Super Connectivity and Diagnosability of Crossed Cubes

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Abstract

A multiprocessor system and an interconnection network have a underlying topology, which usually presented by a graph G. In 2016, Zhang et al. proposed the g-extra diagnosability of G, which restrains that every component of $G-S$ has at least $(g+1)$ vertices. As an important variant of the hypercube, the n-dimensional crossed cube CQ_n has many good properties. In this paper, we prove that CQ_n is tightly $(4n-9)$ super 3-extra connected for $n \geq 7$ and the 3-extra diagnosability of CQ_n is $4n-6$ under the PMC model $(n \geq 5)$ and MM* model $(n \geq 7)$.

Keywords: Interconnection network, Connectivity, Diagnosability, Crossed cube

1 Introduction

A multiprocessor system and an interconnection network (networks for short) have a underlying topology, which usually presented by a graph, where nodes represent processors and links represent communication links between processors. Some processors may be faulty when the system is in operation. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system G is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of G is the maximum value of t such that G is t-diagnosable.

In order to discuss the connectivity and diagnosability in different situations, people put forward the restricted connectivity and diagnosability in the system. In 1996, Fàbrega and Fiol [1] introduced the g-extra connectivity of a system $G=(V(G),E(G))$, which is denoted by $\tilde{\kappa}^{(g)}(G)$. A vertex subset $S \subseteq V(G)$ is called a g-extra vertex cut if $G-S$ is disconnected and every component of $G-S$ has at least $(g+1)$ vertices. $\tilde{\kappa}^{(g)}(G)$ is defined as the cardinality of a minimum g-extra vertex cut. For the hypercube Q_n, Yang et al. [2] determined $\tilde{\kappa}^{(3)}(Q_n)=4n-9$ for $n \geq 4$. For the folded hypercubes FQ_n, Chang et al. [3] determined $\tilde{\kappa}^{(3)}(FQ_n)=4n-5$ for $n \geq 6$. Gu et al. studied the 3-extra connectivity of 3-ary n-cubes [4] and k-ary n-cubes [5]. For the star graph S_n and the bubble-sort graph B_n, Li et al. [6] determined $\tilde{\kappa}^{(3)}(S_n)=4n-10$ for $n \geq 4$, $\tilde{\kappa}^{(3)}(B_n)=4n-12$ for $n \geq 6$. Chang et al. [7] determined the n-dimensional hypercube-like networks 3-extra connectivity, $\tilde{\kappa}^{(3)}(HL_n)=4n-9$ for $n \geq 6$, and so on.

In 2016, Zhang et al. [8] proposed the g-extra diagnosability of a system, which restrains that every fault-free component has at least $(g+1)$ fault-free vertices. They studied that the g-extra diagnosability of the n-dimensional hypercube under the PMC model and MM* model. In 2016, Wang et al. [9] studied the 2-extra diagnosability of the bubble-sort star graph BS_n under the PMC model and MM* model. In 2017, Wang and Yang [10] studied the 2-good-neighbor (2-extra) diagnosability of alternating group graph networks under the PMC model and MM* model. In 2017, Ren and Wang [11], studied the tightly super 2-extra connectivity and 2-extra diagnosability of locally twisted cubes.

Now, there are many topologies on the networks and systems. Hypercube as an important network model has good properties such as lower diameter and node degree, high connectivity, regular, symmetry, and so on. Efe and Member proposed crossed cube [12] by changing the links between some nodes of hypercubes, which have superior properties over hypercubes. For example, its diameter is about half of hypercube with the same dimension, which makes that the communication speed between any two nodes is increased by almost a half.

Several models of diagnosis have been studied in system level diagnosis. Among these models, the most popular two models are the PMC and MM, which are proposed by Preparata et al. [13] and Maeng et al. [14],

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respectively. Under the PMC model, only neighboring processors are allowed to test each other. Under the MM model, a node tests its two neighbors, and then compares their responses. Sengupta and Dahbura [15] suggested a special case of the MM model, namely the MM* model, and each node must test its any pair of adjacent nodes in the MM*.

In this paper, we proved that (1) CQ_n is tightly $(4n-9)$ super 3-extra connected for $n \geq 7$; (2) the 3-extra diagnosability of CQ_n is $4n-6$ under the PMC model for $n \geq 5$; (3) the 3-extra diagnosability of CQ_n is $4n-6$ under the MM* model for $n \geq 7$.

2 Preliminaries

2.1 Notations

A multiprocessor system is modeled as an undirected simple graph $G=(V,E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. The degree $d_G(v)$ of a vertex v in G is the number of neighbors of v in G .For a vertex v, $N_G(v)$ is the set of vertices adjacent to v in G. Let $S \subseteq V(G)$. $N_G(S) = \cup_{v\in S} N_G(v) \setminus S$ and $G[S]$ is the subgraph of G induced by S. A cycle with length n is called an n-cycle. We use $P = v_1 v_2 \cdots v_n$ to denote a path that begins with v_1 and ends with v_n. A path of the length n is denoted by n-path. A bipartite graph is one whose vertex set can be partitioned into two subsets X and Y, so that each edge has one end in X and one end in Y; such a partition (X,Y) is called a bipartition of the graph. A complete bipartite graph is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y; If $|X|=m$ and $|Y|=n$, such a graph is denoted by $K_{m,n}$ The connectivity $\kappa(G)$ of a connected graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left when G is complete. Let F_1 and F_2 be two distinct subsets of V, and let the symmetric difference $F_1 \Delta F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. For graph-theoretical terminology and notation not defined here we follow [16].

A connected graph G is super g-extra connected if every minimum g-extra cut F of G isolates one connected subgraph of order $g+1$. In addition, if $G-F$ has two components, one of which is the connected subgraph of order $g+1$, then G is tightly $|F|$ super g-extra connected.

2.2 The Crossed Cube CQ_n

Definition 2.1. ([17]) Let $R=\{(00,00),(10,10),(01,11),(11,01)\}$. Two digit binary strings $u=u_1u_0$ and $v=v_1v_0$ are pair related, denoted as $u \sim v$, if and only if $(u,v) \in R$.

