# A New Multiple Criteria Decision Making Approach Based on Intuitionistic Fuzzy Sets, the Weighted Similarity Measure, and the Extended TOPSIS Method

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## Abstract

Many real-world multiple criteria decision making (MCDM) problems are rather complicated and uncertain to handle. In recent years, some MCDM methods have been proposed based on intuitionistic fuzzy sets (IFSs). In this paper, we propose a new MCDM method based on IFSs, the weighted similarity measure (WSM), and the extension of the technique for order preference by similarity to ideal solution (TOPSIS) method with completely unknown weights of criteria. Firstly, we calculate the weights of criteria using the normalized intuitionistic fuzzy entropy values (IFEVs) when the weights of criteria was not given by decision maker. Secondly, we propose a novel weighted similarity measure (WSM) between the IFSs that takes the hesitancy degree of elements of IFSs into account. Finally, we combine the WSM with the Extended TOPSIS Method to propose a new MCDM approach based on IFSs which can overcome the drawbacks and limitations of some existing methods that they cannot get the preference order of the alternatives in the context of the "division by zero" ("DBZ") situations. The proposed method provides us an easier way to handle MCDM problems under intuitionistic fuzzy (IF) environments.

**Keywords:** Entropy, Intuitionistic fuzzy sets (IFSs), Multiple criteria decision making (MCDM), TOPSIS, Weighted similarity measure (WSM)

## 1 Introduction

In order to handle rather complicated and uncertain decision making (DM) problems in the real world, there are lots of vague and imprecise methods have been proposed to solve DM problems using fuzzy sets (FSs) [33], interval-valued fuzzy sets (IVFSs) [34], intuitionistic fuzzy sets (IFSs) [1] and interval-valued intuitionistic fuzzy sets (IVIFSs) [2], etc. Owing to time pressure on decision maker under uncertainty, some DM methods have been presented [4, 9-11, 17-20, 23-24, 27- 30, 32] based on IFSs [1] to solve more and more complex and uncertain DM problems. In [3], Baccour *et al.* explored comprehensively many known similarity measures between IFSs and made comparisons between those measures which are ignored the importance of hesitancy degree for different research area. In [4], Chen proposed a Multiple Criteria Decision Making (MCDM) method using IFSs, combined with grey relational analysis (GRA) techniques and entropy-based TOPSIS, to determine the best sustainable building materials supplier. In [5], Chen and Tsai presented a multiattribute decision making (MADM) method based on IVIF ordered weighted geometric averaging (IVIFOWGA) operator and IVIF hybrid geometric averaging (IVIFHGA) operator to solve investment problems. In [6], Chen *et al.* proposed an IVIF MCDM method which can overcome some methods which cannot distinguish the ranking order between alternatives in certain circumstances. In [7], Chen presented a multiple criteria group decision making (MCGDM) method by using an extended TOPSIS method with an inclusion comparison approach in the framework of IVIFSs. In [8], Chen presented an IVIF permutation method with likelihood-based preference functions applying to MCDM analysis problems and compared the result of the proposed method with other MCDM methods in order to select a appropriate bridge construction method. In [9], Chen and Li made a comparative analysis of different intuitionistic fuzzy (IF) entropy measures to determine objective weights for MADM problem and proposed a new objective entropy-based weighting method under the IFS environments. In [10], Dass and Tomar presented three families of IF entropy measures applying in MCDM problrms. In [11], Li and Cheng proposed some new similarity measures for applying to pattern recognitions under IFS environments. In [12], Dugenci presented a MCGDM method by using a generalized distance measure and the extension of TOPSIS method under IVIF environments. In [14], Guo and Song proposed a

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new IF entropy and then developed a new IF entropy measure to solve group DM problems. In [16], Li proposed a TOPSIS-based nonlinear-programming approach for MCDM under IVIF environments. In [17], Li *et al.* presented a MADM method based on weighted induced distance through an IF weighted induced ordered weighted averaging operator for the investment selection problem. In [18], Liu *et al.* presented an integrated entropy-based best-worst method (BWM) for multiple attribute group decision making (MAGDM) problems under IF environments. In [19], Mishra *et al.* proposed an extended multi-attribute border approximation area comparison method for smartphone selection using discrimination measures under IVIF environments. In [20], Phochanikorn and Tan proposed an extended MCDM method under an IF environment for sustainable supplier selection based on sustainable supply chain with DEMATEL combined with an analytic network process (ANP) to identify uncertainties and interdependencies. In [21], Park *et al.* presented a MAGDM method that extended the TOPSIS method with partially known attribute weights under IVIF environments. In [22], Senapati and Yager proposed Fermatean fuzzy sets (FFSs) which are the extension of IFSs. FFSs can handle more uncertain information than IFSs for DM problems. In [23], Singh *et al.* presented an MCDM method based on some knowledge measures of intuitionistic fuzzy sets of second type (IFSST) to overcome the certain limitations of IFSs. In [24], Turk proposed an MCDM method for location selection of pilot area for green roof systems in Igdir province, Turkey, under IF environment. In [25], Wan *et al.* presented a MAGDM method with IVIF values and incomplete attribute weight information that all attributes were determined through considering the similarity degree and proximity degree simultaneously. In [26], Wang and Chen proposed a MADM method based on IVIFSs, linear programming methodology and the extended TOPSIS that can overcome some methods which cannot distinguish the ranking order between alternatives in some situations. In [27], Xia and Xu proposed an Entropy/cross entropy-based weight-determining methods for group DM under IF environments. In [31], Ye presented an extended TOPSIS method for group DM with IVIF numbers to solve the pattern selection problems under incomplete and uncertain information environments. In [32], Ye proposed an IF MCDM method based on an evaluation formula of weighted correlation coefficients using entropy weights for unknown criteria weights. The TOPSIS method has successfully been applied to deal with many kinds of MCDM problems [7, 12-13, 15-16, 18, 21, 26, 31-32] under IF and IVIF environments and receive more attention from both the government and the private sector. In this study, we propose a new MCDM approach in which ratings of alternatives on criteria are all IFSs, the weighted similarity measure (WSM) [28], and the extension of the TOPSIS method with unknown weights of criteria for choosing suitable sustainable building materials supplier in the initial stage of the supply chain. The contributions of this study include: (1) we can objectively determine the weights of criteria using the normalized intuitionistic fuzzy entropy values (IFEVs) when the weights of criteria was not given by decision maker; (2) we propose a novel WSM between the IFSs that takes the hesitancy degree of elements of IFSs into account; (3) by combining the WSM with the Extended TOPSIS Method, we propose a new MCDM approach based on IFSs which can overcome the drawbacks and limitations of some existing methods that they cannot get the preference order of the alternatives in the context of the "division by zero" ("DBZ") situations.

The rest of this paper is organized as follows. In Section 2, we briefly review the definitions of IFSs [1] and the WSM [28] between two IFSs. In Section 3, we propose a new MCDM method based on IFSs, the WSM, and the extended TOPSIS method. In Section 4, we use two examples to compare the proposed method with the Chen's method [4] for DM under IF environments. The conclusions are discussed in Section 5.

## 2 Preliminaries

In this section, we briefly review the definitions of IFSs [1] and the WSM [28] between two IFSs.

**Definition 2.1 [1]:** An IFS $A$ in the universe of discourse $X$, where $X = \{x_1, x_2, \dots, x_n\}$, can be represented by $A = \{\langle x_i, \mu_A(x_i), \upsilon_A(x_i)\rangle | x_i \in X\}$, where $\mu_A$ is the membership function of the IFS $A$, $\mu_A : X \to [0,1]$ and $\upsilon_A$ is the non-membership function of the IFS $A$, $\upsilon_A : X \to [0,1]$, respectively, $\mu_A(x_i)$ and $\upsilon_A(x_i)$ denote the degree of membership and the degree of non-membership of element $x_i$ belonging to the IFS $A$, respectively, $0 \le \mu_A(x_i) \le 1$, $0 \le \upsilon_A(x_i) \le 1$, $0 \le \mu_A(x_i) + \upsilon_A(x_i) \le 1$ and $1 \le i \le n$. $\pi_A(x_i)$ is called the hesitancy degree of element $x_i$ belonging to the IFS $A$, where $\pi_A(x_i) = 1 - \mu_A(x_i) - \upsilon_A(x_i)$ and $1 \le i \le n$.

