# A Joint Iterative Quantization and Channel Estimation Scheme for One-Bit Massive MIMO Systems

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## Abstract

We consider the channel estimation problem for uplink multiuser massive MIMO systems with one-bit analog-to-digital converters (ADCs). The problem of optimal one-bit quantization thresholds is studied in this paper. Our analysis reveals that, if the quantization thresholds are optimally devised, using one-bit ADCs can achieve an estimation error close to (only increase by a factor of π/2) that of an ideal clairvoyant estimator using unquantized data. The optimal quantization thresholds, however, are dependent on the unknown channel parameters. To cope with this difficulty, we propose an Iterative Quantization (IQ) approach in which the thresholds are adaptively adjusted in a way such that the thresholds converge to the optimal thresholds. Simulation results show that our proposed iterative quantization scheme presents a similar performance compared with traditional channel estimation with full resolution ADCs.

**Keywords:** Massive MIMO systems, Channel estimation, One-bit quantization design, Cramér-Rao bound (CRB)

## 1 Introduction

Massive multiple-input multiple-output (MIMO), also known as large-scale or very-large MIMO, is a promising technology to meet the ever growing demands for higher throughput and better quality-of-service of next-generation wireless communication systems [1-4]. Despite its benefits, due to the large number of antennas at the base station, the hardware cost and power consumption could become prohibitively high if we still employ expensive and power hungry high resolution analog to digital convertors (ADCs). To address this obstacle, recent studies considered the use of low-resolution ADCs (e.g. 1-3 bits) for massive MIMO systems. For massive MIMO systems with low-resolution ADCs, the spectral efficiency and the uplink achievable rate were investigated in [5-7], under different assumptions. The theoretical analyses suggest that the use of the low cost and low-resolution ADCs can still provide satisfactory achievable rates and spectral efficiency.

This paper focuses on the problem of channel estimation for uplink multiuser massive MIMO systems, where one-bit ADCs are used at the base station in order to reduce the cost and power consumption. It was shown in [8] that for one-bit massive MIMO systems, a least squares channel estimation scheme and a maximum-ratio combining scheme are sufficient to support both multiuser operation and the use of high-order constellations. A Bayes-optimal joint channel and data estimation scheme was proposed in [9], in which the estimated payload data are utilized to aid channel estimation. In [10], a maximum likelihood channel estimator, along with a near maximum likelihood detector, were proposed for uplink massive MIMO systems with one-bit ADCs.

In this paper, we study one-bit quantizer design and examine the impact of the choice of quantization thresholds on the estimation performance. One-bit quantization threshold design is an interesting and important issue but neglected by existing channel estimation studies. In fact, most channel estimation schemes, e.g. [8-10], assume a fixed, typically zero, quantization threshold. Note that the optimal choice of the quantization threshold has been considered in the context of general estimation problems and distributed sensor networks [11-13] where only the estimation of a scalar parameter is investigated. In our problem, the parameter of interest is a vector and the optimal design of quantization thresholds as well as the training sequences should be considered jointly. Our theoretical results reveal that, if the quantization thresholds are optimally devised, using one-bit ADCs can still achieve an estimation error close to (only increase by a factor of π/2) the minimum achievable estimation error attained by using infinite-precision ADCs. The optimal quantization thresholds, however, are dependent on the unknown channel parameters. To cope with this difficulty, we propose an iterative quantization (IQ) scheme by which the thresholds are dynamically adjusted in a way such that the thresholds converge to the optimal thresholds. The realization of this iterative quantization scheme depends on the channel coherence

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time. Simulation results show that our proposed iterative quantization scheme presents a significant performance improvement over the scheme that use a fixed (say, zero) quantization threshold.

The rest of the paper is organized as follows. The system model and the problem of channel estimation in one-bit massive MIMO systems are discussed in Section 2. In Section 3, we develop a maximum likelihood estimator and carry out a Cramer-Rao bound analysis of the one-bit channel estimation problem. The optimal design of quantization thresholds and the pilot sequences is studied in Section 4. In Section 5, through exploiting the channel coherence time, we develop an iterative quantization scheme for practical threshold design. Simulation results are provided in Section 6, followed by concluding remarks in Section 7.

## 2 System Model

Consider a single-cell uplink multiuser massive MIMO system, where the base station equipped with a uniform linear array with $M$ antennas. The number of users is $K$, where each user equipped with a single antenna. In massive MIMO scenarios, we can assume $M \gg K$. Here the flat block fading channel is assumed, which means that the channel keeps constant during the coherence time. Under the above setup, the received signal at the base station can be expressed as

$$U = HX + W \quad \textbf{(1)}$$

where $X \in \mathbb{C}^{K \times L}$ denotes the combination of the each user's training sequences and the number of pilot symbols is $L$. $H \in \mathbb{C}^{M \times K}$ represents the unknown channel response matrix that to be estimated. $W \in \mathbb{C}^{M \times K}$ denotes the additive white Gaussian noise term. Here we assume the noise follows a zero-mean circularly symmetric complex Gaussian distribution with variance $2\sigma^2$.