Definition 2.2. ([17]) The vertex set of a crossed cube CQ_n is $\{v_{n-1}v_{n-2}\cdots v_0 : 0 \leq i \leq n-1, v_i \in \{0,1\}\}$. Two vertices $u=u_{n-1}u_{n-2}\cdots u_0$ and $v=v_{n-1}v_{n-2}\cdots v_0$ are adjacent if and only if one of the following conditions is satisfied.

1. There exists an integer $l\,(1 \leq l \leq n-1)$ such that (1) $u_{n-1}u_{n-2}\cdots u_l = v_{n-1}v_{n-2}\cdots v_l$;
(2) $u_{l-1} \neq v_{l-1}$;
(3) if l is even, $u_{l-2} = v_{l-2}$;
(4) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$, for $0 \leq i < \left\lfloor \frac{l-1}{2} \right\rfloor$.

2.
(1) $u_{n-1} \neq v_{n-1}$;\\(2)if n is even, $u_{n-2} = v_{n-2}$;
(3) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$ for $0 \leq i < \left\lfloor \frac{n-1}{2} \right\rfloor$.

Let $n \geq 2$. We define two graphs CQ_n^0 and CQ_n^1 as follows. If $u = u_{n-2}u_{n-3}\cdots u_0 \in V(CQ_{n-1})$, then $u^0 = 0u_{n-2}u_{n-3}\cdots u_0 \in V(CQ_n^0)$ and $u^1 = 1u_{n-2}u_{n-3}\cdots u_0 \in V(CQ_n^1)$. If $uv \in E(CQ_{n-1})$, then $u^0v^0 \in E(CQ_n^0)$ and $u^1v^1 \in E(CQ_n^1)$. Then $CQ_n^0 \cong CQ_{n-1}$ and $CQ_n^1 \cong CQ_{n-1}$. Define the edges between the vertices of CQ_n^0 and CQ_n^1 according to the following rules.

The vertex $u = 0u_{n-2}u_{n-3}\cdots u_0 \in V(CQ_n^0)$ and the vertex $v = 1v_{n-2}v_{n-3}\cdots v_0 \in V(CQ_n^1)$ are adjacent if and only if

1. $u_{n-2} = v_{n-2}$ if n is even;
2. $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in R$, for $0 \leq i < \left\lfloor \frac{n-1}{2} \right\rfloor$.

The graphs CQ_2, CQ_3 and CQ_4 are depicted in Figure 1. The edges between the vertices of CQ_n^0 and CQ_n^1 are said to be cross edges.

Proposition 2.1. ([17]) Let CQ_n be the crossed cube. Then all cross edges of CQ_n is a perfect matching.

By Proposition 2.1, CQ_n can be recursively defined as follows.

Definition 2.3. ([17]) Define that $CQ_1 \cong K_2$. For $n \geq 2$, CQ_n is obtained by CQ_n^0 and CQ_n^1, and a perfect matching between the vertices of CQ_n^0 and CQ_n^1 according to the following rules (see Figure 1).

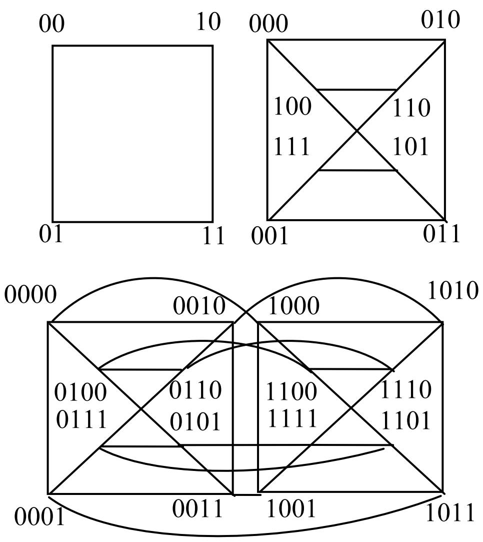

Figure 1. CQ_2, CQ_3 and CQ_4

The vertex $u = 0u_{n-2}u_{n-3}\cdots u_0 \in V(CQ_n^0)$ and the vertex $v = 1v_{n-2}v_{n-3}\cdots v_0 \in V(CQ_n^1)$ are adjacent in CQ_n if and only if

1. $u_{n-2} = v_{n-2}$ if n is even;
2. $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in R$, for $0 \le i < \left\lfloor \frac{n-1}{2} \right\rfloor$.

3 The Connectivity of Crossed Cubes

Lemma 3.1. ([12]) Let CQ_n be the crossed cube. Then $\kappa(CQ_n) = n$ for $n \ge 1$.

Lemma 3.2. ([17]) Let CQ_n be the crossed cube and let $F \subseteq V(CQ_n)$ $(n \ge 3)$ with $n \le |F| \le 2n-3$. If $CQ_n - F$ is disconnected, then $CQ_n - F$ has exactly two components, one of which is an isolated vertex.

Lemma 3.3. ([17]) Let CQ_n be the crossed cube and let CQ_n be the crossed cube and let $F \subseteq V(CQ_n)$ $(n \ge 5)$ with $2n-2 \le |F| \le 3n-6$. If $CQ_n - F$ is disconnected, then $CQ_n - F$ satisfies one of the following conditions:

(1) $CQ_n - F$ has two components, one of which is a K_2;

(2) $CQ_n - F$ has two components, one of which is an isolated vertex;

(3) $CQ_n - F$ has three components, two of which are isolated vertices.

Lemma 3.4. Let CQ_n be the crossed cube and let $A = \{0\cdots0000, 0\cdots0100, 0\cdots0110, 0\cdots0111\}$. If $F = N_{CQ_n}(A)$, then $|F| = 4n-9$ and $CQ_n - (A \cup F)$ is connected for $n \ge 4$.