For convenience, Xu [28] called $\alpha = (a, b)$ an intuitionistic fuzzy number (IFN) or an intuitionistic fuzzy value (IFV), where $0 \le a \le 1$, $0 \le b \le 1$, and $a + b \le 1$.

Atanassov [1] also defined the following operations and relations between the IFSs $A$ and $B$:

(1) $A \subset B$ iff $\forall\ x_i \in X$, $\mu_A(x_i) \le \mu_B(x_i)$ and $\upsilon_A(x_i) \ge \upsilon_B(x_i)$;

(2) $A = B$ iff $\forall\ x_i \in X$, $\mu_A(x_i) = \mu_B(x_i)$ and $\upsilon_A(x_i) = \upsilon_B(x_i)$;

(3) $A \cup B =$
$$\{\langle x_i, max(\mu_A(x_i), \mu_B(x_i)), min(\upsilon_A(x_i), \upsilon_B(x_i))\rangle | x_i \in X\};$$

(4) $A \cap B =$
$$\{\langle x_i, min(\mu_A(x_i), \mu_B(x_i)), max(\upsilon_A(x_i), \upsilon_B(x_i))\rangle | x_i \in X\};$$

(5) The complement of a set $A$ is defined as:

$$\bar{A} = \{\langle x_i, \upsilon_A(x_i), \mu_A(x_i)\rangle | x_i \in X\};$$

(6) $A + B = \{\langle x_i, \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i) \cdot \mu_B(x_i), \upsilon_A(x_i) \cdot \upsilon_B(x_i)\rangle | x_i \in X\}$;

(7) $A \cdot B = \{\langle x_i, \mu_A(x_i) \cdot \mu_B(x_i), \upsilon_A(x_i) + \upsilon_B(x_i) - \upsilon_A(x_i) \cdot \upsilon_B(x_i)\rangle | x_i \in X\}$;

where $X = \{x_1, x_2, \dots, x_n\}$.

**Definition2.2:** Assume that A and B are two IFSs, where $A = \{\langle x_i, \mu_A(x_i), \upsilon_A(x_i)\rangle | x_i \in X\}$, $B = \{\langle x_i, \mu_B(x_i), \upsilon_B(x_i)\rangle | x_i \in X\}$, $X = \{x_1, x_2, \dots, x_n\}$ and $1 \le i \le n$. Let $w_i$ be the weight of element $x_i$, where $0 \le w_i \le 1$, $1 \le i \le n$ and $\sum_{i=1}^{n} w_i = 1$. The weighted similarity measure (WSM) between the IFSs $A$ and $B$ is defined as follows:

$$S_\omega(A,B) = 1 - \sum_{i=1}^{n} w_i \left[\frac{1}{6}(|\mu_A - \mu_B| + |\upsilon_A - \upsilon_B| + |\pi_A - \pi_B|) + \frac{1}{2}(max\{|\mu_A - \mu_B|, |\upsilon_A - \upsilon_B|, |\pi_A - \pi_B|\})\right]. \quad \textbf{(1)}$$

The WSM is based on the weighted Hamming distance and the weighted Hausdorff metric [28] and has the following properties [9]:

(1) $0 \le S_\omega(A,B) \le 1$;

(2) $S_\omega(A,B) = 1$ iff $A = B$;

(3) $S_\omega(A,B) = S_\omega(B,A)$;

(4) If $C$ is an IFS and $A \subseteq B \subseteq C$, then $S_\omega(A,C) \le S_\omega(A,B)$ and $S_\omega(A,C) \le S_\omega(B,C)$.

*Proof of* (4).

Since $A \subseteq B \subseteq C$, we can get $\mu_A \le \mu_B \le \mu_C$ and $\upsilon_A \ge \upsilon_B \ge \upsilon_C$. Therefore,

$$\begin{aligned} &S_\omega(A,C) \\ &= 1 - \sum_{i=1}^{n} w_i \left[\frac{1}{6}(|\mu_A - \mu_C| + |\upsilon_A - \upsilon_C| + |\mu_C - \mu_A + \upsilon_C - \upsilon_A|)\right. \\ &\quad \left. + \frac{1}{2}(max\{|\mu_A - \mu_C|, |\upsilon_A - \upsilon_C|, |\mu_C - \mu_A + \upsilon_C - \upsilon_A|\})\right] \\ &\le 1 - \sum_{i=1}^{n} w_i \left[\frac{1}{6}(|\mu_A - \mu_B| + |\upsilon_A - \upsilon_B| + |\mu_B - \mu_A + \upsilon_B - \upsilon_A|)\right. \\ &\quad \left. + \frac{1}{2}(max\{|\mu_A - \mu_B|, |\upsilon_A - \upsilon_B|, |\mu_B - \mu_A + \upsilon_B - \upsilon_A|\})\right] \\ &= 1 - \sum_{i=1}^{n} w_i \left[\frac{1}{6}(|\mu_A - \mu_B| + |\upsilon_A - \upsilon_B| + |\pi_A - \pi_B|)\right. \\ &\quad \left. + \frac{1}{2}(max\{|\mu_A - \mu_B|, |\upsilon_A - \upsilon_B|, |\pi_A - \pi_B|\})\right] \\ &= S_\omega(A,B). \end{aligned}$$

Similarly, we can prove $S_\omega(A,C) \le S_\omega(B,C)$. Q.E.D.

# 3 The Proposed MCDM Method based on IFSs, the WSM, and the Extended TOPSIS Method

At present, we propose a novel MCDM method based on IFSs, the WSM, and the extended TOPSIS method with unknown weights of criteria for choosing suitable sustainable building materials supplier in the initial stage of the supply chain. Assuming that $A = \{A_1, A_2, \dots, A_m\}$ is a set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ is a set of criteria. Let $K_1$ be a collection of benefit criteria and let $K_2$ be a collection of cost criteria, where $K_1 \cap K_2 = \emptyset$. Let $D = (d_{ij})_{m\times n} = ((x_{ij}, y_{ij}, z_{ij}))_{m\times n}$, where $z_{ij} = 1 - x_{ij} - y_{ij}$, be the IFV decision matrix given by the decision maker. The steps of the proposed MCDM method is shown as follows:

**Step 1:** Construct the IFV decision matrix $D = (d_{ij})_{m\times n} = ((x_{ij}, y_{ij}, z_{ij}))_{m\times n}$, shown as follows:

$$D = (d_{ij})_{m\times n} = ((x_{ij}, y_{ij}, z_{ij}))_{m\times n} =$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} (x_{11}, y_{11}, z_{11}) & (x_{12}, y_{12}, z_{12}) & \cdots & (x_{1n}, y_{1n}, z_{1n}) \\ (x_{21}, y_{21}, z_{21}) & (x_{22}, y_{22}, z_{22}) & \cdots & (x_{2n}, y_{2n}, z_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (x_{m1}, y_{m1}, z_{m1}) & (x_{m2}, y_{m2}, z_{m2}) & \cdots & (x_{mn}, y_{mn}, z_{mn}) \end{bmatrix} \end{array},$$

where $z_{ij} = 1 - x_{ij} - y_{ij}$, $1 \le i \le m$ and $1 \le j \le n$.

**Step 2:** Calculate the weights of criteria. Because the weights of criteria are not given by decision maker subjectively, we can calculate the weights of criteria in objective way. In order to get the weight vector $W = (w_1, w_2, \cdots, w_n)^T$ for the criteria $C_1$, $C_2$, $\cdots$, and $C_n$, we first calculate the normalized intuitionistic fuzzy entropy values (IFEVs) [13, 25, 27, 29] shown as follows:

$$E_j = \frac{1}{m}\sum_{i=1}^{m} z_{ij}, \quad \textbf{(2)}$$

where $z_{ij} = 1 - x_{ij} - y_{ij}$, $1 \le i \le m$ and $1 \le j \le n$. Summarize all the normalized IFEVs into $E_{sum}$:

$$E_{sum} = \sum_{j=1}^{n} E_j, \quad \textbf{(3)}$$

then we can get the weight $w_j$ for criterion $C_j$, shown as follows:

$$w_j = \frac{1 - E_j}{n - E_{sum}}, \quad \textbf{(4)}$$

where $\le i \le m$ and $1 \le j \le n$.

