Structural and State Controllability Study of the Supply Chain Network Based on the Complex Network Theory

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Abstract

Based on the complex network theory, the state and structural controllability conditions of complex supply chain network (SCN) are analyzed in detail. The criterion for the state controllability of SCN is introduced, and the method for estimating the minimum number of control inputs for achieving the complete state/ global structural controllability is proposed. The proposed criterion can effectively assess the global controllability of SCN, while the introduced method can effectively determine the minimum number of control inputs for the global controllability of SCN. The respective theorem and its proof presented in this study yields a quite unexpected corollary: it is not necessary to control the inputs of core enterprises in SCN, to achieve the complete state controllability on the global scale. Alternatively, this goal can be accomplished by a reasonable choice of state nodes of suppliers in the uppermost stream and those of the distributors in the downstream. Strengthening of ties between suppliers can reduce the minimum number of control inputs required for the global structural controllability of SCN.

Keywords: Network, Supply chain, State controllability, Structural controllability, Minimum control input

1 Introduction

The supply chain network (SCN) is a system of supply and demand relationships formed among enterprises in the strategic partnership, which are located in a single or several value chains. In the market environment, the operation of SCN is characterized by high dynamics and complexity, since it is influenced by internal or external factors, which manifest themselves within and beyond the system, respectively, and exhibit large variations. When studying SCN operation, besides the procedural features, one needs to elucidate the measures for the effective control of SCN, which should operate in the desired state spatially and temporally. In this sense, the most critical issue is SCN controllability, which implies that all states in the state space model are affected by the input, or there is no part of this model that is not ultimately affected by the demand signal.

In the 1950s, the classical control theory was applied to the supply chain by establishing a simple production inventory model using the Laplace transform [1] or its discrete version (Z-transform) [2]. In 1982, Towill represented the supply chain with a block diagram for the first time [3]. According to Towill, the supply chain dynamic behavior is control by two main factors, namely (1) the speed of restoring inventory level, and (2) the degree of amplification of orders' fluctuation relative to the actual demand fluctuation. In the 2000s, Disney and Towill applied the transfer function and spectral analysis to a single-level supply chain under normal distribution and derived the analytical expression of the bullwhip effect, which refers to the scenario where the orders to the supplier tend to have larger fluctuations than sales to the buyer and the distortion propagates upstream in an amplified form [4]. Disney obtained the order-demand transfer function using a causal loop diagram, block diagram, and Z-transform, in order to analyze the stability and bullwhip effect of the supply chain [5]. Lalwani et al. represented the state space model of a supply chain system under discrete time and analyzed the stability, controllability, and measurability of the system. [6] Liu studied the stability of the current level of the supply chain under non-returnable conditions by introducing a switched system and simulating its stability, with the analysis of each subsystem [7-8]. The application of large-scale system method to the supply chain management was proposed by Cheng [9], who analyzed the process of information transfer within a typical large-scale supply chain system. Using the decomposition-synthesis methodology, the generalized operator model for the coarse granularity was introduced. Then the layer-by-layer decomposition was performed, and the generalized operator model for finer granularity was elaborated for each decision-making point or control step. Several scholars,

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DOI: 10.3966/160792642019102006018

including Helbing [10], applied the complex network theory to SCN by treating it as a complex adaptive system with emergent, self-organizing, dynamic, non-linear, and evolutionary features. Small variations at any link are likely to invoke changes in other links, which are closely associated with the typology and macroscopic property of SCN. It was assumed that the typology of a complex SCN had a strong impact on the information amplification effect in the supply chain management. A reasonable supply chain structure can relieve the bullwhip effect while enhancing the robustness and anti-risk capacity [10]. Laumanns and Lefeber treated SCN as a process where materials flew dynamically, and each node was equivalent to a converter [11]. Materials passing through a certain node were simulated via the first-order differential equation, while the supply chain optimization was realized by the robust optimal control. Authors analyzed the supply chain control strategy considering the multi-level productivity of suppliers, which was modeled and optimized using the Arena simulation platform [12]. Authors proposed the robust control and optimization model for SCN, respectively, based on actual SCNs [13-14]. Zhao Gang, Yang Ying-bao et al. studied the the topology of supply chain network of agricultural products based on the complex network theory and calculated the main topology parameters of the supply chain networks [15]. Mizgier et al. proposed a model for the quantification of risk in supply chain networks according to Value at Risk and Expected Shortfall and illustrated the mechanics of the model on complex network designs based on a Monte Carlo simulation [16]. Alternatively, Mousavi et al. applied the modified particle swarm optimization algorithm to the optimization control problem of a two-level SCN [17]. Gang Zhao, Shu-li Gong et al. studied the dynamic mechanisms of risk propagation in complex supply chain networks are and topological evolutionary trend of complex supply chain networks [18].