Proof. By the definition of the crossed cube, $CQ_n[A]$ is a 3-path. We proof this lemma by induction on n. In CQ_4 (see Figure 1), $A = \{0000, 0100, 0110, 0111\}$ and $F = \{0001, 0010, 0101, 1000, 1100, 1110, 1101\}$. It is easy to see that $|A| = 4$, $|F| = 7$ and $CQ_4 - (A \cup F)$ is connected. We can decompose CQ_n along dimension $n-1$ into CQ_n^0 and CQ_n^1. Then both CQ_n^0 and CQ_n^1 are isomorphic to CQ_{n-1}. Let $F_0 = F \cap V(CQ_n^0)$ and $F_1 = F \cap V(CQ_n^1)$. Then $|F_0| + |F_1| = |F|$. We assume that the lemma is true for $n-1$, i.e., if $F = N_{CQ_{n-1}}(A)$, then $|F| = 4(n-1) - 9 = 4n-13$ and $CQ_{n-1} - (A \cup F)$ is connected. Now we proof that the lemma is also true for n $(n \ge 5)$. Note that $A \in V(CQ_n^0)$. By the inductive hypothesis, we have $|F_0| = 4n-13$ and $CQ_n^0 - (A \cup F_0)$ is connected. By Proposition 2.1, $|F_1| = 4$. Thus, $|F| = |F_0| + |F_1| = 4n-13+4 = 4n-9$. Now we prove that $CQ_n - (A \cup F)$ is connected for $n \ge 5$.

By Lemma 3.1, $\kappa(CQ_n^1) = n-1 > 4 = |F_1|$ for $n \ge 6$. Thus, $CQ_n^1 - F_1$ is connected for $n \ge 6$. We consider that $n = 5$. Note that $A = \{00000, 00100, 00110, 00111\}$. By Proposition 2.1, $F_1 = \{10000, 11100, 11110, 11101\}$. By the definition of the crossed cube, 11100 is adjacent to 11110. Note that $|F_1| = 4 = 5-1$. By Lemma 3.2, $CQ_5^1 - F_1$ is connected or has two components, one of which is an isolated vertex. Suppose that $CQ_5^1 - F_1$ is disconnected. Let u be the isolated vertex in $CQ_5^1 - F_1$. Since $d_{CQ_5^1}(u) = 4 = |F_1|$, u is connected to every vertex of F_1. So u is adjacent to 11100 and 11110. Then we get that $CQ_5^1[\{u, 11100, 11110\}]$ is a triangle, a contradiction. Thus, $CQ_5^1 - F_1$ is connected. So we can conclude that $CQ_n^1 - F_1$ is connected for $n \ge 5$. Note that $CQ_n^0 - (A \cup F_0)$ is connected. Since $2^{n-1} - (4n-9) \ge 1$ $(n \ge 5)$, by Proposition 2.1, $CQ_n[V(CQ_n^0 - (A \cup F_0)) \cup V(CQ_n^1 - F_1)] = CQ_n - (A \cup F)$ is connected for $n \ge 5$. □

Lemma 3.5. ([18]) Let CQ_n be the crossed cube and let $F \subseteq V(CQ_5)$. If $|F| = 10$, then $CQ_5 - F$ satisfies one of the following conditions:

(1) $CQ_5 - F$ is connected;

(2) $CQ_5 - F$ has two components, one of which is a K_2;

(3) $CQ_5 - F$ has two components, one of which is a 2-path;

(4) $CQ_5 - F$ has two components, one of which is an isolated vertex;

(5) $CQ_5 - F$ has three components, two of which are isolated vertices;

(6) $CQ_5 - F$ has four components, three of which are isolated vertices;

(7) $CQ_5 - F$ has three components, one of which is an isolated vertex and the other is a K_2.

Lemma 3.6. Let CQ_n be the crossed cube and let $F \subseteq V(CQ_n)$ $(n \geq 5)$. If $3n-5 \leq |F| \leq 4n-10$, then $CQ_n - F$ satisfies one of the following conditions:

(1) $CQ_n - F$ is connected;

(2) $CQ_n - F$ has two components, one of which is a K_2;

(3) $CQ_n - F$ has two components, one of which is a 2-path;

(4) $CQ_n - F$ has two components, one of which is an isolated vertex;

(5) $CQ_n - F$ has three components, two of which are isolated vertices;

(6) $CQ_n - F$ has four components, three of which are isolated vertices;

(7) $CQ_n - F$ has three components, one of which is an isolated vertex and the other is a K_2.

Proof. We prove the lemma by induction on n. By Lemma 3.5, the lemma is true for $n=5$. We assume that the lemma is true for $n-1$, i.e., if $3n-8 \leq |F| \leq 4n-14$, then $CQ_{n-1} - F$ satisfies one of the conditions (1)-(7). Now we show that the lemma is also true for n $(n \geq 6)$. We can decompose CQ_n along dimension $n-1$ into CQ_n^0 and CQ_n^1. Then both CQ_n^0 and CQ_n^1 are isomorphic to CQ_{n-1}. Let $F_0 = F \cap V(CQ_n^0)$ and $F_1 = F \cap V(CQ_n^1)$ with $|F_0| \leq |F_1|$. Let B_i be the maximum component of $CQ_n^i - F_i$ (If $CQ_n^i - F_i$ is connected, then let $B_i = CQ_n^i - F_i$) for $i \in \{0,1\}$. Since $3n-5 \leq |F| \leq 4n-10$, we have $0 \leq |F_0| \leq \frac{4n-10}{2} = 2n-5$ and $n \leq \left\lceil \frac{3n-5}{2} \right\rceil \leq |F_1| \leq 4n-10$ $(n \geq 6)$. We consider the following cases.

Case 1. $n \leq |F_1| \leq 2n-5$.

Note that $|F_i| \leq 2n-5 = 2(n-1)-3$ for $i \in \{0,1\}$. By Lemma 2.1, $CQ_n^i - F_i$ is connected or has two components, one of which is an isolated vertex. Since $2^{n-1} - (4n-10) - 2 \geq 1$ $(n \geq 6)$, by Proposition 2.1, $CQ_n[V(B_0) \cup V(B_1)]$ is connected. Thus, $CQ_n - F$ satisfies one of the conditions (1)-(7).