**Step 3:** Based on [10, 13], we can get the intuitionistic fuzzy positive ideal solution (IFPIS) $T^+$, shown as follows:

$$\begin{aligned} T^+ &= \{(\max_i d_{ij} | C_j \in K_1), (\min_i d_{ij} | C_j \in K_2) | \ 1 \le i \le m \\ &\qquad \text{and } 1 \le j \le n\} \\ &= \{t_1^+, t_2^+, \dots, t_n^+\}, \end{aligned} \quad \textbf{(5)}$$

where if $C_j \in K_1$, then let $t_j^+ = \left(\max_i x_{ij}, \min_i y_{ij}, 1 - \max_i x_{ij} - \min_i y_{ij}\right) = (x_j^+, y_j^+, z_j^+)$; if $C_j \in K_2$, then let $t_j^+ = \left(\min_i x_{ij}, \max_i y_{ij}, 1 - \min_i x_{ij} - \max_i y_{ij}\right) = (x_j^+, y_j^+, z_j^+)$, where $1 \le j \le n$. And we can also get the intuitionistic fuzzy negative ideal solution (IFNIS) $T^-$, shown as follows:

$$T^- = \{(\min_i d_{ij} | C_j \in K_1), (\max_i d_{ij} | C_j \in K_2) | 1 \le i \le m \text{ and } 1 \le j \le n\}$$
$$= \{t_1^-, t_2^-, \ldots, t_n^-\}, \tag{6}$$

where if $C_j \in K_1$, then let $t_j^- = \left(\min_i x_{ij}, \max_i y_{ij}, 1 - \min_i x_{ij} - \max_i y_{ij}\right) = (x_j^-, y_j^-, z_j^-)$; if $C_j \in K_2$, then let $t_j^- = \left(\max_i x_{ij}, \min_i y_{ij}, 1 - \max_i x_{ij} - \min_i y_{ij}\right) = (x_j^-, y_j^-, z_j^-)$, where $1 \le j \le n$.

**Step 4:** Based on Eq. (1) and the weight vector $W = (w_1, w_2, \cdots, w_n)^T$ for the criteria $C_1$, $C_2$, $\cdots$, and $C_n$ obtained from **Step 2**, calculate the WSM $S_{\omega_i}^+$ between the elements at the *i*th row of the IFV decision matrix $D = (d_{ij})_{m \times n}$ and the elements in the obtained IFPIS $T^+$, shown as follows:

$$S_{\omega_i}^+ = 1 - \sum_{j=1}^n w_j \left[\frac{1}{6}\left(|x_{ij} - x_j^+| + |y_{ij} - y_j^+| + |z_{ij} - z_j^+|\right) + \frac{1}{2}(max\{|x_{ij} - x_j^+|, |y_{ij} - y_j^+|, |z_{ij} - z_j^+|\})\right], \tag{7}$$

where $0 \le S_{\omega_i}^+ \le 1$, $\sum_{j=1}^n w_j = 1$, $0 \le w_j \le 1$, $1 \le i \le m$ and $1 \le j \le n$.

**Step 5:** Based on Eq. (1) and the weight vector $W = (w_1, w_2, \cdots, w_n)^T$ for the criteria $C_1$, $C_2$, $\cdots$, and $C_n$ obtained from **Step 2**, calculate the WSM $S_{\omega_i}^-$ between the elements at the *i*th row of the IFV decision matrix $D = (d_{ij})_{m \times n}$ and the elements in the obtained IFNIS $T^-$, shown as follows:

$$S_{\omega_i}^- = 1 - \sum_{j=1}^n w_j \left[\frac{1}{6}\left(|x_{ij} - x_j^-| + |y_{ij} - y_j^-| + |z_{ij} - z_j^-|\right) + \frac{1}{2}(max\{|x_{ij} - x_j^-|, |y_{ij} - y_j^-|, |z_{ij} - z_j^-|\})\right], \tag{8}$$

where $0 \le S_{\omega_i}^- \le 1$, $\sum_{j=1}^n w_j = 1$, $0 \le w_j \le 1$, $1 \le i \le m$ and $1 \le j \le n$.

**Step 6:** Calculate the relative closeness of alternative $A_i$ with respect to the IFPIS $T^+$, shown as follows:

$$R_i^* = \frac{S_{\omega_i}^+}{S_{\omega_i}^+ + S_{\omega_i}^-}, \tag{9}$$

where $0 \le R_i^* \le 1$ and $1 \le i \le m$. The larger the value of the relative closeness to the ideal solution $R_i^*$, the better the preference order of alternative $A_i$, where $1 \le i \le m$.

## 4 Illustrative Examples

We use two examples to compare the proposed method with Chen's method [4] for DM under IF environments shown as below.

***Example 4.1*** [4]: Suppose that a company want to choose the most appropriate sustainable building materials supplier for their future development. Assuming that there are five alternatives: $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ and assuming that there are four criteria in the assessment, shown as follows:

$C_1$: Business credit,
$C_2$: Technical capability,
$C_3$: Quality level, and
$C_4$: Price,

where the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion. Assume that the IFV decision matrix $D = (d_{ij})_{5 \times 4}$ is given by the decision maker.

**[Step 1]:** Construct the IFV decision matrix $D = (d_{ij})_{5 \times 4}$, shown as follows:

$$D = (d_{ij})_{5\times4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} (0.712, 0.157, 0.131) & (0.491, 0.263, 0.246) & (0.627, 0.183, 0.190) & (0.635, 0.217, 0.148) \\ (0.628, 0.239, 0.133) & (0.562, 0.197, 0.241) & (0.582, 0.195, 0.223) & (0.619, 0.205, 0.176) \\ (0.537, 0.296, 0.167) & (0.612, 0.189, 0.199) & (0.631, 0.209, 0.160) & (0.597, 0.196, 0.207) \\ (0.691, 0.162, 0.147) & (0.582, 0.201, 0.217) & (0.609, 0.253, 0.138) & (0.681, 0.192, 0.127) \\ (0.586, 0.177, 0.237) & (0.627, 0.125, 0.248) & (0.573, 0.181, 0.246) & (0.592, 0.182, 0.226) \end{bmatrix} \end{array}.$$

**[Step 2]:** Caculatee the objective weights of criteria. Based on Eq. (2), calculate the normalized IFEVs, shown as follow:

$$E_1 = 0.1630, E_2 = 0.2302, E_3 = 0.1914, E_4 = 0.1768.$$

Based on Eq. (3), summarize all the normalized IFEVs into $E_{sum}$:

$$E_{sum} = 0.7614.$$

Based on Eq. (4), we can get the weight $w_j$ for criterion $C_j$, where $1 \le j \le 4$, shown as follow:

$$w_1 = 0.2584, w_2 = 0.2377, w_3 = 0.2497, w_4 = 0.2542.$$

**[Step 3]:** Based on Eq. (5), the IFPIS $T^+$ can be shown as follows. Because the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion, i.e., $K_1 = \{C_1, C_2, C_3\}$ and $K_2 = \{C_4\}$, we can get the IFPIS $T^+ = \{t_1^+, t_2^+, t_3^+, t_4^+\}$, where

$$\begin{aligned} t_1^+ = (&\max(0.712, 0.628, 0.537, 0.691, 0.586), \\ &\min(0.157, 0.239, 0.296, 0.162, 0.177), \\ &(1 - \max(0.712, 0.628, 0.537, 0.691, 0.586) \\ &- \min(0.157, 0.239, 0.296, 0.162, 0.177))) \\ = (&0.712, 0.157, 0.131), \end{aligned}$$