To achieve a fast response of the supply chain, effective customer response, and balance of a supply chain in SCN, it is necessary to determine the main control nodes and to formulate the scheme for achieving global controllability of SCN. However, only a few studies have been conducted regarding the controllability and minimum control inputs of SCN. Therefore, we analyze the state and structural controllability conditions of SCN. We then consider and prove the theorem for determining the minimum control inputs to achieve the structural controllability of SCN and the determination method of the minimum number of control inputs to achieve the state controllability of SCN.

2 SCN State Controllability and its Rank Criterion

Consider an SCN consisting of N enterprise nodes and involving only forward logistics. Thus, we disregard the reverse logistics, which covers any operations related to the reuse of products and materials, including remanufacturing and refurbishing activities or any processes of moving goods from their typical final destination for capturing value or proper disposal. Given this, the SCN under study is a directed network consisting of N nodes. The state equation for the enterprise nodes in this network has the following form:

$$\frac{dx_i}{dt} = \sum_{j=1}^{N} a_{ij} x_j + \sum_{j=1}^{R} b_{ij} u_j \tag{1}$$

The state equation for SCN can be written as follows:

$$\dot{x} = Ax + Bu, \quad x \in \Re^N, u \in \Re^M \tag{2}$$

where x is the state vector of the enterprise node, $x = (x_1, x_2, \cdots, x_N)^T$; $A = (a_{ij})_{N\times N}$ is the system matrix; $u = (u_1, u_2, \cdots, u_R)$ is the control input vector; $B = (b_{ij})_{N\times R}, (R \le N)$ is the control input matrix, while N enterprise nodes in this SCN are called state nodes. The system parameter a_{ij} of SCN under control varies with different configurations of the state variables. For example, a_{ij} is apparently different when setting the state variable as the amount of working capital between the state nodes or as the logistics volume operating between the state nodes. The coordination and control of SCN usually imply the control of the volume of materials flow as the main target. So the volume of materials flow is treated as the variable of the state nodes. For the controllability study of an assembly SCN, variables of the state nodes may include volumes of producer goods, parts, and finished goods' flow at the enterprise nodes converted according to the bill of materials (BOM). For the local SCN in the upstream of core enterprises, the parameter a_{ij} is related to BOM of core enterprises and the number of suppliers of the same category of parts. For the local SCN in the downstream of core enterprises, the parameter a_{ij} is related to the number of distributors, regional market sales, and historical sales performance of distributors. If the control inputs act on a specific state node of an enterprise, such node is referred to as a controlled enterprise node.

In turn, a controlled SCN system is treated as state controllable, if for the initial $x(t_0) = x_0$ and final x_f states at any initial time t_0, there exist a finite time t_f and an unconstrained control input u, for which the equality $x(t_f)=x_f$ is valid. According to Kalman's controllability condition, the necessary and sufficient condition for the system in (2) to be treated as state controllable is as follows. The rank of matrix $M_C=[B,AB,A^2B,\cdots,A^{N-1}B]$ should satisfy the system state controllability criterion (3):

$$rank(M_C)=rank[B,AB,A^2B,\cdots,A^{N-1}B]=N \quad \textbf{(3)}$$

where rank(.) is the rank of and is the controllability judgment matrix.

The state controllability of SCN depends not only on the control input mechanism of the network and structural features of the controlled objects but also on parameters of the network system and control input matrix. The structural scheme design of a controlled SCN and the structural link study of the controlled objects are usually focused on the SCN typology. Therefore, noteworthy is the network structure effect on the structural controllability of SCN.