Case 2. $2n-4 \leq |F_1| \leq 3n-9$.

In this case, $|F_0| \leq 4n-10-(2n-4) = 2n-6 < 2n-5$. By Lemma 3.2, $CQ_n^0 - F_0$ is connected or has two components, one of which is an isolated vertex. By Lemma 3.3, $CQ_n^1 - F_1$ satisfies one of the following conditions:

(a) $CQ_n^1 - F_1$ has two components, one of which is a K_2;

(b) $CQ_n^1 - F_1$ has two components, one of which is an isolated vertex;

(c) $CQ_n^1 - F_1$ has three components, two of which are isolated vertices.

Since $2^{n-1} - (4n-10) - 3 \geq 1$ $(n \geq 6)$, by Proposition 2.1, $CQ_n[V(B_0) \cup V(B_1)]$ is connected. Thus, $CQ_n - F$ satisfies one of the conditions (1)-(7).

Case 3. $3n-8 \leq |F_1| \leq 4n-14$.

By the inductive hypothesis, $CQ_n^1 - F_1$ satisfies one of the conditions (1)-(7). In this case, $|F_0| \leq 4n-10-(3n-8) = n-2$. By Lemma 3.1, $CQ_n^0 - F_0$ is connected. Since $2^{n-1} - (4n-10) - 3 \geq 1$ $(n \geq 6)$, by Proposition 2.1, $CQ_n[V(B_0) \cup V(B_1)]$ is connected. Thus, $CQ_n - F$ satisfies one of the conditions (1)-(7).

Case 4. $4n-13 \leq |F_1| \leq 4n-10$.

In this case, $|F_0| \leq 4n-10-(4n-13) = 3$. By Lemma 3.1, $CQ_n^0 - F_0$ is connected. Suppose that $CQ_n^1 - F_1$ is connected. Since $2^{n-1} - (4n-10) \geq 1$ $(n \geq 6)$, by Proposition 2.1, $CQ_n[V(CQ_n^0 - F_0) \cup V(CQ_n^1 - F_1)] = CQ_n - F$ is connected. Then we suppose that $CQ_n^1 - F_1$ is disconnected. Let the components of $CQ_n^1 - F_1$ be C_1, C_2, …, C_k $(k \geq 2)$. Note that $|F_0| \leq 3$. If every component C_i of $CQ_n^1 - F_1$ such that $|V(C_i)| \geq 4$ for $i \in \{1,\ldots,k\}$, then $CQ_n[V(CQ_n^0 - F_0) \cup V(C_1) \cup \cdots \cup V(C_k)]$ $= CQ_n - F$ is connected. Suppose that there is a components C_i such that $|V(C_i)| \leq 3$ for $i \in \{1,\ldots,k\}$. If $N_{CQ_n}(V(C_i)) \cap V(CQ_n^0) \subseteq F_0$, then C_i is a component of $CQ_n - F$ with $|V(C_i)| \leq 3$. Combining $|F_0| \leq 3$, we get that $CQ_n - F$ satisfies one of the conditions (1)-(7).

Theorem 3.1. ([19]) Let CQ_n be the crossed cube. Then $\tilde{\kappa}^{(3)}(CQ_n) = 4n-9$ for $n \geq 5$.

Lemma 3.7. ([18]) Let $F \subseteq V(CQ_4)$. If $|F| = 6$, then $CQ_4 - F$ satisfies one of the following conditions:

(1) $CQ_4 - F$ is connected;

(2) $CQ_4 - F$ has two components, one of which is a K_2;

(3) $CQ_4 - F$ has two components, one of which is an isolated vertex;

(4) $CQ_4 - F$ has three components, two of which

are isolated vertices;

(5) $CQ_4 - F$ has two components, which are two components of order 5.

Theorem 3.2. Let CQ_4 be the crossed cube. Then the 3-extra connectivity of CQ_4 is not 7.

Proof. Let F be the minimum 3-extra cut of CQ_4. If $|F| = 6$, by Lemma 3.7, then $CQ_4 - F$ satisfies one of the conditions (1)-(5) in Lemma 3.7. If $CQ_4 - F$ satisfies the condition (5), then F is a 3-extra cut. By the definition of 3-extra connectivity, $\tilde{\kappa}^{(3)}(CQ_4) \le |F| = 6$. Thus, the 3-extra connectivity of CQ_4 is not 7.

We give an example such that the 3-extra connectivity of CQ_4 is not 7. In CQ_4 (see Figure 1), let $F = \{0100, 0111, 0011, 1000, 1110, 1011\}$. Then $CQ_4 - F$ has two components A and B, where $V(A) = \{0001, 0000, 0010, 0110, 1010\}$ and $V(B) = \{0101, 1111, 1001, 1101, 1100\}$.

It is easy to see that $|F| = 6$ and $|V(A)| = |V(B)| = 5$.

Thus, F is a 3-extra cut of CQ_4. By the definition of 3-extra connectivity, $\tilde{\kappa}^{(3)}(CQ_4) \le |F| = 6$. In other words, the 3-extra connectivity of CQ_4 is not 7.

Lemma 3.8. ([18]) Let $F \subseteq V(CQ_n)$ $(n \ge 6)$. If $3n-4 \le |F| \le 4n-9$, then $CQ_n - F$ satisfies one of the following conditions:

(1) $CQ_n - F$ is connected;

(2) $CQ_n - F$ has two components, one of which is a K_2;

(3) $CQ_n - F$ has two components, one of which is a $K_{1,3}$;

(4) $CQ_n - F$ has two components, one of which is a 2-path;

(5) $CQ_n - F$ has two components, one of which is a 3-path;

(6) $CQ_n - F$ has two components, one of which is an isolated vertex;

(7) $CQ_n - F$ has three components, two of which are isolated vertices;

(8) $CQ_n - F$ has four components, three of which are isolated vertices;

(9) $CQ_n - F$ has three components, one of which is an isolated vertex and the other is a K_2;

(10) $CQ_n - F$ has three components, one of which is an isolated vertex and the other is a 2-path.