$$\begin{aligned} t_2^+ = (&\max(0.491, 0.562, 0.612, 0.582, 0.627), \\ &\min(0.263, 0.197, 0.189, 0.201, 0.125), \\ &(1 - \max(0.491, 0.562, 0.612, 0.582, 0.627) \\ &- \min(0.263, 0.197, 0.189, 0.201, 0.125))) \\ = (&0.627, 0.125, 0.248), \end{aligned}$$

$$\begin{aligned} t_3^+ = (&\max(0.627, 0.582, 0.631, 0.609, 0.573), \\ &\min(0.183, 0.195, 0.209, 0.253, 0.181), \\ &(1 - (\max(0.627, 0.582, 0.631, 0.609, 0.573) \\ &- \min(0.183, 0.195, 0.209, 0.253, 0.181))) \\ = (&0.631, 0.181, 0.188), \end{aligned}$$

$$\begin{aligned} t_4^+ = (&\min(0.635, 0.619, 0.597, 0.681, 0.592), \\ &\max(0.217, 0.205, 0.196, 0.192, 0.182), \\ &(1 - \min(0.635, 0.619, 0.597, 0.681, 0.592) \end{aligned}$$

$$- \max(0.217,0.205,0.196,0.192,0.182))) = (0.592,0.217,0.191).$$

And based on Eq. (6), the IFNIS $T^-$ can be shown as follows. Because the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion, i.e., $K_1 = \{C_1, C_2, C_3\}$ and $K_2 = \{C_4\}$, we can get the IFNIS $T^- = \{t_1^-, t_2^-, \ldots, t_4^-\}$, where

$$t_1^- = (\min(0.712,0.628,0.537,0.691,0.586), \max(0.157,0.239,0.296,0.162,0.177), (1 - \min(0.712,0.628,0.537,0.691,0.586) - \max(0.157,0.239,0.296,0.162,0.177))) = (0.537,0.296,0.167).$$

$$t_2^- = (\min(0.491,0.562,0.612,0.582,0.627), \max(0.263,0.197,0.189,0.201,0.125), (1 - \min(0.491,0.562,0.612,0.582,0.627) - \max(0.263,0.197,0.189,0.201,0.125))) = (0.491,0.263,0.246).$$

$$t_3^- = (\min(0.627,0.582,0.631,0.609,0.573), \max(0.183,0.195,0.209,0.253,0.181), (1 - \min(0.627,0.582,0.631,0.609,0.573) - \max(0.183,0.195,0.209,0.253,0.181))) = (0.573,0.253,0.174).$$

$$t_4^- = (\max(0.635,0.619,0.597,0.681,0.592), \min(0.217,0.205,0.196,0.192,0.182), (1 - \max(0.635,0.619,0.597,0.681,0.592) - \min(0.217,0.205,0.196,0.192,0.182))) = (0.681,0.182,0.137).$$

**[Step 4]:** Based on Eq. (7) and the weight vector $W = (w_1, w_2, \cdots, w_4)^T = (0.2584,0.2377,0.2497,0.2542)^T$ for the criteria $C_1$, $C_2$, $C_3$ and $C_4$ obtained from **[Step 2]**, calculate the WSM $S_{\omega_i}^+$ between the elements at the $i$th row of the IFV decision matrix $D = (d_{ij})_{5\times 4}$ and the elements in the obtained IFPIS $T^+$, where $1 \le i \le 5$ and $1 \le j \le 4$, shown as follows:

$$S_{\omega_1}^+ = 1 - 0.2584 \times \left[\tfrac{1}{6}(|0.712 - 0.712| + |0.157 - 0.157| + |0.131 - 0.131|) + \tfrac{1}{2}(\max\{|0.712 - 0.712|, |0.157 - 0.157|, |0.131 - 0.131|\})\right] - 0.2377 \times \left[\tfrac{1}{6}(|0.491 - 0.627| + |0.263 - 0.125| + |0.246 - 0.248|) + \tfrac{1}{2}(\max\{|0.491 - 0.627|, |0.263 - 0.125|, |0.246 - 0.248|\})\right] - 0.2497 \times \left[\tfrac{1}{6}(|0.627 - 0.631| + |0.183 - 0.181| + |0.190 - 0.188|) + \tfrac{1}{2}(\max\{|0.627 - 0.631|, |0.183 - 0.181|, |0.190 - 0.188|\})\right] - 0.2542 \times \left[\tfrac{1}{6}(|0.635 - 0.592| + |0.217 - 0.217| + |0.148 - 0.191|) + \tfrac{1}{2}(\max\{|0.635 - 0.592|, |0.217 - 0.217|, |0.148 - 0.191|\})\right] 1 - (0.2584 \times 0.0000 + 0.2377 \times 0.1150 + 0.2497 \times 0.0033 + 0.2542 \times 0.0358) = 0.9627,$$

$$S_{\omega_2}^+ = 1 - 0.2584 \times \left[\tfrac{1}{6}(|0.628 - 0.712| + |0.239 - 0.157| + |0.133 - 0.131|) + \tfrac{1}{2}(\max\{|0.628 - 0.712|, |0.239 - 0.157|, |0.133 - 0.131|\})\right] - 0.2377 \times \left[\tfrac{1}{6}(|0.562 - 0.627| + |0.197 - 0.125| + |0.241 - 0.248|) + \tfrac{1}{2}(\max\{|0.562 - 0.627|, |0.197 - 0.125|, |0.241 - 0.248|\})\right] - 0.2497 \times \left[\tfrac{1}{6}(|0.582 - 0.631| + |0.195 - 0.181| + |0.223 - 0.188|) + \tfrac{1}{2}(\max\{|0.582 - 0.631|, |0.195 - 0.181|, |0.223 - 0.188|\})\right] - 0.2542 \times \left[\tfrac{1}{6}(|0.619 - 0.592| + |0.205 - 0.217| + |0.176 - 0.191|) + \tfrac{1}{2}(\max\{|0.619 - 0.592|, |0.205 - 0.217|, |0.176 - 0.191|\})\right] = 1 - (0.2584 \times 0.0700 + 0.2377 \times 0.0600 + 0.2497 \times 0.0408 + 0.2542 \times 0.0225) = 0.9517,$$

$$S_{\omega_3}^+ = 1 - 0.2584 \times \left[\tfrac{1}{6}(|0.537 - 0.712| + |0.296 - 0.157| + |0.167 - 0.131|) + \tfrac{1}{2}(\max\{|0.537 - 0.712|, |0.296 - 0.157|, |0.167 - 0.131|\})\right] - 0.2377 \times \left[\tfrac{1}{6}(|0.612 - 0.627| + |0.189 - 0.125| + |0.199 - 0.248|) + \tfrac{1}{2}(\max\{|0.612 - 0.627|, |0.189 - 0.125|, |0.199 - 0.248|\})\right] - 0.2497 \times \left[\tfrac{1}{6}(|0.631 - 0.631| + |0.209 - 0.181| + |0.160 - 0.188|) + \tfrac{1}{2}(\max\{|0.631 - 0.631|, |0.209 - 0.181|, |0.160 - 0.188|\})\right] - 0.2542 \times \left[\tfrac{1}{6}(|0.597 - 0.592| + |0.196 - 0.217| + |0.207 - 0.191|) \tfrac{1}{2}(\max\{|0.597 - 0.592|, |0.196 - 0.217|, |0.207 - 0.191|\})\right] = 1 - (0.2584 \times 0.1458 + 0.2377 \times 0.0533 + 0.2497 \times 0.0233 + 0.2542 \times 0.0175) = 0.9394,$$