In nowadays high-rate communication systems, to reduce the heavy burden in power consumption and hardware cost, a new kind of massive MIMO system were proposed, which is called one-bit massive MIMO. The architecture of the multi-user one-bit massive MIMO communication system is illustrated in Figure 1.

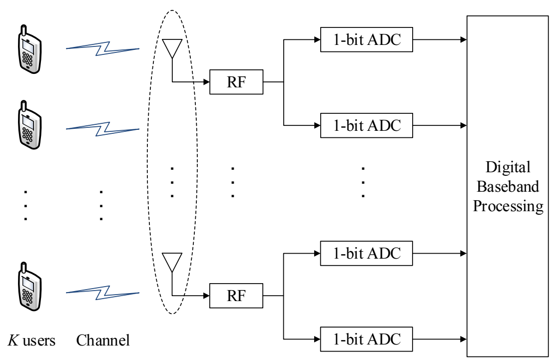


**Figure 1.** The architecture of the multi-user one-bit massive MIMO communication system

Different with the traditional one, in one-bit massive MIMO systems, only one-bit ADCs are equipped in the base station to quantize the received signal. For example, at each antenna, we deploy two one-bit ADCs. One is for the quantization of the real parts of the received signal, and the other is for the imaginary parts. Therefore, under this setup, the total number of one-bit ADCs we need is $2M$. The output data from the quantized received signal could be expressed as

$$V = \delta(U) \quad \textbf{(2)}$$

where $\delta(U)$ denotes an entry-wise function that to quantize the analog data matrix $U$. Specifically, for the $(m,l)$ th entry of $U$, we have

$$\delta(U_{m,l}) = q(\Re(U_{m,l})) + q(\Im(U_{m,l})) \cdot j \quad \textbf{(3)}$$

where $\Re(x)$ and $\Im(x)$ represents the real and imaginary parts of $x$, respectively. The quantization function $q(x)$ could be defined as

$$q(x) = \begin{cases} \rho, & x \geq 0 \\ -\rho, & x < 0 \end{cases} \quad \textbf{(4)}$$

where $\rho > 0$ is a constant to represent the sign of the received analog signal $x$. Here, we can assume $\rho = 1$. From these definitions, the output data after quantization satisfied

$$V_{m,l} \in \{\rho + j\rho, -\rho + j\rho, \rho - j\rho, -\rho - j\rho\}$$

These above discussions are all based on the traditional one-bit massive MIMO concepts, that the one-bit quantization threshold is fixed to zero (as shown in (2)). However, using an identical zero threshold for quantization at all antennas and time slots is not necessarily an optimal choice. To investigate the impact of the one-bit quantization threshold on the channel estimation performance, we here introduce a nonzero quantization threshold matrix in (2). Now the output data from the quantized received signal could be expressed as

$$V = \delta(U - \Theta) \quad \textbf{(5)}$$

where $\Theta \in \mathbb{C}^{M \times L}$ represents the threshold matrix for one-bit quantization. For simplicity, we can first rewrite (1) into a real-valued counterpart as

$$\bar{U} = \bar{\Phi}\bar{H} + \bar{W} \quad \textbf{(6)}$$

where

$$\bar{U} = [\Re(U), \Im(U)]^T$$
$$\bar{H} = [\Re(H), \Im(H)]^T$$
$$\bar{W} = [\Re(W), \Im(W)]^T$$
$$\bar{\Phi} = \begin{bmatrix} \Re(X) & \Im(X) \\ -\Im(X) & \Re(X) \end{bmatrix}^T$$

Reshaping the real matrix $\bar{U}$ into a column vector, we can obtain a more simplified matrix-vector expression as

$$u = \Phi h + w \tag{7}$$

where

$$\begin{aligned} &u = \text{vec}(\bar{U}), h = \text{vec}(\bar{H}), \\ &w = \text{vec}(\bar{W}), \Phi = I_M \otimes \bar{\Phi} \end{aligned} \tag{8}$$

and $\text{vec}(X)$ denotes the vectorization of the matrix $X$ formed by stacking its columns into a single column vector. Through these definition, we can easily justify that

$$\begin{aligned} &u \in \mathbb{R}^{2ML}, h \in \mathbb{R}^{2MK}, \\ &w \in \mathbb{R}^{2ML}, \Phi \in \mathbb{R}^{2ML \times 2MK} \end{aligned}$$

Thus the corresponding single-bit observed data can be expressed as

$$v = q(u - \theta) \tag{9}$$

where $q(x)$ is defined in (4). Here

$$\theta = \text{vec}\left([\Re(\Theta), \Im(\Theta)]^T\right) \in \mathbb{R}^{2ML}$$

For simplicity, we denote $N = 2ML$.