3 Structural Controllability of SCN

The structural controllability of SCN depends on the typology of a network, but not on the values of its parameters. Moreover, it applies not only to the linear approximation of the state equation of the network but also to the non-linearity and time variability of parameters of the state equation for the network, which are more typical for real SCN. If the structure of the complex SCN can guarantee the existence of the input state control information, which can achieve the network controllability, this network is considered to be controllable in the structural sense. This condition can be formulated as follows.

If for the controlled SCN system described by equation (2), there exist arbitrary non-zero values of matrices A and B that make the system controllable, this network system is considered to possess structural controllability.

For SCN system, the matrix is $\Pi=(\pi_{ij})\in\Re^{N\times N}$, which reflects the relational structure and coupling relationship between enterprises in the network. Hereinafter, Π is treated as a relational structure matrix of the enterprise nodes, which is a direct pathway of the control information transmission between state nodes of the enterprises in the network. For a directed SCN with no reverse logistics, we assume that

$$\pi_{ij}=\begin{cases}1\\0\end{cases}\text{when parameter } a_{ij}\begin{cases}\neq 0\\=0\end{cases},\ i\neq j$$

Let $\Gamma=(\gamma_{ij})\in\Re^{N\times M}$ be the relational structure matrix after removing the parameter information from the control input matrix of SCN. Then, we assume that the following condition holds:

$$\gamma_{ij}=\begin{cases}1\\0\end{cases}\text{when } b_{ij}\begin{cases}\neq 0\\=0\end{cases},\ i\neq j\ ,\ \gamma_{ii}=0$$

Next, we consider Γ as the structural matrix of the control input of the network and use symbols $N_A\gg\langle k_B\rangle$ and $N_A\gg\langle k_B\rangle$ as addition and multiplication operators, respectively, while $\xi_A=\dfrac{\pi_A}{\pi_A+\tau_A}$ and $\partial_t\xi_B(t)=0$ correspond to the addition and multiplication operators in Boolean algebra.

For matrices $P_C(\lambda_B)=\dfrac{(\xi_B\langle I_C\rangle)^{\lambda_B}}{\lambda_B!}\bullet e^{-\xi_B\langle I_C\rangle}$ and $\dfrac{\partial\xi_A(t)}{\partial t}=-\tau_A\xi_A(t)+\pi_A(1-\xi_A(t))$, their addition is applied as follows:

$$U\oplus Z=[u_{ij}\oplus z_{ij}]_{m\times n}=[u_{ij}\vee z_{ij}]_{m\times n}$$

Similarly, their multiplication implies

$$\begin{aligned}U\otimes Z&=[\mathop{\oplus}_{k=1}^{n}(u_{ik}\otimes z_{kj})]_{m\times s}\\&=[(u_{i1}\wedge z_{1j})\vee(u_{i2}\wedge z_{2j})\vee\cdots\vee(u_{in}\wedge z_{nj})]_{m\times s}\end{aligned}$$

Let the reachability matrix of the relational structure of the state be C_A for SCN, so there is

$$C_A=\Pi\oplus\Pi^2\oplus\cdots\oplus\Pi^{N-1}=\mathop{\oplus}_{m=1}^{N-1}\Pi^m=\Pi\oplus\mathop{\oplus}_{m=2}^{N-1}\Pi^m \quad \textbf{(4)}$$

According to the above definition, Π is the direct pathway for control information transmission between state nodes. Accordingly, Π^m is the -th logical power of Π. In the above formula, $\Pi^m(m=2,3\cdots,N-1)$ is the indirect pathway formed by series connections of *m* direct pathways between the state nodes.

Using the relational structure matrix Π and the input structural matrix Γ, one can calculate the structural reachability matrix of the control input information transmission in the network:

$$C_{AB}=\Gamma\oplus(C_A\otimes\Gamma) \quad \textbf{(5)}$$

As indicated by C_{AB} for SCN, the pathway that transmits the control information contained in the control input *u* to the state *x* of the enterprise node is structurally comprised of two parts, namely Γ and $C_A\otimes\Gamma$. Here Γ is the direct control information transmission pathway for control input *B*, while $C_A\otimes\Gamma$ is the indirect control information transmission pathway formed by the series connection of control input *B* and the controlled SCN system *A* via C_A. In formula (4), the control information transmission pathway is composed of two parts, namely, the direct linking pathway Π between state nodes and indirect linking pathway $\mathop{\oplus}_{m=2}^{N-1}\Pi^m$ formed by multiple series connections of direct pathways.