Theorem 3.3. For $n \ge 7$, the crossed cube CQ_n is tightly $(4n-9)$ super 3-extra connected.

Proof. Let F be a minimum 3-extra cut of CQ_n. By Theorem 3.1, $|F| = 4n-9$. We can decompose CQ_n along dimension $n-1$ into CQ_n^0 and CQ_n^1. Then both CQ_n^0 and CQ_n^1 are isomorphic to CQ_{n-1}. Let $F_0 = F \cap V(CQ_n^0)$ and $F_1 = F \cap V(CQ_n^1)$ with $|F_0| \le |F_1|$. Then $|F_1| \ge \left\lceil \frac{4n-9}{2} \right\rceil = 2n-4$. We consider the following cases.

Case 1. $2n-4 \le |F_1| \le 3n-9$.

In this case, $|F_0| \le 4n-9-(2n-4) = 2n-5$. By Lemma 3.2, $CQ_n^0 - F_0$ is connected or has two components, one of which is an isolated vertex.

By Lemma 3.3, $CQ_n^1 - F_1$ satisfies one of the following conditions:

(1) $CQ_n^1 - F_1$ is connected;

(2) $CQ_n^1 - F_1$ has two components, one of which is a K_2;

(3) $CQ_n^1 - F_1$ has two components, one of which is an isolated vertex;

(4) $CQ_n^1 - F_1$ has three components, two of which are isolated vertices.

Let B_i be the maximum component of $CQ_n^i - F_i$ for $i \in \{0,1\}$ (if $CQ_n^i - F_i$ is connected, then $B_i = CQ_n^i - F_i$). Since $2^{n-1} - (4n-9) - 3 \ge 1$ $(n \ge 7)$, by Proposition 2.1, $CQ_n[V(B_0) \cup V(B_1)]$ is connected. Thus, F is not a 3-extra cut of CQ_n. This is a contradiction to that F is a minimum 3-extra cut of CQ_n.

Case 2. $3n-8 \le |F_1| \le 4n-14$.

In this case, $|F_0| = 4n-9-(3n-8) = n-1$. By Lemma 3.2, $CQ_n^0 - F_0$ is connected or has two components, one of which is an isolated vertex. Note that $3(n-1)-5 = 3n-8 \le |F_1| \le 4n-14 = 4(n-1)-10$. By Lemma 3.6, $CQ_n^1 - F_1$ satisfies one of the following conditions:

(1) $CQ_n^1 - F_1$ is connected;

(2) $CQ_n^1 - F_1$ has two components, one of which is a K_2;

(3) $CQ_n^1 - F_1$ has two components, one of which is a 2-path;

(4) $CQ_n^1 - F_1$ has two components, one of which is an isolated vertex;

(5) $CQ_n^1 - F_1$ has three components, two of which are isolated vertices;

(6) $CQ_n^1 - F_1$ has four components, three of which are isolated vertices;

(7) $CQ_n^1 - F_1$ has three components, one of which is an isolated vertex and the other is a K_2.

If $CQ_n^0 - F_0$ is connected, by Proposition 2.1, then F is not a 3-extra cut of CQ_n. We suppose that $CQ_n^0 - F_0$ is disconnected. Let u be the isolated vertex and B_0 be the other component in $CQ_n^0 - F_0$. If $CQ_n^1 - F_1$ satisfies the condition (3), then let P be the 2-path and B_1 be the other component in $CQ_n^1 - F_1$. Since $2^{n-1} - (4n-9) - 4 \geq 1$ $(n \geq 7)$, by Proposition 2.1, $CQ_n[V(B_0) \cup V(B_1)]$ is connected. If u is connected to P, then $CQ_n - F$ has two components, one of which is $CQ_n[V(B_0) \cup V(B_1)]$ and the other is $CQ_n[\{u\} \cup V(P)]$ with $|\{u\} \cup V(P)| = 1 + 3 = 4$. If $CQ_n^1 - F_1$ satisfies one of the conditions (1)-(7) except (3), then F is not a minimum 3-extra cut of CQ_n, a contradiction.

Case 3. $|F_1| = 4n - 13$.

In this case, $|F_0| = 4n - 9 - (4n - 13) = 4$. By Lemma 3.1, $CQ_n^0 - F_0$ is connected. Note that $|F_1| = 4n - 13 = 4(n-1) - 9$. By Lemma 3.8, $CQ_n^1 - F_1$ satisfies one of the following conditions:

(1) $CQ_n^1 - F_1$ is connected;

(2) $CQ_n^1 - F_1$ has two components, one of which is a K_2;

(3) $CQ_n^1 - F_1$ has two components, one of which is a $K_{1,3}$;

(4) $CQ_n^1 - F_1$ has two components, one of which is a 2-path;

(5) $CQ_n^1 - F_1$ has two components, one of which is a 3-path;

(6) $CQ_n^1 - F_1$ has two components, one of which is an isolated vertex;

(7) $CQ_n^1 - F_1$ has three components, two of which are isolated vertices;

(8) $CQ_n^1 - F_1$ has four components, three of which are isolated vertices;

(9) $CQ_n^1 - F_1$ has three components, one of which is an isolated vertex and the other is a K_2;

(10) $CQ_n^1 - F_1$ has three components, one of which is an isolated vertex and the other is a 2-path.

Suppose that $CQ_n^1 - F_1$ satisfies the condition (3), then $CQ_n^1 - F_1$ has two components, one of which is a $K_{1,3}$. Note that $|V(K_{1,3})| = 4$. If $N_{CQ_n}(V(K_{1,3})) \cap V(CQ_n^0) = F_0$, then $CQ_n - F$ has two components, one of which is a $K_{1,3}$. Similarly, if $CQ_n^1 - F_1$ satisfies the condition (5), then $CQ_n - F$ has two components, one of which is a subgraph H of CQ_n with $|V(H)| = 4$. If $CQ_n^1 - F_1$ satisfies one of the conditions (1)-(10) except (3) and (5), then F is not a minimum 3-extra cut of CQ_n, a contradiction.

Case 4. $4n - 12 \leq |F_1| \leq 4n - 9$.