$$S_{\omega_4}^+ = 1 - 0.2584 \times \left[\tfrac{1}{6}(|0.691 - 0.712| + |0.162 - 0.157| + |0.147 - 0.131|) + \tfrac{1}{2}(\max\{|0.691 - 0.712|, |0.162 - 0.157|, |0.147 - 0.131|\})\right] - 0.2377 \times \left[\tfrac{1}{6}(|0.582 - 0.627| + |0.201 - 0.125| + |0.217 - 0.248|) + \tfrac{1}{2}(\max\{|0.582 - 0.627|, |0.201 - 0.125|, |0.217 - 0.248|\})\right] - 0.2497 \times \left[\tfrac{1}{6}(|0.609 - 0.631| + |0.253 - 0.181| + |0.138 - 0.188|) + \tfrac{1}{2}(\max\{|0.609 - 0.631|, |0.253 - 0.181|, |0.138 - 0.188|\})\right] - 0.2542 \times \left[\tfrac{1}{6}(|0.681 - 0.592| + |0.192 - 0.217| + |0.127 - 0.191|) + \tfrac{1}{2}(\max\{|0.681 - 0.592|, |0.192 - 0.217|, |0.127 - 0.191|\})\right] = 1 - (0.2584 \times 0.0175 + 0.2377 \times 0.0633 + 0.2497 \times 0.0600 + 0.2542 \times 0.0742) = 0.9466,$$

$$S_{\omega_5}^+ = 1 - 0.2584 \times \left[\tfrac{1}{6}(|0.586 - 0.712| + |0.177 - 0.157| + |0.237 - 0.131|) + \tfrac{1}{2}(\max\{|0.586 - 0.712|, |0.177 - 0.157|, |0.237 - 0.131|\})\right] - 0.2377 \times \left[\tfrac{1}{6}(|0.627 - 0.627| + |0.125 - 0.125| + |0.248 - 0.248|) + \tfrac{1}{2}(\max\{|0.627 - 0.627|, |0.125 - 0.125|, |0.248 - 0.248|\})\right] - 0.2497 \times \left[\tfrac{1}{6}(|0.573 - 0.631| +\right.$$

$$|0.181-0.181|+|0.246-0.188|) +\frac{1}{2}(\max\{|0.573-0.631|,|0.181-0.181|,|0.246-0.188|\})\Big]-\ 0.2542\times\Big[\frac{1}{6}(|0.592-0.592|+|0.182-0.217|+|0.226-0.191|)+\frac{1}{2}(\max\{|0.592-0.592|,|0.182-0.217|,|0.226-0.191|\})\Big]=1-(0.2584\times 0.1050+0.2377\times 0.0000+0.2497\times 0.0483+0.2542\times 0.0292)=0.9534.$$

**[Step 5]:** Based on Eq. (8) and the weight vector $W=(w_1,w_2,\cdots,w_4)^T=(0.2584,0.2377,0.2497,0.2542)^T$ for the criteria $C_1$, $C_2$, $C_3$ and $C_4$ obtained from **[Step 2]**, calculate the WSM $S^-_{\omega_i}$ between the elements at the $i$th row of the IFV decision matrix $D=(d_{ij})_{5\times 4}$ and the elements in the obtained IFNIS $T^-$, where $1\le i\le 5$ and $1\le j\le 4$, shown as follows:

$$S^-_{\omega_1}=1-0.2584\times\Big[\frac{1}{6}(|0.712-0.537|+|0.157-0.296|+|0.131-0.167|)+\frac{1}{2}(\max\{|0.712-0.537|,|0.157-0.296|,|0.131-0.167|\})\Big]-0.2377\times\Big[\frac{1}{6}(|0.491-0.491|+|0.263-0.263|+|0.246-0.246|)+\frac{1}{2}(\max\{|0.491-0.491|,|0.263-0.263|,|0.246-0.246|\})\Big]-0.2497\times\Big[\frac{1}{6}(|0.627-0.573|+|0.183-0.253|+|0.190-0.174|)+\frac{1}{2}(\max\{|0.627-0.573|,|0.183-0.253|,|0.190-0.174|\})\Big]-0.2542\times\Big[\frac{1}{6}(|0.635-0.681|+|0.217-0.182|+|0.148-0.137|)+\frac{1}{2}(\max\{|0.635-0.592|,|0.217-0.217|,|0.148-0.191|\})\Big]=1-(0.2584\times 0.1458+0.2377\times 0.0000+0.2497\times 0.0583+0.2542\times 0.0383)=0.9380,$$

$$S^-_{\omega_2}=1-0.2584\times\Big[\frac{1}{6}(|0.628-0.537|+|0.239-0.296|+|0.133-0.167|)+\frac{1}{2}(\max\{|0.628-0.537|,|0.239-0.296|,|0.133-0.167|\})\Big]-0.2377\times\Big[\frac{1}{6}(|0.562-0.491|+|0.197-0.263|+|0.241-0.246|)+\frac{1}{2}(\max\{|0.562-0.491|,|0.197-0.263|,|0.241-0.246|\})\Big]-0.2497\times\Big[\frac{1}{6}(|0.582-0.573|+|0.195-0.253|+|0.223-0.174|)+\frac{1}{2}(\max\{|0.582-0.573|,|0.195-0.253|,|0.223-0.174|\})\Big]-0.2542\times\Big[\frac{1}{6}(|0.619-0.681|+|0.205-0.182|+|0.176-0.137|)+\frac{1}{2}(\max\{|0.619-0.681|,|0.205-0.182|,|0.176-0.137|\})\Big]=1-(0.2584\times 0.0758+0.2377\times 0.0592+0.2497\times 0.0483+0.2542\times 0.0517)=0.9411,$$

$$S^-_{\omega_3}=1-0.2584\times\Big[\frac{1}{6}(|0.537-0.537|+|0.296-0.296|+|0.167-0.167|)+\frac{1}{2}(\max\{|0.537-0.537|,|0.296-0.296|,|0.167-0.167|\})\Big]-0.2377\times\Big[\frac{1}{6}(|0.612-0.491|+|0.189-0.263|+|0.199-0.246|)+\frac{1}{2}(\max\{|0.612-0.491|,|0.189-0.263|,|0.199-0.246|\})\Big]-0.2497\times\Big[\frac{1}{6}(|0.631-0.573|+|0.209-0.253|+|0.160-0.174|)+\frac{1}{2}(\max\{|0.631-0.573|,|0.209-0.253|,|0.160-0.174|\})\Big]-0.2542\times\Big[\frac{1}{6}(|0.597-0.681|+|0.196-0.182|+|0.207-0.137|)+\frac{1}{2}(\max\{|0.597-0.681|,|0.196-0.182|,|0.207-0.137|\})\Big]=1-(0.2584\times 0.0000+0.2377\times 0.1008+0.2497\times 0.0483+0.2542\times 0.0700=0.9462,$$

$$S^-_{\omega_4}=1-0.2584\times\Big[\frac{1}{6}(|0.691-0.537|+|0.162-0.296|+|0.147-0.167|)+\frac{1}{2}(\max\{|0.691-0.537|,|0.162-0.296|,|0.147-0.167|\})\Big]-0.2377\times\Big[\frac{1}{6}(|0.582-0.491|+|0.201-0.263|+|0.217-0.246|)+\frac{1}{2}(\max\{|0.582-0.491|,|0.201-0.263|,|0.217-0.246|\})\Big]-0.2497\times\Big[\frac{1}{6}(|0.609-0.573|+|0.253-0.253|+|0.138-0.174|)+\frac{1}{2}(\max\{|0.609-0.573|,|0.253-0.253|,|0.138-0.174|\})\Big]-0.2542\times\Big[\frac{1}{6}(|0.681-0.681|+|0.192-0.182|+|0.127-0.137|)+\frac{1}{2}(\max\{|0.681-0.681|,|0.192-0.182|,|0.127-0.137|\})\Big]=1-(0.2584\times 0.1283+0.2377\times 0.0758+0.2497\times 0.0300+0.2542\times 0.0083)=0.9392,$$

$$S^-_{\omega_5}=1-0.2584\times\Big[\frac{1}{6}(|0.586-0.537|+|0.177-0.296|+|0.237-0.167|)+\frac{1}{2}(\max\{|0.586-0.537|,|0.177-0.296|,|0.237-0.167|\})\Big]-0.2377\times\Big[\frac{1}{6}(|0.627-0.491|+|0.125-0.263|+|0.248-0.246|)+\frac{1}{2}(\max\{|0.627-0.491|,|0.125-0.263|,|0.248-0.246|\})\Big]-0.2497\times\Big[\frac{1}{6}(|0.573-0.573|+|0.181-0.253|+|0.246-0.174|)+\frac{1}{2}(\max\{|0.573-0.573|,|0.181-0.253|,|0.246-0.174|\})\Big]-0.2542\times\Big[\frac{1}{6}(|0.592-0.681|+|0.182-0.182|+|0.226-0.137|)+\frac{1}{2}(\max\{|0.592-0.681|,|0.182-0.182|,|0.226-0.137|\})\Big]=1-(0.2584\times 0.0992+0.2377\times 0.1150+0.2497\times 0.0600+0.2542\times 0.0742)=0.9132,$$