For SCN system, if the state equation (2) has non-zero values of matrices A and B that make the system controllable, such SCN is considered to be structurally controllable. In other words, the extended system comprising SCN and input system provides at least one controllable information link. This satisfies the structural pre-condition that the state control input information can reach the state nodes, which is crucial for the network system to be structurally controllable. If the supply chain system is structural controllable, any variations of the non-zero parameter a_{ij} will not violate the system controllability. If matrix C_{AB} of SCN system has no rows where all elements are zero, the number of non-zero rows will be equal to the number of dimensions of the state vectors:

$$\begin{aligned} R(C_{AB}) &= R[\Gamma \oplus (C_A \otimes \Gamma)] \\ &= R\left\{[\Gamma \oplus [(\Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{N-1}) \otimes \Gamma]\right\} = N \end{aligned} \quad \textbf{(6)}$$

This implies that SCN is structurally controllable. In the above formula, $R(C_{AB})$ is the number of non-zero rows in matrix C_{AB}, while N is the number of dimensions of state vectors in SCN, i.e., the number of enterprises. In formula (6), each state node of the enterprise has at least one control information transmission pathway. Thus, it is ensured structurally that the flow of control input information reaches every state node and that the control input B matches with SCN A. Formula (6) can be used as the state controllability criterion for SCN. The structural controllability of SCN is a structural feature of the state controllability. Noteworthy is that SCN with no state controllability may still possess the structural controllability. Therefore, the state controllability of SCN is a sufficient but not necessary condition of its structural controllability.

4 The Minimum Number of Control Inputs Required for the SCN Global Controllability

The main objective of multiple studies on SCN is to exert effective control over it, so that SCN can operate in the desired way. If control inputs act on each enterprise node in SCN (i.e., all enterprises are treated as control nodes), SCN is inevitably controllable. However, it is quite problematic to control all nodes in a complex SCN, which makes the choice of its control nodes a primary issue. An operation manager may need to control the entire SCN effectively by controlling the minimum number of enterprise nodes. However, this may be accomplished by controlling all enterprise nodes in SCN, which is a quite cost-, time-, and labor-consuming issue. This study makes an attempt to find a method of achieving the global control of SCN through the minimum number of control inputs.

4.1 Minimum Control inputs Required for the SCN Structural Controllability

As indicated by the available research results, in order to achieve the structural controllability, it is more expedient to take upstream suppliers as control nodes (i.e., control inputs acting on suppliers), instead of core enterprises and distributors.

To determine the minimum number of control inputs (i.e., enterprise nodes) required for the complete structural controllability of SCN, we will discuss the respective theorem and provide its proof.

Theorem. Assume that $S(X,E)$ is SCN consisting of N enterprise nodes, where X is the set of state nodes and E is the set of directed edges of the network topology. Let enterprise nodes x_i and x_j be the i-th state und upstream enterprise nodes, respectively. If $E_{ji}=\{e_{ji}\}=\varnothing$ or $X_j=\{x_j\}=\varnothing$, then $X_i=\{x_i \mid i=1,2,\cdots,m\}$ is the set of minimum control nodes required for the SCN global controllability of $S(X,E)$, where m is the minimum number of control inputs required for the global structural controllability of $S(X,E)$.

Proof. Consider a case where $S(X,E)$ achieves the global structural controllability, although x_i is a non-controlled enterprise node.

Insofar as $E_{ji}=\{e_{ji}\}=\varnothing$ or $X_j=\{x_j\}=\varnothing$, in the state equation of SCN system, $a_{ji}=0$.

$\therefore$ In the relational structure matrix Π, $\pi_{ji}=0$, $j=1,2,\cdots,N$, then all elements in the i-th row of $C_A=\overset{N-1}{\underset{m=1}{\oplus}}\Pi^m$ have zero values.

Since $\because$ x_i is a non-controlled node, all elements in the i-th row of the control input structure matrix Γ are zero.