In this case, $|F_0| \leq 4n - 9 - (4n - 12) = 3$. By Lemma 3.1, $CQ_n^0 - F_0$ is connected. Suppose that $CQ_n^1 - F_1$ is connected. Since $2^{n-1} - (4n-9) \geq 1$ $(n \geq 7)$, by Proposition 2.1, $CQ_n[V(CQ_n^0 - F_0) \cup V(CQ_n^1 - F_1)] = CQ_n - F$ is connected. Then we suppose that $CQ_n^1 - F_1$ is disconnected. Let the components of $CQ_n^1 - F_1$ be C_1, C_2, …, C_k $(k \geq 2)$. Note that $|F_0| \leq 3$. If every component C_i of $CQ_n^1 - F_1$ such that $|V(C_i)| \geq 4$ for $i \in \{1, \ldots, k\}$, then $CQ_n[V(CQ_n^0 - F_0) \cup V(C_1) \cup \cdots \cup V(C_k)] = CQ_n - F$ is connected. Suppose that there is a components C_i such that $|V(C_i)| \leq 3$ for $i \in \{1, \ldots, k\}$. If $N_{CQ_n}(V(C_i)) \cap V(CQ_n^0) \subseteq F_0$, then C_i is a component of $CQ_n - F$ with $|V(C_i)| \leq 3$. Thus, F is not a 3-extra cut of CQ_n. This is a contradiction to that F is a minimum 3-extra cut of CQ_n.

4 The 3-extra Diagnosaility of Crossed Cubes Under the PMC and MM* Model

Theorem 4.1. ([8, 20-21]) A system $G = (V, E)$ is g-extra t-diagnosable under the PMC model if and only if there is an edge $uv \in E$ with $u \in V \setminus (F_1 \cup F_2)$ and $v \in F_1 \triangle F_2$ for each distinct pair of g-extra faulty subsets F_1 and F_2 of $V(CQ_n)$ with $|F_1| \leq t$ and $|F_2| \leq t$ (see Figure 2).

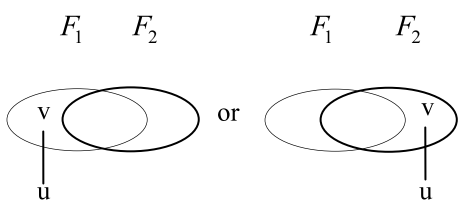

Figure 2. u diagnosis v under the PMC model

Theorem 4.2. ([8, 20-21]) A system $G = (V, E)$ is g-extra t-diagnosable under the MM* model if and only if each distinct pair of g-extra faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions (see Figure 3):

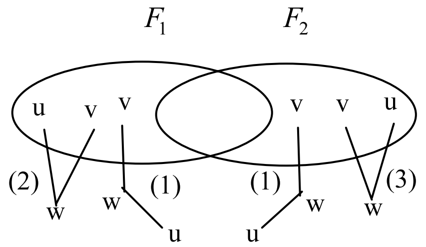

Figure 3. w diagnosis u and v under the MM^* model

(1) There exist two vertices $u, w \in V(G) \setminus (F_1 \cup F_2)$ and there exists a vertex $v \in F_1 \Delta F_2$ such that $uw, vw \in E(G)$.

(2) There exist two vertices $u, v \in F_1 \setminus F_2$ and there exists a vertex $w \in V(G) \setminus (F_1 \cup F_2)$ such that *uw*, $vw \in E(G)$.

(3) There exist two vertices $u, v \in F_2 \setminus F_1$ and there exists a vertex $w \in V(G) \setminus (F_1 \cup F_2)$ such that *uw*, $vw \in E(G)$.

Lemma 4.1. Let $n \geq 4$. Then the 3-extra diagnosability of the crossed cube CQ_n under the PMC and MM* model is less than or equal to $4n-6$, i.e., $\widetilde{t}_3(CQ_n) \leq 4n-6$.

Proof. Let A be defined in Lemma 3.4, $F_1 = N_{CQ_n}(A)$ and $F_2 = A \cup F_1$. By Lemma 3.4, $|F_1| = 4n-9$, $|F_2| = 4n-5$ and $CQ_n - F_2$ is connected. Thus, $CQ_n - F_1$ has two components $CQ_n - F_2$ and $CQ_n[A]$. Note that $|A| = 4$ and $|V(CQ_n - F_2)| = 2^n - (4n-5) \geq 4$ for $n \geq 4$. By the definition of 3-extra connectivity, F_1 is a 3-extra cut of CQ_n. Thus, F_1 and F_2 are both 3-extra faulty sets of CQ_n with $|F_1| = 4n-9$ and $|F_2| = 4n-5$. Since $A = F_1 \Delta F_2$ and $N_{CQ_n}(A) = F_1 \subset F_2$, there is no edge of CQ_n between $V(CQ_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$. By Theorems 4.1 and 4.2, CQ_n is not 3-extra $(4n-5)$-diagnosable under PMC and MM* model, respectively. By the definition of 3-extra diagnosability, we can deduce that the 3-extra diagnosability of CQ_n is less than or equal to $4n-6$, i.e., $\widetilde{t}_3(CQ_n) \leq 4n-6$.

Lemma 4.2. Let $n \geq 5$. Then the 3-extra diagnosability of the crossed cube CQ_n under the PMC model is more than or equal to $4n-6$, i.e., $\widetilde{t}_3(CQ_n) \geq 4n-6$.

Proof. By the definition of the 3-extra diagnosability, it is sufficient to show that CQ_n is 3-extra $(4n-6)$-diagnosable. By Theorem 4.1, we need to prove that there is an edge $uv \in E$ with $u \in V(CQ_n) \setminus (F_1 \cup F_2)$ and $v \in F_1 \Delta F_2$ for each distinct pair of 3-extra faulty subsets F_1 and F_2 of $V(CQ_n)$ with $|F_1| \leq 4n-6$ and $|F_2| \leq 4n-6$.