Under the above one-bit massive MIMO signal model, our goal of this paper is to recover the unknown channel coefficient vector $h$, after collecting the one-bit observed data $v$ in the base station. Furthermore, we also want to investigate the impact of the one-bit quantization threshold vector $\theta$ on the channel estimation performance. We seek to find the optimal design of the one-bit quantization threshold vector $\theta$, as well as the optimal training matrix $X$. In the following sections, we will develop a method for channel estimation based on maximum likelihood framework, and at the same time give a corresponding Cramer-Rao bound (CRB) analysis. Based on the CRB of the maximum likelihood estimation problem, we can easily find the optimal design of the one-bit quantization threshold vector $\theta$, as well as the optimal training matrix $X$.

## 3 Maximum Likelihood Method and CRB Analysis

In this section, we will develop a maximum likelihood method for one-bit massive MIMO channel estimation. Then, a corresponding Cramer-Rao bound analysis will followed.

### 3.1 Maximum Likelihood Method

From (7) and (9), we can get

$$\begin{aligned} v_n &= q(u_n - \theta_n) \\ &= q(\varphi_n^T h + w_n - \theta_n) \end{aligned} \tag{10}$$

where we denote $v_n$, $u_n$, $w_n$ and $\theta_n$ as the $n$ th element of vector $v$, $u$, $w$ and $\theta$, respectively. The vector $\varphi_n$ represents the $n$th row vector of matrix $\Phi$. From these definition, we can easily obtain

$$\begin{aligned} P(v_n = \rho) &= P(w_n \geq -(\varphi_n^T h - \theta_n)) \\ &= F_w(\varphi_n^T h - \theta_n) \end{aligned} \tag{11}$$

and

$$\begin{aligned} P(v_n = -\rho) &= P(w_n < -(\varphi_n^T h - \theta_n)) \\ &= 1 - F_w(\varphi_n^T h - \theta_n) \end{aligned} \tag{12}$$

where the notation $P(\cdot)$ denotes the probability and $F_w(\cdot)$ denotes the cumulative density function of $w_n$. Through the definition the Section 2, we know that $w_n$ follows the real Gaussian distribution with zero mean and variance $\sigma^2$. So we can calculate the probability mass function of $v_n$ as

$$\begin{aligned} p(v_n) = &\left[1 - F_w(\varphi_n^T h - \theta_n)\right]^{(\rho - v_n)/2\rho} \\ &\times \left[F_w(\varphi_n^T h - \theta_n)\right]^{(\rho + v_n)/2\rho} \end{aligned} \tag{13}$$

Because all the $v_n$ are uncorrelated, we can calculate the log probability mass function (or log likelihood function) as

$$\begin{aligned} L(h) &= \log p(v_1, \ldots, v_N) \\ &= \sum_{n=1}^{N} \left\{ \begin{aligned} &\frac{\rho - v_n}{2\rho} \log\left[1 - F_w(\varphi_n^T h - \theta_n)\right] \\ &+ \frac{\rho + v_n}{2\rho} \log\left[F_w(\varphi_n^T h - \theta_n)\right] \end{aligned} \right\} \end{aligned}$$

From this derivation, we can have the maximum likelihood estimate of the unknown channel coefficient vector $h$ as

$$\hat{h} = \arg\max_h L(h) \tag{14}$$

Furthermore, we can prove that this maximum likelihood objective function is a concave function. Hence this problem is turned to be a convex optimization problem, which can be solved and reach the global minimum with some computationally efficient search algorithms. We have the following theorem to prove the concavity of the function $L(h)$.

*Theorem 1*: The maximum likelihood function $L(h)$ is a concave function for the channel coefficient vector $h$.

*Proof*: It can be easily verified that $f_w(\varphi_n^T h - \theta_n)$ is log-concave in channel response vector $h$ since the Hessian matrix of $\log f_w(\varphi_n^T h - \theta_n)$, which is given by

$$\frac{\partial^2 \log f_w(\varphi_n^T h-\theta_n)}{\partial h \partial h^T}=-\frac{\varphi_n \varphi_n^T}{\sigma^2}$$

is negative semi-definite. Consequently the corresponding cumulative density function (CDF) and complementary CDF (CCDF), which are integrals of the log-concave function $f_w(\varphi_n^T h-\theta_n)$ over convex sets $(-\infty,\theta_n)$ and $(\theta_n,+\infty)$ respectively, are also log-concave, and their logarithms are concave. Since summation preserves concavity, $L(h)$ is a concave function of the channel response vector $h$. □

### 3.2 CRB Analysis

We now carry out a CRB analysis of the one-bit channel estimation problem. The CRB result helps us to understand the impact of different system parameters, including one-bit quantization thresholds $\theta$ as well as training matrix $X$, on the estimation performance. With the definition of Fisher information matrix and CRB, we can easily derive the following theorem.