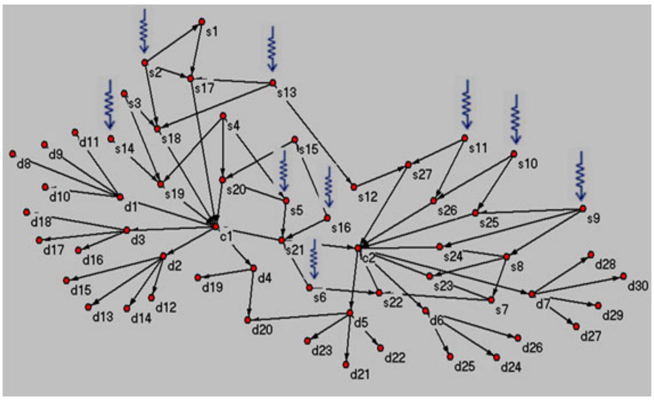
$\therefore$ All elements in the i-th row of $C_{AB}=\Gamma\oplus(C_A\otimes\Gamma)$ are zero.

Then $R(C_{AB})=R[\Gamma\oplus(C_A\otimes\Gamma)]<N$

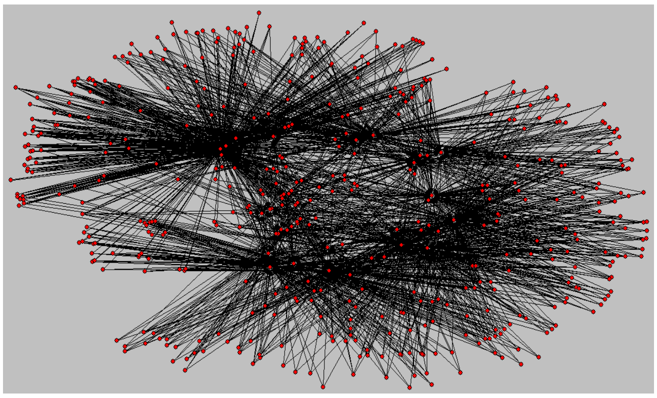
However, according to the structural controllability criterion (6), $S(X,E)$ is not structurally controllable, which is contradictory to the above assumption. This implies that the theorem is proven (QED).

Therefore, to determine the minimum number of control inputs to achieve the complete structural controllability, control nodes should be chosen among suppliers in the uppermost stream. Let $X_S=\{x_{si}\}$, $X_C=\{x_{ci}\}$ and $X_D=\{x_{di}\}$ be the sets of suppliers, core enterprises and distributors chosen as the control nodes, respectively. $X_K=\{x_{ki}\}$ is the set of minimum control nodes required for the complete structural controllability of SCN, i.e., the control scheme. This yields: $X_K \subseteq X_S$.

Using this theorem, we can analyze a specific large SCN under the survey (Figure 1). The scheme of minimum control inputs required for the global structural controllability of the supply chain is then constructed. Figure 1(a) shows the typology of SCN consisting of 59 enterprises, which includes sixteen second-tier suppliers (s1-s16), eleven first-tier suppliers (s17-s27), two core enterprises (c1-c2), seven first-tier distributors (d1-d7), and twenty-three second-tier distributors (d8-d30). Figure 1(b) depicts the typology of the agricultural SCN consisting of 526 business entities (including 497 farmers' households).



(a) SCN control scheme



(b) Complex agricultural SCN

Figure 1. Structural controllability of a complex SCN

According to the above theorem, the minimum number of control inputs for the global controllability depicted in Figure 1(a) is nine. The respective value of $R(C_{AB})$ is calculated as follows:

$$R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)]$$
$$= R\left\{[\Gamma \oplus [(\Pi \oplus \Pi^2 \oplus \cdots \oplus \Pi^{58}) \otimes \Gamma]\right\} = 59$$

Under this control scheme, SCN has achieved the structural controllability. If there is one node, which is not controlled as specified by the control scheme in Figure 1(a), then $R(C_{AB}) = R[\Gamma \oplus (C_A \otimes \Gamma)] < 59$. This envisages two possible options. The first one is that the required number of control inputs is less than nine. In this case, at least one enterprise node is not under control as specified by the control scheme in Figure 1(a). Another option is that the number of controlled inputs is no less than nine, but not all nodes are put under control as specified in Figure 1(a). In other words, at least one control input does not act on the enterprise node as specified in Figure 1(a). According to the above theorem, to achieve the global structural controllability for the case depicted in Figure 1(b), at least 487 enterprise nodes should be controlled. This example elucidates poor controllability of agricultural SCNs, (e.g., in China). The coordination and control of agricultural SCNs at a certain scale are quite problematic, and their cost is quite high.