Suppose, on the contrary, that there are two distinct 3-extra faulty subsets F_1 and F_2 of $V(CQ_n)$ with $|F_1| \leq 4n-6$ and $|F_2| \leq 4n-6$, but there is no edge between $V(CQ_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$. Without loss of generality, assume that $F_2 \setminus F_1 \neq \varnothing$.

Claim 1. $V(CQ_n) \neq F_1 \cup F_2$.

On the contrary, we suppose that $V(CQ_n) = F_1 \cup F_2$. We get $2^n = |V(CQ_n)| = |F_1 \cup F_2| = |F_1| + |F_2| - |F_1 \cap F_2| \leq |F_1| + |F_2| \leq 2(4n-6) = 8n-12$, a contradiction to $n \geq 5$. Therefore, $V(CQ_n) \neq F_1 \cup F_2$. The proof of Claim 1 is complete.

Since there is no edge between $V(CQ_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$, $CQ_n - F_1$ has two parts $CQ_n \setminus (F_1 \cup F_2)$ and $CQ_n[F_2 \setminus F_1]$. Note that F_1 is a 3-extra faulty set. Thus, every component B_i of $CQ_n \setminus (F_1 \cup F_2)$ such that $|V(B_i)| \geq 4$ and every component C_i of $CQ_n[F_2 \setminus F_1]$ such that $|V(C_i)| \geq 4$. If $F_1 \setminus F_2 = \varnothing$, then $F_1 \cap F_2 = F_1$. Thus, $F_1 \cap F_2$ is also a 3-extra faulty set. If $F_1 \setminus F_2 \neq \varnothing$, similarly, every component D_i of $CQ_n[F_1 \setminus F_2]$ such that $|V(D_i)| \geq 4$. Thus, $F_1 \cap F_2$ is also a 3-extra faulty set. Since there is no edge between $V(CQ_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$, $F_1 \cap F_2$ is a 3-extra cut of CQ_n. By Theorem 3.1, $|F_1 \cap F_2| \geq 4n-9$. Thus, $|F_2| = |F_2 \setminus F_1| + |F_1 \cap F_2| \geq 4 + 4n - 9 = 4n-5$. This is a contradiction to that $|F_2| \leq 4n-6$. Therefore, CQ_n is 3-extra $(4n-6)$-diagnosable, i.e., $\widetilde{t}_3(CQ_n) \geq 4n-6$.

Combining Lemmas 4.1 and 4.2, we have the following theorem.

Theorem 4.3. Let $n \geq 5$. Then the 3-extra diagnosability of the crossed cube CQ_n under the PMC model is $4n-6$, i.e., $\widetilde{t}_3(CQ_n) = 4n-6$.

A component of a graph G is odd according as it has an odd number of vertices. We denote by $o(G)$ the number of odd components of G.

Lemma 4.3. ([16]) A graph $G = (V, E)$ has a perfect matching if and only if $o(G - S) \leq |S|$ for all $S \subseteq V$.

Lemma 4.4. Let $n \geq 7$. Then the 3-extra diagnosability of the crossed cube CQ_n under the MM* model is more than or equal to $4n-6$, i.e., $\widetilde{t}_3(CQ_n) \geq 4n-6$.

Proof. By the definition of 3-extra diagnosability, it is sufficient to show that CQ_n is 3-extra $(4n-6)$-diagnosable. On the contrary, there are two distinct 3-extra faulty subsets F_1 and F_2 of CQ_n with $|F_1| \leq 4n-6$ and $|F_2| \leq 4n-6$, but the vertex set pair (F_1, F_2) is not satisfied with any one condition in

Theorem 4.2. Without loss of generality, assume that $F_2 \setminus F_1 \neq \emptyset$.

By Claim 1 in Lemma 4.2, we get $V(CQ_n) \neq F_1 \cup F_2$.

Claim 1. $CQ_n - (F_1 \cup F_2)$ has no isolated vertex.

On the contrary, we suppose that $CQ_n - (F_1 \cup F_2)$ has at least one isolated vertex w. Since F_1 is one 3-extra faulty set, there is a vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w. Note that the vertex set pair (F_1, F_2) is not satisfied with any one condition in Theorem 4.2. By the condition (3) of Theorem 4.2., there is at most one vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w. Thus, there is just a vertex $u \in F_2 \setminus F_1$ such that u is adjacent to w. If $F_1 \setminus F_2 = \emptyset$, then $F_1 \subseteq F_2$. Since F_2 is a 3-extra faulty set, every component G_i of $CQ_n - F_2$ has $|V(G_i)| \geq 4$. Note that $CQ_n - F_2 = CQ_n - (F_1 \cup F_2)$. So $CQ_n - (F_1 \cup F_2)$ has no isolated vertex. It is contradict with the hypothesis. Thus, $F_1 \setminus F_2 \neq \emptyset$. Similarly, we can deduce that there is just a vertex $v \in F_1 \setminus F_2$ such that v is adjacent to w. Let $W \subseteq V(CQ_n) \setminus (F_1 \cup F_2)$ be the set of isolated vertices in $CQ_n[V(CQ_n) \setminus (F_1 \cup F_2)]$, and let H be the induced subgraph by the vertex set $V(CQ_n) \setminus (F_1 \cup F_2 \cup W)$. Then for any vertex $w \in W$, we can get that w has $(n-2)$ neighbors in $F_1 \cap F_2$. By Lemma 4.3 and Proposition 2.1, $|W| \leq o(CQ_n - (F_1 \cup F_2)) \leq |F_1 \cup F_2| = |F_1| + |F_2| - |F_1 \cap F_2| \leq 2(4n-6) - (n-2) = 7n - 10$. We assume that $V(H) = \emptyset$. Then $2^n = |V(CQ_n)| = |F_1 \cup F_2| + |W| = |F_1| + |F_2| - |F_1 \cap F_2| + |W| \leq 2(4n-6) - (n-2) + (7n-10) = 14n - 20$, a contradiction to that $n \geq 7$. Therefore, $V(H) \neq \emptyset$.

