4.1 [8] the symmetric matrix *A* of order *n* is a positive definite matrix if and only if there exists a triangular invertible matrix *T* on the main line, make

$$A = T^T T$$

Prove: (sufficiency)

Let $T = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ & t_{22} & \cdots & t_{2n} \\ & & \ddots & \vdots \\ & & & t_{nn} \end{pmatrix}$ $t_{ii} > 0 \ \ (i = 1, 2, \cdots, n)$ be a triangular invertible matrix on the main line, then $|T| = \prod_{i=1}^{n} t_{ii} > 0$ is reversible, then there exists T^{-1}, make

$$(T^T)^{-1} A T^{-1} = E$$

Therefore, the symmetric matrix *A* of order *n* is a positive definite matrix.

(necessity)

Since the symmetric matrix *A* of order *n* is a positive definite matrix, there exists an invertible matrix *B*, make

$$A = B^T E B = B^T B$$

From lemma 1

$$B = QT$$

So $\quad A = T^T Q^T Q T = T^T T$

2.3.2 Non Degenerate Matrix Factorization of Positive Definite Matrices

Theorem 2.4.2 the symmetric matrix *A* of order *n* is positive definite if and only if there are invertible matrix *B* and orthogonal matrix *H*, make $A = HB$.

Among them, invertible matrix *B* and orthogonal matrix *H* are pairwise existence and uniqueness.

Prove: (sufficiency)

First, let's prove such a lemma

Lemma 2 [9] any real invertible matrix *B* must be expressed as the product of a positive definite matrix and an orthogonal matrix

$$B = Q_1 H_1 = H_2 Q_2$$

Where Q_1, Q_2 are positive definite matrices and H_1, H_2 are orthogonal matrices.

Since *B* is a real invertible matrix, BB^T is a positive definite matrix in theorem3.4.1. Therefore, from the inference in theorem 3.1.5, we can see that there exists a positive definite matrix A_1, such that

$$BB^T = A_1^2$$

Let $\quad A_1^{-1} B = H_1, \ B A_1^{-1} = H_2$

then $\quad B = Q_1 H_1 = H_2 Q_2 \ \ (Q_1 = Q_2)$

and $\quad H_1 H_1^T = (A_1^{-1} B)(A_1^{-1} B)^T = A_1^{-1} B B^T (A_1^{-1})^T$

$$= A_1^{-1} A_1^2 (A_1^{-1})^T = E$$

That is to say, H_1 is proved to be an orthogonal matrix. Similarly, H_2 is proved to be an orthogonal matrix.

Now let's prove the sufficiency of theorem 2.4.1.

By using lemma 2, an invertible matrix *B* can be decomposed into the product of a positive definite matrix with an orthogonal matrix, Let the orthogonal matrix be H^T, then there exists a positive definite matrix A_1, which satisfies

$$B = A_1 H^T$$
$$A = HB = H A_1 H^T$$

Since matrix *A* is orthogonal to positive definite matrix A_1, matrix *A* is positive definite matrix.

(necessity)

Since the symmetric matrix A of order n is a positive definite matrix, according to the Jordan canonical form of positive definite matrix, there can be an orthogonal matrix H, so that

$$H^T AH = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$$

establish, then

$$A = Hdiag(\lambda_1, \lambda_2, \cdots, \lambda_n)H^T$$

let $$B = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)H^T$$

Because of the uniqueness of the orthogonal matrix determined by Schmidt orthogonalization, the existence and uniqueness of the reversible matrix B and the orthogonal matrix H are pairwise.

3 Conclusion

In this paper, the determination method of positive definite matrix is expounded in detail from the four aspects of definition, normal form, matrix subform and special decomposition of positive definite matrix, and the corresponding sufficient and necessary conditions are obtained. On this basis, the properties of the relative deformation matrix, the partition matrix and the decomposition of the positive definite matrix are discussed respectively, and then the application of the positive definite matrix in some practical problems is given, which fully reflects the particularity and universality of the positive definite matrix.

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