The minimum number of control inputs (or control nodes) required for the global structural controllability of SCN can be reduced by core enterprises via strengthening their ties and business contacts with suppliers (especially those in the uppermost stream). Since the control of agricultural SCN is quite problematic, the strengthening of ties between planting households (or enterprises) or increasing the scale of planting households (or enterprises) is lucrative for the improved controllability of SCN.

4.2 Minimum Control inputs for SCN State Controllability

It seems plausible to formulate the complete controllability scheme based on the structural controllability, operating parameters, and control inputs of SCN. However, this may lead to the incomplete state controllability that fluctuates between controllable and uncontrollable states due to parameter variations. To avoid the instability of SCN control system, we need to find a more feasible method that determines the minimum number of control inputs required for the complete state controllability at the global scale.

Assume that SCN is converted into a direct bipartite graph, and the matching subset is calculated. All nodes in the matching subset are upstream ones. If all nodes outside the matching subset are controlled, the complete controllability of SCN can be achieved. Therefore, to determine the minimum number of control nodes for the global controllability of SCN, we should identify all non-matching enterprise nodes outside the largest possible matching subset. The minimum number of control inputs is equivalent to the number of enterprise nodes outside the largest possible matching subset. Consider a typology of SCN $S(V_A, E_A)$ consisting of N enterprise nodes, where $V_A = (x_1, x_2, \cdots, x_N)$, $E_A = \left\{(x_j, x_i) \mid a_{ij} \neq 0\right\}$. The bipartite graph of the typology S of SCN is $G(X, E, Y)$, where $X = \left\{x_1^{\ c}, x_2^{\ c}, \cdots, x_N^{\ c}\right\}$ is the set of state nodes in the N-

th column of SCN system matrix; $Y=\{x_1^{\,r},x_2^{\,r},\cdots,x_N^{\,r}\}$ is the set of state nodes in the N-th column of SCN system matrix; the set of edges is $E=\{x_j^{\,c},x_i^{\,r}\}$. Below is an illustration for a simple SCN, which has five enterprise nodes, including two supplier nodes, one core enterprise node, and two distributor nodes, as shown in Figure 2(a). Figure 2(b) is a bipartite graph converted from a simple SCN.

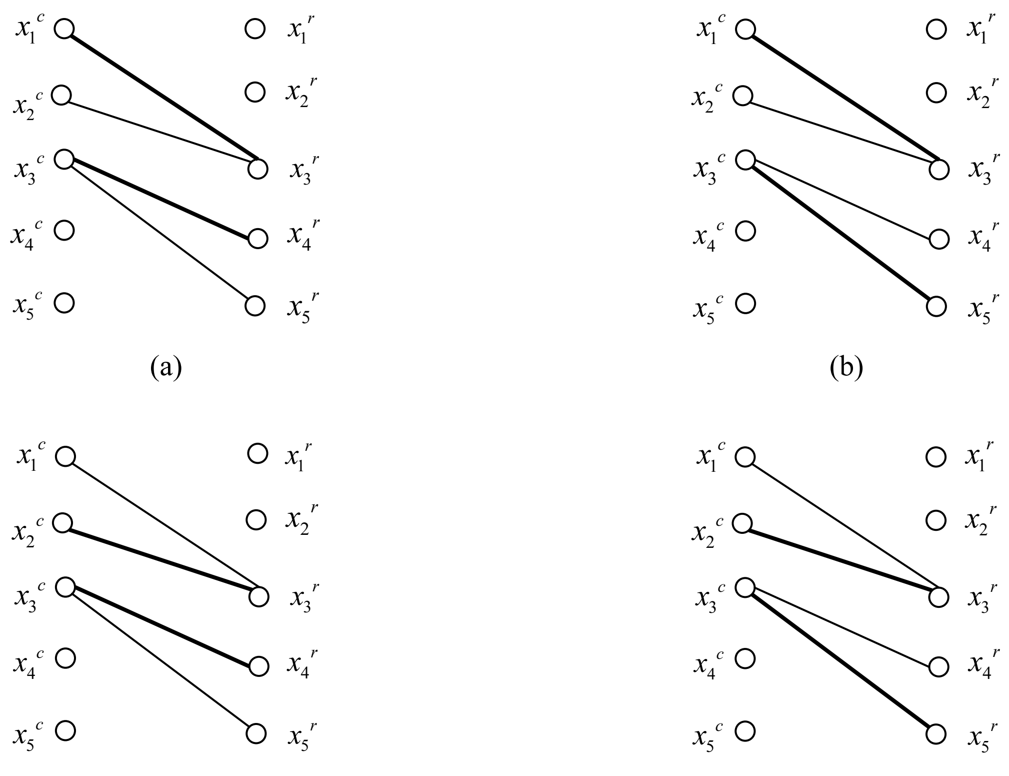
For the above bipartite graph, the maximum matching is achieved at $x_1^{\,c}$—$x_3^{\,r}$, $x_3^{\,c}$—$x_4^{\,r}$, or $x_1^{\,c}$—$x_3^{\,r}$, $x_3^{\,c}$—$x_5^{\,r}$, or $x_2^{\,c}$—$x_3^{\,r}$, $x_3^{\,c}$—$x_4^{\,r}$, or $x_2^{\,c}$—$x_3^{\,r}$, $x_3^{\,c}$—$x_5^{\,r}$, which links are depicted by thick lines in Figure 3.