Since the vertex set pair (F_1, F_2) is not satisfied with the condition (1) of Theorem 4.2 and any vertex of $V(H)$ is not isolated in H, we induce that there is no edge between $V(H)$ and $F_1 \triangle F_2$. If $F_1 \cap F_2 = \emptyset$, then $F_1 \triangle F_2 = F_1 \cup F_2$. Since there is no edge between $V(H)$ and $F_1 \triangle F_2$, CQ_n is disconnected, a contradiction to that CQ_n is connected. Thus, $F_1 \cap F_2 \neq \emptyset$. Note that $CQ_n - (F_1 \cap F_2)$ has two parts: H and $CQ_n[(F_1 \setminus F_2) \cup (F_2 \setminus F_1) \cup W]$. Thus, $F_1 \cap F_2$ is a vertex cut of CQ_n. Since F_1 is a 3-extra faulty set of CQ_n, we have that every component H_i of H has $|V(H_i)| \geq 4$ and every component B_i^2 of $CQ_n[(F_2 \setminus F_1) \cup W]$ has $|V(B_i^2)| \geq 4$. Similarly, every component B_i^1 of $CQ_n[(F_1 \setminus F_2) \cup W]$ has $|V(B_i^1)| \geq 4$. By the definition of 3-extra cut, $F_1 \cap F_2$ is a 3-extra cut of CQ_n. By Theorem 3.1, $|F_1 \cap F_2| \geq 4n-9$.

Note that any vertex $w \in W$ has two neighbors u and v such that $u \in V(F_1 \setminus F_2)$ and $v \in V(F_2 \setminus F_1)$. If there is not a vertex w such that $w \in V(B_i^2)$, then B_i^2 is a component of $F_2 \setminus F_1$. We get that $|F_2 \setminus F_1| \geq |V(B_i^2)| \geq 4$. If there is a vertex w such that $w \in V(B_i^2)$, then $B_i^2 - w$ is a component of $F_2 \setminus F_1$. We get that $|F_2 \setminus F_1| \geq |V(B_i^2) \setminus \{u\}| \geq 3$. Thus, $|F_2 \setminus F_1| \geq 3$. Similarly, $|F_1 \setminus F_2| \geq 3$. Since $|F_1 \cap F_2| = |F_2| - |F_2 \setminus F_1| \leq (4n-6) - 3 = 4n-9$, we have $|F_1 \cap F_2| = 4n-9$. Then we get that $|F_2 \setminus F_1| = 3$ and $|F_2| = 4n-6$. Similarly, we have $|F_1 \setminus F_2| = 3$ and $|F_1| = 4n-6$. By Theorem 3.3, the crossed cube CQ_n is tightly $(4n-9)$ super 3-extra connected, i.e., $CQ_n - (F_1 \cap F_2)$ has two components, one of which is a subgraph of order 4. Thus, $CQ_n - (F_1 \cap F_2)$ has two components, one of which is $CQ_n[(F_1 \setminus F_2) \cup (F_2 \setminus F_1) \cup W]$ and the other one is H with $|V(H)| = 4$ and $|(F_1 \setminus F_2) \cup (F_2 \setminus F_1) \cup W| \geq 3 + 3 + 1 = 7$. Note that $|W| \leq 7n - 10$.

$2^n = |V(CQ_n)| = |F_1 \setminus F_2| + |F_2 \setminus F_1| + |F_1 \cap F_2| + |W| + |V(H)| \leq 3 + 3 + (4n-9) + (7n-10) + 4 = 11n - 9$, a contradiction to $n \geq 6$. The proof of Claim 1 is complete.

Let $u \in V(CQ_n) \setminus (F_1 \cup F_2)$. By Claim 1 in Lemma 4.4, u has at least one neighbor in $CQ_n - (F_1 \cup F_2)$. Since (F_1, F_2) is not satisfied with any one condition in Theorem 4.2, u has no neighbor in $F_1 \triangle F_2$. By the arbitrariness of u, there is no edge between $V(CQ_n) \setminus (F_1 \cup F_2)$ and $F_1 \triangle F_2$. Since F_1 and F_2 are two 3-extra faulty set, every component H_i of $CQ_n - (F_1 \cup F_2)$ has $|V(H_i)| \geq 4$, every component B_i of $CQ_n([F_2 \setminus F_1])$ has $|V(B_i)| \geq 4$, and every component C_i of $CQ_n([F_1 \setminus F_2])$ has $|V(C_i)| \geq 4$ when $F_1 \setminus F_2 \neq \emptyset$. Thus, $F_1 \cap F_2$ is also a 3-extra faulty set. Since there is no edge between $V(CQ_n \setminus (F_1 \cup F_2))$ and $F_1 \triangle F_2$, we have $F_1 \cap F_2$ is a 3-extra cut of CQ_n. By Theorem 3.1, we have $|F_1 \cap F_2| \geq 4n-9$. Since B_i is a component of $CQ_n([F_2 \setminus F_1])$ with $|V(B_i)| \geq 4$. we have $|F_2 \setminus F_1| \geq |V(B_i)| \geq 4$. Therefore, $|F_2| = |F_2 \setminus F_1| + |F_1 \cap F_2| \geq 4 + (4n-9) = 4n-5$, which contradicts $|F_2| \leq 4n-6$. Thus, CQ_n is 3-extra $(4n-6)$-diagnosable, i.e., $\widetilde{t}_3(CQ_n) \geq 4n-6$. The proof is complete.

Combining Lemmas 4.1 and 4.4, we can get the following theorem.

Theorem 4.4. Let $n \geq 7$. Then the 3-extra diagnosability of the crossed cube CQ_n under the MM* model is $4n-6$, i.e., $\widetilde{t_2}(CQ_n) = 4n-6$.

4 Conclusion

We prove that the 3-extra connectivity of CQ_n is $4n-9$ for $n \geq 5$. Moreover, CQ_n is tightly $(4n-9)$ super 3-extra connected for $n \geq 7$. Then we determine that the 3-extra diagnosability of CQ_n is $4n-6$ under the PMC model $(n \geq 5)$ and MM* model $(n \geq 7)$. On the basis of this study, the researchers can continue to study the g-extra connectivity and diagnosability of networks [22].

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