**[Step 6]**: Based on Eq. (9), we can get the relative closeness of alternative $A_i$ with respect to the IFPIS $T^+$, where $1 \le i \le 5$, shown as follows:

$$R_1^* = \frac{S_{\omega_1}^+}{S_{\omega_1}^+ + S_{\omega_1}^-} = \frac{0.9627}{0.9627 + 0.9380} = 0.5065,$$

$$R_2^* = \frac{S_{\omega_2}^+}{S_{\omega_2}^+ + S_{\omega_2}^-} = \frac{0.9517}{0.9517 + 0.9411} = 0.5028,$$

$$R_3^* = \frac{S_{\omega_3}^+}{S_{\omega_3}^+ + S_{\omega_3}^-} = \frac{0.9394}{0.9394 + 0.9462} = 0.4982,$$

$$R_4^* = \frac{S_{\omega_4}^+}{S_{\omega_4}^+ + S_{\omega_4}^-} = \frac{0.9466}{0.9466 + 0.9392} = 0.5020,$$

$$R_5^* = \frac{S_{\omega_5}^+}{S_{\omega_5}^+ + S_{\omega_5}^-} = \frac{0.9534}{0.9534 + 0.9132} = 0.5108.$$

Because $R_5^* \succ R_1^* \succ R_2^* \succ R_4^* \succ R_3^*$, the preference order of the alternatives $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ is: $A_5 \succ A_1 \succ A_2 \succ A_4 \succ A_3$. The appropriate alternative, i.e., $A_5$, obtained by the proposed method is coincided with Chen's method [4]. The advantage of the proposed method is that it is simpler than Chen's method [4] for MCDM under IF environments. However, Chen's method [4] has the drawback of getting an unreasonable preference order of the alternatives in the context of the "division by zero" ("DBZ") situations, illustrated in ***Example 4.2***.

***Example 4.2***: Assuming that there are five alternatives: $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ and assuming that there are four criteria: $C_1$, $C_2$, $C_3$, and $C_4$, where the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion. Assume that the IFV decision matrix $D = (d_{ij})_{5\times4}$ is given by the decision maker.

**[Step 1]:** Construct the IFV decision matrix $D = (d_{ij})_{5\times4}$, shown as follows:

$$D = (d_{ij})_{5\times4} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{bmatrix} (1.000, 0.000,0.000) & (0.491, 0.263,0.246) & (0.627, 0.183,0.190) & (0.635, 0.217,0.148) \\ (1.000, 0.000,0.000) & (0.562,0.197,0.241) & (0.582,0.195,0.223) & (0.619,0.205,0.176) \\ (1.000, 0.000,0.000) & (0.612,0.189,0.199) & (0.631,0.209,0.160) & (0.597,0.196,0.207) \\ (1.000, 0.000,0.000) & (0.582,0.201,0.217) & (0.609,0.253,0.138) & (0.681,0.192,0.127) \\ (1.000, 0.000,0.000) & (0.627,0.125,0.248) & (0.573,0.181,0.246) & (0.592,0.182,0.226) \end{bmatrix} \end{array}.$$

**[Step 2]:** Calculate the objective weights of criteria. Based on Eq. (2), calculate the normalized IFEVs, shown as follow:

$$E_1 = 0.000, E_2 = 0.2302, E_3 = 0.1914, E_4 = 0.1768.$$

Based on Eq. (3), summarize all the normalized IFEVs into $E_{sum}$:

$$E_{sum} = 0.5984.$$

Based on Eq. (4), we can get the weight $w_j$ for criterion $C_j$, where $1 \le j \le 4$, shown as follow:

$$w_1 = 0.2940, w_2 = 0.2263, w_3 = 0.2377, w_4 = 0.2420.$$

**[Step 3]:** Based on Eq. (5), the IFPIS $T^+$ can be obtained shown as follows. Because the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion, i.e., $K_1 = \{C_1, C_2, C_3\}$ and $K_2 = \{C_4\}$, we can get the IFPIS $T^+ = \{t_1^+, t_2^+, t_3^+, t_4^+\}$, where

$$\begin{aligned} t_1^+ = {} & (\max(1.000,1.000,1.000,1.000,1.000), \\ & \min(0.000,0.000,0.000,0.000,0.000), \\ & (1 - \max(1.000,1.000,1.000,1.000,1.000) \\ & - \min(0.000,0.000,0.000,0.000,0.000))) \\ = {} & (1.000,0.000,0.000), \end{aligned}$$

$$\begin{aligned} t_2^+ = {} & (\max(0.491,0.562,0.612,0.582,0.627), \\ & \min(0.263,0.197,0.189,0.201,0.125), \\ & (1 - \max(0.491,0.562,0.612,0.582,0.627) \\ & - \min(0.263,0.197,0.189,0.201,0.125))) \\ = {} & (0.627,0.125,0.248), \end{aligned}$$

$$\begin{aligned} t_3^+ = {} & (\max(0.627,0.582,0.631,0.609,0.573), \\ & \min(0.183,0.195,0.209,0.253,0.181), \\ & (1 - \max(0.627,0.582,0.631,0.609,0.573) \\ & - \min(0.183,0.195,0.209,0.253,0.181))) \\ = {} & (0.631,0.181,0.188), \end{aligned}$$

$$\begin{aligned} t_4^+ = {} & (\min(0.635,0.619,0.597,0.681,0.592), \\ & \max(0.217,0.205,0.196,0.192,0.182), \\ & (1 - \min(0.635,0.619,0.597,0.681,0.592) \\ & - \max(0.217,0.205,0.196,0.192,0.182))) \\ = {} & (0.592,0.217,0.191). \end{aligned}$$

And based on Eq. (6), the IFNIS $T^-$ can be obtained shown as follows. Because the criteria $C_1$, $C_2$ and $C_3$ are benefit criteria and the criterion $C_4$ is cost criterion, i.e., $K_1 = \{C_1, C_2, C_3\}$ and $K_2 = \{C_4\}$, we can get the IFNIS $T^- = \{t_1^-, t_2^-, \ldots, t_4^-\}$, where

$$\begin{aligned} t_1^- = {} & (\min(1.000,1.000,1.000,1.000,1.000), \\ & \max(0.000,0.000,0.000,0.000,0.000), \\ & (1 - \min(1.000,1.000,1.000,1.000,1.000) \\ & - \max(0.000,0.000,0.000,0.000,0.000))) \\ = {} & (1.000,0.000,0.000), \end{aligned}$$

$$\begin{aligned} t_2^- = {} & (\min(0.491,0.562,0.612,0.582,0.627), \\ & \max(0.263,0.197,0.189,0.201,0.125), \\ & (1 - \min(0.491,0.562,0.612,0.582,0.627) \\ & - \max(0.263,0.197,0.189,0.201,0.125))) \\ = {} & (0.491,0.263,0.246), \end{aligned}$$

$$\begin{aligned} t_3^- = {} & (\min(0.627,0.582,0.631,0.609,0.573), \\ & \max(0.183,0.195,0.209,0.253,0.181), \\ & (1 - \min(0.627,0.582,0.631,0.609,0.573) \\ & - \max(0.183,0.195,0.209,0.253,0.181))) \\ = {} & (0.573,0.253,0.174), \end{aligned}$$

$$\begin{aligned} t_4^- = {} & (\max(0.635,0.619,0.597,0.681,0.592), \\ & \min(0.217,0.205,0.196,0.192,0.182), \\ & (1 - \max(0.635,0.619,0.597,0.681,0.592) \\ & - \min(0.217,0.205,0.196,0.192,0.182))) \\ = {} & (0.681,0.182,0.137). \end{aligned}$$

