A Fuel-Efficient Route Plan Method Based on Game Theory

Chi-Lun Lo1,2, Chi-Hua Chen3, Jin-Li Hu4, Kuen-Rong Lo1, Hsun-Jung Cho2

1 Telecommunication Laboratories, Chunghwa Telecom Co., Ltd., Taiwan
2 Department of Transportation and Logistics Management, National Chiao Tung University, Taiwan
3 College of Mathematics and Computer Science, Fuzhou University, China
4 Institute of Business and Management, National Chiao Tung University, Taiwan
clo@cht.com.tw, chihua0826@gmail.com, jinlihu@mail.nctu.edu.tw, lo@cht.com.tw, hjcho001@gmail.com

Abstract

This study adopts a fuel consumption estimation method to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with individual behaviors. Furthermore, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan the most fuel-efficient route according to the consumed fuel quantity. The numerical analysis results show that the proposed fuel-efficient route plan app can effectively support fuel-saving for logistics industries.

Keywords: Fuel efficiency, Route plan, Game theory

1 Introduction

In recent years, the prices of diesel fuel and unleaded fuel have been increased to lead to higher cost of transportation for logistics industries [1]. For instance, the fuel cost of logistics industries was increased up to 35.8 billion dollars in Taiwan in 2015 [2]. Therefore, saving fuel consumption of fleet vehicles is an important challenge for logistics.

For fleet management, commercial vehicle operation systems (CVOSs) have been designed and implemented to collect the movement records of vehicles. These movement records can be periodically reported and used to track the location and speed of vehicle. Furthermore, the fuel invoices including the fuel quantity information after refueling can be uploaded into CVOS by driver. A fuel consumption estimation method (FCEM) based on a generic algorithm is hence proposed to analyze the movement records and the fuel quantity information for measuring the relationship the driver’s behaviors and fuel consumption [3].

Although the fuel consumption can be estimated to detect fuel-wasting based on driver’s behavior, some fuel-saving strategies (e.g., fuel-efficient route plans) should be developed and performed for reducing fuel cost. Therefore, this study adopts the proposed FCEM to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with each individual’s optimal behavior. Moreover, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan the most fuel-efficient route based on the game theory.

The remainder of the paper is organized as follows. Section 2 discusses the related work of route plan methods. Section 3 remarks the detail processes of a FCEM. Section 4 presents the design of proposed fuel-efficient route plan app and the game model of route plan apps. Section 5 gives a numerical analysis to evaluate the performance of the propose route plan app. Finally, Section 6 concludes this paper and discusses suggestions for future work.

2 Literature Reviews

Several studies proposed and designed the route plans based on (1) the shortest distance or (2) the shortest travel time [4-17].

For route plans based on the shortest distance, Jia al. used Particle Swarm Optimization (PSO) to minimize the total travel distance of all vehicles and solve the dynamic capacitated vehicle route problem [4]. This study assumed that the solution of shorter travel time and lower travel cost could be found in accordance with the shortest distance route plan. However, the traffic condition is dynamically changed. Therefore, the shortest distance route may be driven with longer travel time while the vehicle density of the shortest distance route is higher.

For route plans based on the shortest travel time, previous studies estimated the average travel time of each road segment and minimized the total travel time of each route [5-9]. Furthermore, some studies considered more factors (e.g., charge time of electric vehicle [9]) for a variety of applications. Although the traffic conditions were considered and analyzed in...
these studies [5-9], the fuel consumption estimation was not investigated. Therefore, the route plan method based on fuel consumption estimation is proposed and compared with the shortest distance route and the shortest travel time route.

3 Fuel Consumption Estimation Method

A FCEM based on a generic algorithm was proposed and evaluated to analyze the consumed fuel quantity of each vehicle speed interval [3] for individual driver. The method can generate a fuel consumption estimation function $g(u_i)$ in accordance with the vehicle speed $u_i$ in Route $i$ to estimate the fuel quantity in each 30 seconds. The details of process are illustrated as follows (shown in Figure 1).