*Theorem 2*: The Fisher information matrix for the estimation problem (14) can be calculated as

$$J(h)=\sum_{n=1}^{N} g(\varphi_n^T h-\theta_n)\varphi_n\varphi_n^T \quad \textbf{(15)}$$

where $g(x)$ is defined as

$$g(x)=\frac{f_w^2(x)}{F_w(x)(1-F_w(x))} \quad \textbf{(16)}$$

and $f_w(x)$ represents the probability distribution function of $w_n$. The corresponding CRB matrix for the estimation problem (14) is the inverse of the Fisher information matrix, which is given by

$$\mathrm{CRB}(h,\theta)=\left(\sum_{n=1}^{N} g(\varphi_n^T h-\theta_n)\varphi_n\varphi_n^T\right)^{-1} \quad \textbf{(17)}$$

*Proof*: See Appendix A.

We observe that the CRB matrix of $h$ depends on the quantization thresholds $\theta$ as well as the matrix $\Phi$ which is constructed from the training matrix $X$. Obviously, we seek to jointly optimize the quantization thresholds $\theta$ and the training matrix $X$, through minimizing the trace of the CRB matrix, i.e. the overall estimation error asymptotically achieved by the maximum likelihood estimator. The optimization therefore can be formulated as follows:

$$\begin{aligned}\min_{X,\theta}\quad & \mathrm{Tr}\left\{\left(\sum_{n=1}^{N} g(\varphi_n^T h-\theta_n)\varphi_n\varphi_n^T\right)^{-1}\right\}\\ & \Phi=I_M\otimes\bar{\Phi}\\ s.t.\quad & \bar{\Phi}=\begin{bmatrix}\Re(X) & \Im(X)\\ -\Im(X) & \Re(X)\end{bmatrix}^T\\ & \mathrm{Tr}(XX^H)\le P\end{aligned} \quad \textbf{(18)}$$

where $\mathrm{Tr}(XX^H)\le P$ represents a transmit power constraint imposed on the pilot matrix. Such an optimization is examined in the following section, where it is shown that the optimization of pilot matrix $X$ can be decoupled from the optimization of the one-bit quantization threshold vector $\theta$.

## 4 Optimal Design of Quantization Thresholds and Pilot Matrix

Before proceeding, we first introduce the following result.

*Theorem 3*: If the random variable $w_n$ follows the real-valued Gaussian distribution with zero mean, $g(x)$ defined in (16) is a positive and symmetric function. It can attatin its maximum if $x=0$.

*Proof*: See Appendix B.

From this Theorem 3, we can give the following theorem to discuss the optimal choice of the quantization threshold $\theta$.

*Theorem 4*: Given a fixed pilot matrix $X$, the solution of the optimization problem

$$\min_{\theta}\quad \mathrm{Tr}\left\{\left(\sum_{n=1}^{N} g(\varphi_n^T h-\theta_n)\varphi_n\varphi_n^T\right)^{-1}\right\}$$

where $g(x)$ is defined in (16), is

$$\tilde{\theta}_n=\varphi_n^T h,\quad n=1,\cdots,N \quad \textbf{(19)}$$

*Proof*: Since $g(x)$ defined in (16) is a unimodel, positive and symmetric function attaining its maximum when $x=0$. We have

$$\sum_{n=1}^{N} g(\varphi_n^T h-\tilde{\theta}_n)\varphi_n\varphi_n^T-\sum_{n=1}^{N} g(\varphi_n^T h-\theta_n)\varphi_n\varphi_n^T\succ 0$$

From convex theory, $\mathrm{Tr}\{(\cdot)^{-1}\}$ is a convex function over the set of positive definite matrix, which means that for any $U\succ 0$, $V\succ 0$, and $U-V\succ 0$, the inequality

$$\mathrm{Tr}\{(U)^{-1}\}\le \mathrm{Tr}\{(V)^{-1}\}$$

holds. Combining the above two equations, the result (19) comes directly. □

We see that the optimal choice of the quantization threshold $\tilde{\theta}_n$ is rely on the unknown channel coeffient vector $h$. To facilitate our analysis, we, for the time being, suppose the unknown channel coeffient vector $h$ is known. Substituting (19) into (18), the optimization problem reduces to

$$\begin{aligned} \min_{X} \quad & \frac{\pi\sigma^2}{2}\mathrm{Tr}\left\{\left(\Phi^T\Phi\right)^{-1}\right\} \\ s.t. \quad & \Phi = I_M \otimes \bar{\Phi} \\ & \bar{\Phi} = \begin{bmatrix} \Re(X) & \Im(X) \\ -\Im(X) & \Re(X) \end{bmatrix}^T \\ & \mathrm{Tr}(XX^H) \le P \end{aligned} \quad \textbf{(20)}$$

which is only related to training pilot matrix $X$. We have the following theorem regarding the solution to the optimization (20). We omit the proof of this result since it can be obtained in the same way as that of [14].

*Theorem 5*: The minimum achievable objective function value of (20) can be attained if the pilot matrix $X$ satisfies

$$XX^H = (P/K)I_K \quad \textbf{(21)}$$

Theorem 5 reveals that, for one-bit massive MIMO systems, users should employ orthogonal pilot sequences in order to minimize channel estimation errors. Although it is a convention to use orthogonal pilots to facilitate channel estimation for conventional massive MIMO systems, to our best knowledge, its optimality in one-bit massive MIMO systems has not been established before.