**[Step 4]:** Based on Eq. (7) and the weight vector $W = (w_1, w_2, \cdots, w_4)^T = (0.2584,0.2377,0.2497,0.2542)^T$ for the criteria $C_1$, $C_2$, $C_3$ and $C_4$ obtained from **[Step 2]**, calculate the WSM $S_{\omega_i}^+$ between the elements at the $i$th row of the IFV decision matrix $D = (d_{ij})_{5\times4}$ and the elements in the obtained IFPIS $T^+$, where $1 \le i \le 5$ and $1 \le j \le 4$, shown as follows:

$$S_{\omega_1}^+ = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.491 - 0.627| + |0.263 - 0.125| + |0.246 - 0.248|) + \frac{1}{2}(\max\{|0.491 - 0.627|, |0.263 - 0.125|, |0.246 - 0.248|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.627 - 0.631| + |0.183 - 0.181| + |0.190 - 0.188|) + \frac{1}{2}(\max\{|0.627 - 0.631|, |0.183 - 0.181|, |0.190 - 0.188|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.635 - 0.592| + |0.217 - 0.217| + |0.148 - 0.191|) + \frac{1}{2}(\max\{|0.635 - 0.592|, |0.217 - 0.217|, |0.148 - 0.191|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.1150 + 0.2377 \times 0.0033 + 0.2420 \times 0.0358) = 0.9645,$$

$$S_{\omega_2}^+ = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.562 - 0.627| + |0.197 - 0.125| + |0.241 - 0.248|) + \frac{1}{2}(\max\{|0.562 - 0.627|, |0.197 - 0.125|, |0.241 - 0.248|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.582 - 0.631| + |0.195 - 0.181| + |0.223 - 0.188|) + \frac{1}{2}(\max\{|0.582 - 0.631|, |0.195 - 0.181|, |0.223 - 0.188|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.619 - 0.592| + |0.205 - 0.217| + |0.176 - 0.191|) + \frac{1}{2}(\max\{|0.619 - 0.592|, |0.205 - 0.217|, |0.176 - 0.191|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0600 + 0.2377 \times 0.0408 + 0.2420 \times 0.0225) = 0.9713,$$

$$S_{\omega_3}^+ = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.612 - 0.627| + |0.189 - 0.125| + |0.199 - 0.248|) + \frac{1}{2}(\max\{|0.612 - 0.627|, |0.189 - 0.125|, |0.199 - 0.248|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.631 - 0.631| + |0.209 - 0.181| + |0.160 - 0.188|) + \frac{1}{2}(\max\{|0.631 - 0.631|, |0.209 - 0.181|, |0.160 - 0.188|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.597 - 0.592| + |0.196 - 0.217| + |0.207 - 0.191|) + \frac{1}{2}(\max\{|0.597 - 0.592|, |0.196 - 0.217|, |0.207 - 0.191|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0533 + 0.2377 \times 0.0233 + 0.2420 \times 0.0175) = 0.9781,$$

$$S_{\omega_4}^+ = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.582 - 0.627| + |0.201 - 0.125| + |0.217 - 0.248|) + \frac{1}{2}(\max\{|0.582 - 0.627|, |0.201 - 0.125|, |0.217 - 0.248|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.609 - 0.631| + |0.253 - 0.181| + |0.138 - 0.188|) + \frac{1}{2}(\max\{|0.609 - 0.631|, |0.253 - 0.181|, |0.138 - 0.188|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.681 - 0.592| + |0.192 - 0.217| + |0.127 - 0.191|) + \frac{1}{2}(\max\{|0.681 - 0.592|, |0.192 - 0.217|, |0.127 - 0.191|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0633 + 0.2377 \times 0.0600 + 0.2420 \times 0.0742) = 0.9535,$$

$$S_{\omega_5}^+ = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.627 - 0.627| + |0.125 - 0.125| + |0.248 - 0.248|) + \frac{1}{2}(\max\{|0.627 - 0.627|, |0.125 - 0.125|, |0.248 - 0.248|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.573 - 0.631| + |0.181 - 0.181| + |0.246 - 0.188|) \frac{1}{2}(\max\{|0.573 - 0.631|, |0.181 - 0.181|, |0.246 - 0.188|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.592 - 0.592| + |0.182 - 0.217| + |0.226 - 0.191|) + \frac{1}{2}(\max\{|0.592 - 0.592|, |0.182 - 0.217|, |0.226 - 0.191|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0000 + 0.2377 \times 0.0483 + 0.2420 \times 0.0292) = 0.9815,$$

**[Step 5]:** Based on Eq. (8) and the weight vector $W = (w_1, w_2, \cdots, w_4)^T = (0.2584,0.2377,0.2497,0.2542)^T$ for the criteria $C_1$, $C_2$, $C_3$ and $C_4$ obtained from **[Step 2]**, calculate the WSM $S_{\omega_i}^-$ between the elements at the $i$th row of the IFV decision matrix $D = (d_{ij})_{5\times 4}$ and the elements in the obtained IFNIS $T^-$, where $1 \le i \le 5$ and $1 \le j \le 4$, shown as follows:

$$S_{\omega_1}^- = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.491 - 0.491| + |0.263 - 0.263| + |0.246 - 0.246|) + \frac{1}{2}(\max\{|0.491 - 0.491|, |0.263 - 0.263|, |0.246 - 0.246|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.627 - 0.573| + |0.183 - 0.253| + |0.190 - 0.174|) + \frac{1}{2}(\max\{|0.627 - 0.573|, |0.183 - 0.253|, |0.190 -$$

$0.174|\})\big] - 0.2420 \times \big[\frac{1}{6}(|0.635 - 0.681| + |0.217 - 0.182| + |0.148 - 0.137|) + \frac{1}{2}(\max\{|0.635 - 0.681|, |0.217 - 0.182|, |0.148 - 0.137|\})\big] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0000 + 0.2377 \times 0.0583 + 0.2420 \times 0.0383) = 0.9769,$

$$S_{\omega_2}^- = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.562 - 0.491| + |0.197 - 0.263| + |0.241 - 0.246|) + \frac{1}{2}(\max\{|0.562 - 0.491|, |0.197 - 0.263|, |0.241 - 0.246|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.582 - 0.573| + |0.195 - 0.253| + |0.223 - 0.174|) + \frac{1}{2}(\max\{|0.582 - 0.573|, |0.195 - 0.253|, |0.223 - 0.174|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.619 - 0.681| + |0.205 - 0.182| + |0.176 - 0.137|) + \frac{1}{2}(\max\{|0.619 - 0.681|, |0.205 - 0.182|, |0.176 - 0.137|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0592 + 0.2377 \times 0.0483 + 0.2420 \times 0.0517) = 0.9626,$$

$$S_{\omega_3}^- = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.612 - 0.491| + |0.189 - 0.263| + |0.199 - 0.246|) + \frac{1}{2}(\max\{|0.612 - 0.491|, |0.189 - 0.263|, |0.199 - 0.246|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.631 - 0.573| + |0.209 - 0.253| + |0.160 - 0.174|) + \frac{1}{2}(\max\{|0.631 - 0.573|, |0.209 - 0.253|, |0.160 - 0.174|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.597 - 0.681| + |0.196 - 0.182| + |0.207 - 0.137|) + \frac{1}{2}(\max\{|0.597 - 0.681|, |0.196 - 0.182|, |0.207 - 0.137|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.1008 + 0.2377 \times 0.0483 + .2420 \times 0.0700) = 0.9488,$$

$$S_{\omega_4}^- = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.582 - 0.491| + |0.201 - 0.263| + |0.217 - 0.246|) + \frac{1}{2}(\max\{|0.582 - 0.491|, |0.201 - 0.263|, |0.217 - 0.246|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.609 - 0.573| + |0.253 - 0.253| + |0.138 - 0.174|) + \frac{1}{2}(\max\{|0.609 - 0.573|, |0.253 - 0.253|, |0.138 - 0.174|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.681 - 0.681| + |0.192 - 0.182| + |0.127 - 0.137|) + \frac{1}{2}(\max\{|0.681 - 0.681|, |0.192 - 0.182|, |0.127 - 0.137|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.0758 + 0.2377 \times 0.0300 + 0.2420 \times 0.0083) = 0.9737,$$