Thus, to achieve the complete state controllability of SCN, as shown in Figure 2(a), the minimum number of control inputs should be equal to three. That is, nodes x_1, x_2 and x_4, or x_1, x_2 and x_5 are controlled. The control scheme is depicted in Figure 4.

The control scheme presented in Figure 4 is then verified by exerting the control, as shown in Figure 5(a), Figure 5(b) and Figure 5(c).

(a) (b)

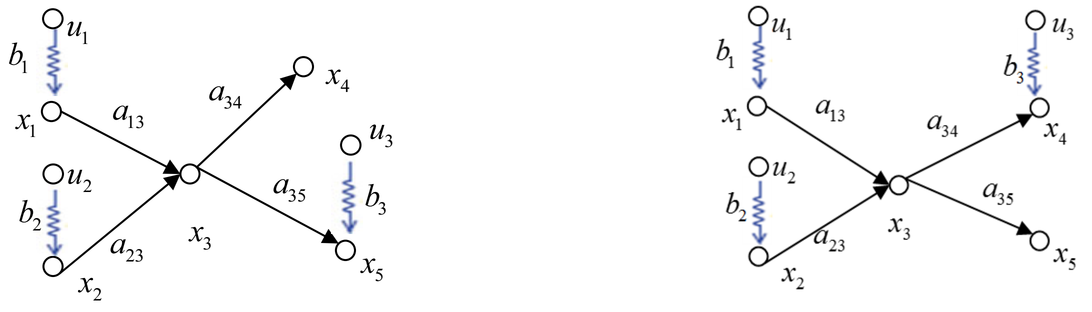
Figure 2. SCN and the corresponding bipartite graph



(a) (b)

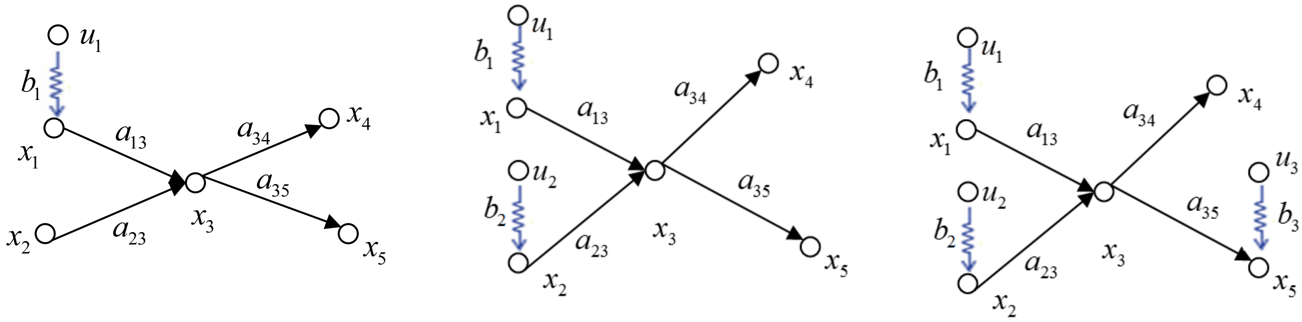
(c) (d)