## 5 Performance Analysis and Practical Threshold Design Strategy

### 5.1 Performance Analysis

Substituting the optimal thresholds into the CRB matrix, we can see that using one-bit ADCs for channel estimation incurs only a mild performance loss relative to using infinite-precision ADCs, with the CRB increasing by only a factor of $\pi/2$, i.e.,

$$\mathrm{CRB}(h,\tilde{\theta}) = \frac{\pi}{2}\mathrm{CRB}_{\mathrm{NQ}}(h) \quad \textbf{(22)}$$

where the subscript NQ denotes the estimation scheme employing unquantized observations with full resolution ADCs. For the CRB of NQ, it can be readily verified that

$$\mathrm{CRB}_{\mathrm{NQ}}(h) = \sigma^2\left(\Phi^T\Phi\right)^{-1} \quad \textbf{(23)}$$

The above results point out that a careful design of quantization thresholds can help improve the estimation performance substantially, and help achieve an estimation accuracy close to an ideal clairvoyant estimator which has access to the raw observations $u$. However, the problem lies in that the optimal thresholds $\tilde{\theta}$ are a function of $h$, as described in (19). Since $h$ is unknown and to be estimated, the optimal thresholds $\tilde{\theta}$ are also unknown.

### 5.2 Practical Threshold Design Strategy

Our strategy to overcome the above difficulty is to use an iterative algorithm in which the thresholds are iteratively refined based on the previous estimate of $h$. Specifically, at iteration $i$, we use the current quantization thresholds $\theta^{(i)}$ to generate the one-bit observation data $v^{(i)}$. Then a new estimate $\hat{h}^{(i)}$ is obtained from the ML estimator (14). This estimate is then plugged in (19) to obtain updated quantization thresholds, i.e.

$$\theta^{(i+1)} = \Phi\hat{h}^{(i)} \quad \textbf{(24)}$$

for subsequent iteration. When computing the ML estimate $\hat{h}^{(i)}$, not only the quantized data from the current iteration but also from all previous iterations can be used. The maximum likelihood estimator (14) can be easily adapted to accommodate these quantized data since the data are independent across different iterations. Due to the consistency of the ML estimator for large data records, this iterative process will asymptotically lead to optimal quantization thresholds, i.e.

$$i \to +\infty, \quad \theta^{(i)} \to \Phi h \quad \textbf{(25)}$$

In fact, our simulation results show that the iterative quantization scheme yields quantization thresholds close to the optimal values within only a few iterations. For clarity, we summarize the iterative quantization (IQ) strategy as follows.

**Iterative Quantization Scheme**

1. Given an initialization $\theta^{(0)}$ and the maximum number of iterations $i_{\max}$.
2. In each iteration $i \in \mathbb{N}^+$: Based on $u$ and $\theta^{(i)}$, quantize the analog signal $u$ and compute the one-bit data as
$$v^{(i)} = q(u - \theta^{(i)})$$
3. Calculate the new estimate of channel coefficient $\hat{h}^{(i)}$, through solving the maximum likelihood problem (14).
4. Update the new threshould vector as
$$\theta^{(i+1)} = \Phi\hat{h}^{(i)}$$
5. Return to Step 2, if
$$i < i_{\max}$$
6. Output $\theta^{(i_{\max})}$ and $\hat{h}^{(i_{\max})}$

Now we discuss the practical strategy for iterative quantization scheme based on channel coherence time. Note that during the iterative process, in a coherence time, the channel response vector $h$ is assumed constant over time. Thus the iterative quantization scheme can be used to estimate channels that are unchanged or slowly time-varying across a number of consecutive frames. For example, for the scenario where the relative speeds between the mobile terminals and the base station are slow, say, 2 meters per second, the channel coherence time could be up to tens of milliseconds, more precisely, about 60 milliseconds if the carrier frequency is set to 1GHz, according to the Clarke's model. Suppose the time duration of each frame is 10 milliseconds which is a typical value for practical LTE systems. In this case, the channel remains unchanged across 6 consecutive frames. We can use the iterative quantization scheme to update the quantization thresholds at each frame based on the channel estimate obtained from the previous frame. In this way, we can expect that the quantization thresholds will come closer and closer to the optimal values from one frame to the next, and as a result, a more and more accurate channel estimate can be obtained. In Figure 2, the practical strategy for iterative quantization scheme based on channel coherence time is illustrated.