$$S_{\omega_5}^- = 1 - 0.2940 \times \left[\frac{1}{6}(|1.000 - 1.000| + |0.000 - 0.000| + |0.000 - 0.000|) + \frac{1}{2}(\max\{|1.000 - 1.000|, |0.000 - 0.000|, |0.000 - 0.000|\})\right] - 0.2263 \times \left[\frac{1}{6}(|0.627 - 0.491| + |0.125 - 0.263| + |0.248 - 0.246|) + \frac{1}{2}(\max\{|0.627 - 0.491|, |0.125 - 0.263|, |0.248 - 0.246|\})\right] - 0.2377 \times \left[\frac{1}{6}(|0.573 - 0.573| + |0.181 - 0.253| + |0.246 - 0.174|) + \frac{1}{2}(\max\{|0.573 - 0.573|, |0.181 - 0.253|, |0.246 - 0.174|\})\right] - 0.2420 \times \left[\frac{1}{6}(|0.592 - 0.681| + |0.182 - 0.182| + |0.226 - 0.137|) + \frac{1}{2}(\max\{|0.592 - 0.681|, |0.182 - 0.182|, |0.226 - 0.137|\})\right] = 1 - (0.2940 \times 0.0000 + 0.2263 \times 0.1150 + 0.2377 \times 0.0600 + 0.2420 \times 0.0742) = 0.9418,$$

**[Step 6]**: Based on Eq. (9), we can get the relative closeness of alternative $A_i$ with respect to the IFPIS $T^+$, where $1 \le i \le 5$, shown as follows:

$$R_1^* = \frac{S_{\omega_1}^+}{S_{\omega_1}^+ + S_{\omega_1}^-} = \frac{0.9645}{0.9645 + 0.9769} = 0.4968,$$

$$R_2^* = \frac{S_{\omega_2}^+}{S_{\omega_2}^+ + S_{\omega_2}^-} = \frac{0.9713}{0.9713 + 0.9626} = 0.5022,$$

$$R_3^* = \frac{S_{\omega_3}^+}{S_{\omega_3}^+ + S_{\omega_3}^-} = \frac{0.9781}{0.9781 + 0.9488} = 0.5076,$$

$$R_4^* = \frac{S_{\omega_4}^+}{S_{\omega_4}^+ + S_{\omega_4}^-} = \frac{0.9535}{0.9535 + 0.9737} = 0.4948,$$

$$R_5^* = \frac{S_{\omega_5}^+}{S_{\omega_5}^+ + S_{\omega_5}^-} = \frac{0.9815}{0.9815 + 0.9418} = 0.5103.$$

Because $R_5^* \succ R_3^* \succ R_2^* \succ R_1^* \succ R_4^*$, the preference order of the alternatives $A_1$, $A_2$, $A_3$, $A_4$ and $A_5$ is: $A_5 \succ A_3 \succ A_2 \succ A_1 \succ A_4$. The appropriate alternative obtained by the proposed method is $A_5$. The advantage of the proposed method is that it is simpler than Chen's method [4] for MCDM under IF environments. However, Chen's method [4] has the drawback of getting an unreasonable preference order of the alternatives in the context of the "DBZ" situations.

In the following, we explain the reason why the Chen's method [4] gets an unreasonable preference order of the alternatives of ***Example 4.2***. In ***Example 4.2***, the decision matrix $D = (d_{ij})_{5\times 4}$ represented by IFVs is shown as follows:

$D = (d_{ij})_{5\times 4} =$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{bmatrix} (1.000, 0.000, 0.000) & (0.491, 0.263, 0.246) & (0.627, 0.183, 0.190) & (0.635, 0.217, 0.148) \\ (1.000, 0.000, 0.000) & (0.562, 0.197, 0.241) & (0.582, 0.195, 0.223) & (0.619, 0.205, 0.176) \\ (1.000, 0.000, 0.000) & (0.612, 0.189, 0.199) & (0.631, 0.209, 0.160) & (0.597, 0.196, 0.207) \\ (1.000, 0.000, 0.000) & (0.582, 0.201, 0.217) & (0.609, 0.253, 0.138) & (0.681, 0.192, 0.127) \\ (1.000, 0.000, 0.000) & (0.627, 0.125, 0.248) & (0.573, 0.181, 0.246) & (0.592, 0.182, 0.226) \end{bmatrix} \end{array}.$$

Based on **[Step 2]** of the method presented in [4], we calculate the IFEV for alternative $A_1$ in $C_1$, shown as below:

$$E_{11}(x_1) = \frac{\{\min[1.000, 0.000] + \min[1 - 0.000, 1 - 1.000]\}}{\{\max[1.000, 0.000] + \max[1 - 0.000, 1 - 1.000]\}} = 0.000.$$

In the same way, we can use the above equation to calculate the other elements in decision matrix $D$. Therefore, the decision matrix $D$ is transformed into the following one:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & \quad C_2 & \quad C_3 & \quad C_4 \end{array} \\ \begin{bmatrix} 0.000 & 0.629 & 0.385 & 0.410 \\ 0.000 & 0.465 & 0.442 & 0.414 \\ 0.000 & 0.405 & 0.406 & 0.428 \\ 0.000 & 0.448 & 0.475 & 0.343 \\ 0.000 & 0.332 & 0.437 & 0.418 \end{bmatrix} \end{array}.$$

Based on **[Step 3]** of the method presented in [4], we can get normalized IFEVs, shown as follows:

$$e_{11} = \frac{0.000}{\max[0.000, 0.000, 0.000, 0.000, 0.000]} \rightarrow$$

occurs the "DBZ" problem,

$$e_{12} = \frac{0.629}{\max[0.629, 0.465, 0.405, 0.448, 0.332]} = 1.000,$$

$$e_{13} = \frac{0.385}{\max[0.385, 0.442, 0.406, 0.475, 0.437]} = 0.811,$$

$$e_{14} = \frac{0.000}{\max[0.000, 0.000, 0.000, 0.000, 0.000]} = 0.960.$$

In the same way, we can use the above equation to calculate the other elements in decision matrix $D$. Therefore, the decision matrix $D$ is now transformed into the following one:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & \quad C_2 & \quad C_3 & \quad C_4 \end{array} \\ \begin{bmatrix} \text{"DBZ"} & 1.000 & 0.811 & 0.960 \\ \text{"DBZ"} & 0.740 & 0.931 & 0.969 \\ \text{"DBZ"} & 0.645 & 0.856 & 1.000 \\ \text{"DBZ"} & 0.713 & 1.000 & 0.803 \\ \text{"DBZ"} & 0.527 & 0.920 & 0.979 \end{bmatrix} \end{array}.$$

It occurs the "DBZ" problem in **[Step 3]** of Chen's method [4] and then we cannot proceed with the following steps of the method presented in [4]. Therefore, Chen's method [4] has the drawback of getting an unreasonable preference order of the alternatives in the context of the "DBZ" situations.

## 5 Conclusion

In this paper, we have proposed a new MCDM method based on IFSs, the WSM [28], and the extension of the TOPSIS method with completely unknown weights of criteria for selecting appropriate sustainable building materials supplier in the initial stage of the supply chain. We have also used two examples to compare the experimental results of the proposed method with Chen's method [4]. The experimental results reveal that the proposed method is simpler than Chen's method [4] for MCDM and can overcome the drawbacks of Chen's method [4] that it cannot get the preference order of the alternatives in the context of the "DBZ" situations. Therefore, the proposed method provides us a good way to handle MCDM problems under IF environments. In this regard, it is confident that the proposed method can be improved to handle more real MCDM problems in the near future. It is worth of future research to expand the proposed method to further develop MCDM methods and MCGDM methods for more uncertain problems under IVIF and Fermatean fuzzy [22] environments.

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