Figure 3. The best fit/maximum matching of SCN



(a) (b)

Figure 4. The control scheme of SCN



(a) (b) (c)

Figure 5. SCN control system

The state controllability judgment matrix of SCN control system in Figure 5(a) can be represented as follows:

$$M_C = \left[B, AB, A^2B, A^3B, A^4B\right] = b_1 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & a_{13} & 0 & 0 & 0 \\ 0 & 0 & a_{13}a_{34} & 0 & 0 \\ 0 & 0 & a_{13}a_{35} & 0 & 0 \end{pmatrix}$$

The maximum rank of this matrix with arbitrary non-zero parameters is equal to three, i.e., $\max[Rank(M_C)] = 3$. This implies that the system depicted in Figure 5(a) fails to achieve the state controllability.

The state controllability judgment matrix of SCN control system in Figure 5(b) has the following form:

$$M_C = \left[B, AB, A^2B, A^3B, A^4B\right]$$
$$= \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{13}b_1 & a_{23}b_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{13}a_{34}b_1 & a_{23}a_{34}b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{13}a_{35}b_1 & a_{23}a_{35}b_2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The rank of the matrix with varying parameters can reach only two possible values, namely $Rank(M_C) = 5$ or $Rank(M_C) = 4$. If the values of parameters a_{13}, a_{23}, a_{34} and a_{35} are properly chosen, the matrix rank is $Rank(M_C) = 4$. In other words, under the variation of system parameters, SCN in Figure 5(b) does not necessarily achieve the state controllability. Therefore, it is not completely controllable.

The state equation of SCN control system in Figure 5(c) can be written as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ a_{13} & a_{23} & 0 & 0 & 0 \\ 0 & 0 & a_{34} & 0 & 0 \\ 0 & 0 & a_{35} & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

The state controllability judgment matrix of this SCN control system can be reduced to the following form:

$$M_C = \left[B, AB, A^2B, A^3B, A^4B\right]$$

$$= \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{13}b_1 & a_{23}b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{13}a_{34}b_1 & a_{23}a_{34}b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 & 0 & 0 & a_{13}a_{35}b_1 & a_{23}a_{35}b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This matrix with arbitrary non-zero parameters always has the same rank $Rank(M_C) = 5$. This implies that SCN in Figure 5(c) can achieve the complete state controllability. The controllability analysis and computing are then performed for SCN control system depicted in Figure 4(b), with the same results being obtained. Therefore, at least three control inputs are required to act on three enterprise nodes to achieve the complete state controllability of SCN. The state controllability scheme implies that three control inputs, respectively, act on supplier nodes x_1 and x_2, as well as distributor node x_4, or on supplier nodes x_1 and x_2, as well as distributor node x_5. This result is consistent with the above control scheme.

5 Conclusion

The results obtained in this study make it possible to draw the following conclusions.

(1) The state and structural controllability conditions of a complex supply chain network (SCN) are analyzed within the framework of the complex network theory. The controllability criterion is proposed and validated with the respective theorem and its proof.

(2) The method for estimating the minimum number of control inputs required for the complete state and global structural controllability of SCN is introduced. Its application revealed that, in order to achieve the complete state controllability on the global scale, the control inputs do not have to act on the core enterprises of SCN. Instead, an appropriate choice of the state nodes of upstream suppliers and distributor nodes is required.

(3) Since the control of agricultural SCN is quite problematic, the strengthening of ties between planting households (or enterprises) or increasing the scale of planting households (or enterprises) is lucrative for the improved controllability of SCN.

(4) Exerting the control over the uppermost stream suppliers can significantly strengthen the structural controllability of SCN. On the other hand, strengthening the ties between suppliers can reduce the minimum number of control inputs/nodes required for the global structural controllability of SCN.

Future scope of the work focuses on the robust control of time-delay complex network systems and the dynamic behavior of complex network systems with time-varying topology.

Acknowledgements

This work is supported by the Humanity and Social Science Youth Foundation of the Ministry of Education of China under Grant No. 15YJC630185, and China Postdoctoral Science Foundation funded project under Grant No. 2016M591845.

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