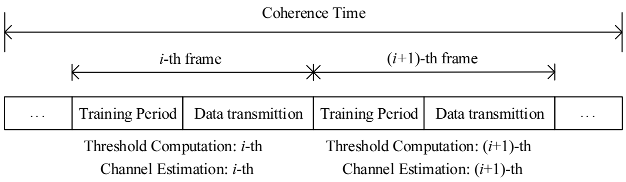


**Figure 2.** The practical strategy for iterative quantization scheme based on channel coherence time

## 6 Simulation Results

We now carry out experiments to corroborate our theoretical analysis and to illustrate the performance of our proposed one-bit quantization schemes, i.e. the iterative quantization schemes. We compare our schemes with the conventional zero quantization (ZQ) scheme which employs a fixed zero threshold for one-bit quantization, and a non-quantization scheme (referred to as NQ) which uses the original unquantized data for channel estimation. For the non-quantization scheme, it can be easily verified that its maximum likelihood estimate is given by

$$\hat{h} = (\Phi^T \Phi)^{-1} \Phi^T u$$

In our simulations, we assume independent and identically distributed (i.i.d.) rayleigh fading channels, i.e. all elements of the channel matrix $H$ follow a circularly symmetric complex Gaussian distribution with zero mean and unit variance. Training matrix $X$ which satisfied (21) is randomly generated. The signal to noise ratio (SNR) is defined as the ratio of the signal component to the noise component, i.e.,

$$\text{SNR} = \frac{\text{Tr}(XX^H)}{KL\sigma^2}$$

The mean squared error (MSE) is calculated as

$$\text{MSE} = \frac{\|H - H\|_F^2}{KM}$$

To better illustrate the effectiveness of the iterative quantization scheme, we include the CRB results in Figure 3, where the CRB-IQ, given by (22), represents the theoretical lower bound on the estimation errors of any unbiased estimator using optimal thresholds for one-bit quantization, and the CRB-NQ, given by (23), represents the lower bound on the estimation errors of any unbiased estimator which has access to the non-quantization observations. From Figure 3, we see that our proposed iterative quantization scheme approaches the theoretical lower bound CRB-IQ within only several iterations, and achieves performance close to the CRB associated with the non-quantization scheme. This result demonstrates the effectiveness of the iterative quantization scheme in searching for the optimal thresholds. In the rest of our simulations, we set the maximum number of iterations, which is denoted as $i_{\max}$, equal to 5 for the iterative quantization scheme.

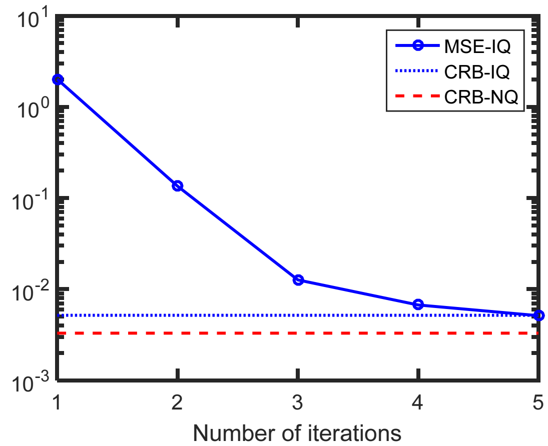


**Figure 3.** Performance of the iterative quantization scheme as a function of the number of iterations

We now compare the estimation performance of different schemes. Figure 4 plots the MSEs of respective schemes as a function of the number of pilot symbols, where we set $K$=4 and SNR=5 dB. The corresponding CRBs of these schemes are also included. Results are averaged over 100 independent runs, with the channel and the pilot sequences randomly generated for each run. From Figure 4, we can see that proposed iterative quantization scheme outperforms the zero quantization scheme by a big margin. This result corroborates our analysis that an optimal choice of the quantization thresholds helps achieve a substantial performance improvement. In Figure 5, we plot the MSEs of respective schemes under different SNRs, where we set $K$=4 and $L$=16. We can made similar conclusions about the performance advantage from Figure 5.

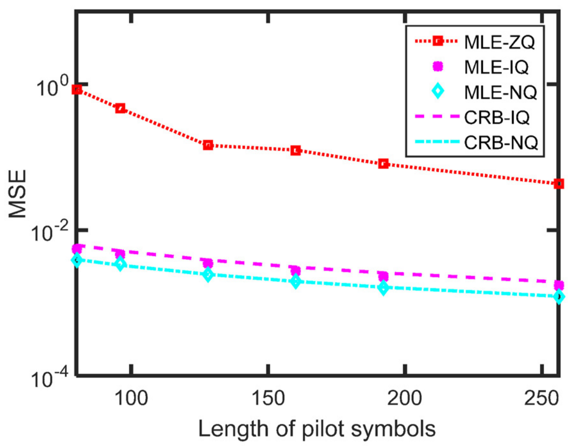


**Figure 4.** Channel estimation MSEs vs. Length of pilot symbols with $K = 4$ and SNR = 5 dB

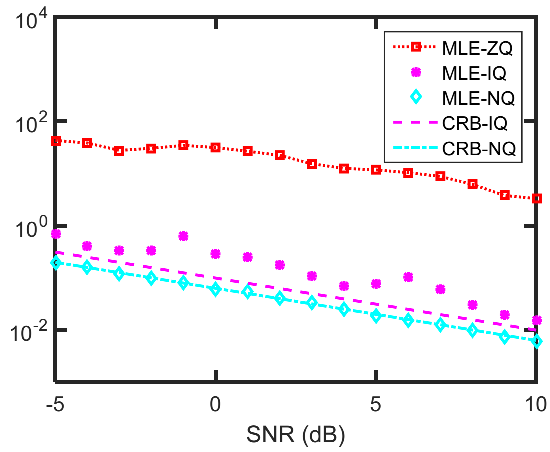


**Figure 5.** Channel estimation MSEs vs. SNR (in dB) with $K = 4$ and $L = 16$

## 7 Conclusion

Assuming one-bit ADCs at the base station, we studied the problem of one-bit quantization design and channel estimation for uplink multiuser massive MIMO systems. Specifically, based on the derived CRB matrix, we examined the impact of quantization thresholds on the channel estimation performance. Our theoretical analysis revealed that using one-bit ADCs can achieve an estimation error close to that attained by using infinite-precision ADCs, given that the quantization thresholds are optimally set. We developed an iterative quantization scheme which adaptively adjusts the thresholds such that the thresholds converge to the optimal thresholds. Simulation results showed that the proposed iterative quantization scheme achieved a significant performance improvement over the conventional zero quantization scheme.

## References


[1] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, F. Tufvesson, Scaling up MIMO: Opportunities and Challenges with Very Large Arrays, *IEEE Signal Processing Magazine*, Vol. 30, No. 1, pp. 40-60, January, 2013.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, T. L. Marzetta, Massive MIMO for Next Generation Wireless Systems, *IEEE Communications Magazine*, Vol. 52, No. 2, pp. 186-195, February, 2014.

[3] T. L. Marzetta, Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas, *IEEE Transactions on Wireless Communications*, Vol. 9, No. 11, pp. 3590-3600, November, 2010.

[4] C. Dai, X. Liu, J. Lai, P. Li, H.-C. Chao, Human Behavior Deep Recognition Architecture for Smart City Applications in the 5G Environment, *IEEE Network*, Vol. 33, No. 5, pp. 206-211, September-October, 2019.

[5] L. Fan, S. Jin, C.-K. Wen, H. Zhang, Uplink Achievable Rate for Massive MIMO Systems with Low-resolution ADC, *IEEE Communications Letters*, Vol. 19, No. 12, pp. 2186-2189, December, 2015.

[6] J. Zhang, L. Dai, S. Sun, Z. Wang, On the Spectral Efficiency of Massive MIMO Systems with Low-resolution ADCs, *IEEE Communications Letters*, Vol. 20, No. 5, pp. 842-845, May, 2016.

[7] C. Mollén, J. Choi, E. G. Larsson, R. W. Heath, Uplink Performance of Wideband Massive MIMO with One-bit ADCs, *IEEE Transactions on Wireless Communications*, Vol. 16, No. 1, pp. 87-100, January, 2017.

[8] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, C. Studer, One-bit Massive MIMO: Channel Estimation and High-order Modulations, *IEEE International Conference on Communication Workshop (ICCW)*, London, UK, 2015, pp. 1304-1309.

[9] C.-K. Wen, C.-J. Wang, S. Jin, K.-K. Wong, P. Ting, “Bayes-optimal Joint Channel-and-data Estimation for Massive MIMO with Low-precision ADCs, *IEEE Transactions on Signal Processing*, Vol. 64, No. 10, pp. 2541-2556, May, 2016.

[10] J. Choi, J. Mo, R. W. Heath, Near Maximum-likelihood Detector and Channel Estimator for Uplink Multiuser Massive MIMO Systems with One-bit ADCs, *IEEE Transactions on Communications*, Vol. 64, No. 5, pp. 2005-2018, May, 2016.

[11] A. Ribeiro, G. B. Giannakis, Bandwidth-constrained Distributed Estimation for Wireless Sensor Networks- Part I: Gaussian Case, *IEEE Transactions on Signal Processing*, Vol. 54, No. 3, pp. 1131-1143, March, 2006.

[12] A. Ribeiro and G. B. Giannakis, Bandwidth-constrained Distributed Estimation for Wireless Sensor Networks- Part II: Unknown Probability Density Function, *IEEE Transactions on Signal Processing*, Vol. 54, No. 7, pp. 2784-2796, July, 2006.

[13] J. Fang, H. Li, Distributed Adaptive Quantization for Wireless Sensor Networks: From Delta Modulation to Maximum Likelihood, *IEEE Transactions on Signal Processing*, Vol. 56, No. 10, pp. 5246-5257, October, 2008.
[14] I. Barhumi, G. Leus, M. Moonen, Optimal Training Design for MIMO OFDM Systems in Mobile Wireless Channels, *IEEE Transactions on Signal Processing*, Vol. 51, No. 6, pp. 1615-1624, June, 2003.
[15] G. Zeitler, G. Kramer, A. C. Singer, Bayesian Parameter Estimation Using Single-bit Dithered Quantization, *IEEE Transactions on Signal Processing*, Vol. 60, No. 6, pp. 2713-2726, June, 2012.

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## Appendix A: Proof of Theorem 2

Let us define a new variable $\xi_n = \varphi_n^T h$ and define

$$l(\xi_n) = \frac{\rho - v_n}{2\rho} log[1 - F_w(\xi_n - \theta_n)] + \frac{\rho + v_n}{2\rho} log[F_w(\xi_n - \theta_n)]$$

The first and second-order derivative of $L(h)$ can be calculated as

$$\frac{\partial L(h)}{\partial h} = \sum_{n=1}^{N} \frac{\partial l(\xi_n)}{\partial \xi_n} \frac{\partial \xi_n}{\partial h} = \sum_{n=1}^{N} \frac{\partial l(\xi_n)}{\partial \xi_n} \varphi_n$$

and

$$\frac{\partial^2 L(h)}{\partial h \partial h^T} = \sum_{n=1}^{N} \varphi_n \frac{\partial^2 l(\xi_n)}{\partial \xi_n^2} \frac{\partial \xi_n}{\partial h^T} = \sum_{n=1}^{N} \frac{\partial^2 l(\xi_n)}{\partial \xi_n^2} \varphi_n \varphi_n^T$$

where

$$\frac{\partial l(\xi_n)}{\partial \xi_n} = \frac{\rho - v_n}{2\rho} \frac{f_w(\xi_n - \theta_n)}{F_w(\xi_n - \theta_n) - 1} + \frac{\rho + v_n}{2\rho} \frac{f_w(\xi_n - \theta_n)}{F_w(\xi_n - \theta_n)}$$

and

$$\frac{\partial^2 l(\xi_n)}{\partial \xi_n^2} = \frac{\rho - v_n}{2\rho} \Big[ \frac{f_w'(\xi_n - \theta_n)}{F_w(\xi_n - \theta_n) - 1} - \frac{f_w^2(\xi_n - \theta_n)}{(F_w(\xi_n - \theta_n) - 1)^2} \Big] + \frac{\rho + v_n}{2\rho} \cdot \Big[ \frac{f_w'(\xi_n - \theta_n)}{F_w(\xi_n - \theta_n)} - \frac{f_w^2(\xi_n - \theta_n)}{F_w^2(\xi_n - \theta_n)} \Big]$$

Here the $f_w(\cdot)$ represents the probability distribution function of $w_n$, and $f_w'(\cdot)$ denotes the derivative of the probability distribution function.

Therefore, the Fisher information matrix (FIM) of the estimation problem is given as

$$J(h) = -E\left[ \frac{\partial^2 L(h)}{\partial h \partial h^T} \right] = -\sum_{n=1}^{N} E_{v_n} \left[ \frac{\partial^2 l(\xi_n)}{\partial \xi_n^2} \right] \varphi_n \varphi_n^T = \sum_{n=1}^{N} \frac{f_w^2(\varphi_n^T h - \theta_n)}{F_w(\varphi_n^T h - \theta_n)(1 - F_w(\varphi_n^T h - \theta_n))} \varphi_n \varphi_n^T$$

where $E_{v_n}[\cdot]$ denotes the expectation with respect to the distribution of $v_n$, and the last equation follows from the fact that $v_n$ is a binary random variable with

$$P(v_n = \rho \mid \theta_n, \xi_n) = F_w(\xi_n - \theta_n)$$

and

$$P(v_n = -\rho \mid \theta_n, \xi_n) = 1 - F_w(\xi_n - \theta_n)$$

This completes the proof.

## Appendix B: Proof of Theorem 3

Before proceeding, we first introduce the following lemma.

*Lemma*: For $x \geq 0$, define

$$\bar{F}(x) \triangleq \int_0^x f(u) du$$

where $f(\cdot)$ denotes the probability distribution function of a real-valued Gaussian random variable with zero-mean and unit variance. We have $\bar{F}(x)$ upper bounded by

$$\bar{F}(x) \leq \frac{1}{2}\sqrt{1 - e^{-2x^2/\pi}}$$

*Proof*: The proof can be found in [15].
Define the function

$$\bar{g}(x) = \frac{f^2(x)}{F(x)(1 - F(x))}$$

where $f(\cdot)$ and $F(\cdot)$ denotes the probability density function and cumulative density function of a real-valued Gaussian random variable with zero-mean and unit variance respectively. Invoking the above Lemma, we have

$$\bar{g}(x) = \frac{f^2(x)}{\frac{1}{4} - \bar{F}^2(x)} \leq \frac{2}{\pi} e^{-\left(1 - \frac{2}{\pi}\right)x^2} \leq \frac{2}{\pi}$$

and $\bar{g}(x) = 2/\pi$ if and only if $x = 0$. Noting that

$$\frac{1}{\sigma^2} \bar{g}\left( \frac{\varphi_n^T h - \theta_n}{\sigma} \right) = g(\varphi_n^T h - \theta_n)$$

Therefore $g(\varphi_n^T h - \theta_n)$ attains its maximum when

$$\theta_n = \varphi_n^T h$$

The proof is completed here.