Figure 1. The process of fuel consumption estimation method based on a generic algorithm [3]

1. The movement records (e.g., vehicle speed) and the fuel invoices (e.g., fuel quantity) are retrieved and analyzed.
2. A fitness function model and the score of each DNA (deoxyribonucleic acid) sequence are defined as Equations (1) and (2) to estimate the values of consumed fuel quantities $\{q_1, q_2, ..., q_{14}\}$. For instance, Driver 1 drove a car which was equipped with OBU 1 during 2016; $c_i$ records idle speed (i.e., the value of $u_i$ is zero) reported by OBU 1 during 2016, and average fuel consumption is $q_1$ liters in each 30 seconds while the average speed is idle speed; $c_2$ records the speed between 0 km/h and 10 km/h reported by OBU 1 during 2016, and average fuel consumption is $q_2$ liters in each 30 seconds while the average speed is between 0 km/h and 10 km/h; consequently, $c_{14}$ records the speed higher than 120 km/h reported by OBU 1 during 2016, and average fuel consumption is $q_{14}$ liters in each 30 seconds while the average speed is higher than 120 km/h. Furthermore, the summation of fuel quantities of OBU 1 during January 2016 is $F$ liters.

$$\sum_{k=1}^{14} c_i \times q_k \approx F \quad (1)$$

$$E = \left| \sum_{k=1}^{14} c_i \times q_k - F \right| \quad (2)$$

3. The sets of initial DNA sequences (i.e., the sets of consumed fuel quantities) can be randomly generated, and the score of each DNA sequence can be measured by using Equation (2). The value of $E$ is the fuel consumption estimation errors. Therefore, this study used a generic algorithm to generate the adatable DNA sequences for minimizing the absolute value of sum of estimation errors.

4. The process of the convergence check can be performed according to the maximum number of iterations, and an adaptable DNA sequence is outputted as the estimated results of the fuel consumption.

5. The processes of gene crossover and gene mutation can be performed to generate child’s DNA sequences.

6. The processes of gene reproduction can be performed to support that the generated child’s DNA sequences are substituted for original maternal DNA sequences for evolution. The score of each DNA sequence is calculated, and the generic algorithm is performed again.

A fuel consumption estimation function $g(u_i)$ can be obtained by the FCEM. The vehicle speed $u_i$ can be adopted into the function $g(u_i)$ to query the consumed fuel quantity $c_i$ for individual driver. A case study of fuel consumption estimation function $g(u_i)$ is showed in Table 1.
Table 1. A case study of fuel consumption estimation function [3]

<table>
<thead>
<tr>
<th>Vehicle speed interval (unit: km/hr)</th>
<th>Consumed fuel quantity in each 30 seconds (unit: liter)</th>
<th>Consumed fuel quantity in each hour (unit: liter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_i = 0$</td>
<td>0.007</td>
<td>0.840</td>
</tr>
<tr>
<td>$0 &lt; u_i \leq 10$</td>
<td>0.020</td>
<td>2.400</td>
</tr>
<tr>
<td>$10 &lt; u_i \leq 20$</td>
<td>0.033</td>
<td>3.960</td>
</tr>
<tr>
<td>$20 &lt; u_i \leq 30$</td>
<td>0.055</td>
<td>6.600</td>
</tr>
<tr>
<td>$30 &lt; u_i \leq 40$</td>
<td>0.013</td>
<td>1.560</td>
</tr>
<tr>
<td>$40 &lt; u_i \leq 50$</td>
<td>0.038</td>
<td>4.560</td>
</tr>
<tr>
<td>$50 &lt; u_i \leq 60$</td>
<td>0.069</td>
<td>8.280</td>
</tr>
<tr>
<td>$60 &lt; u_i \leq 70$</td>
<td>0.150</td>
<td>18.000</td>
</tr>
<tr>
<td>$70 &lt; u_i \leq 80$</td>
<td>0.142</td>
<td>17.040</td>
</tr>
<tr>
<td>$80 &lt; u_i \leq 90$</td>
<td>0.080</td>
<td>9.600</td>
</tr>
<tr>
<td>$90 &lt; u_i \leq 100$</td>
<td>0.048</td>
<td>5.760</td>
</tr>
<tr>
<td>$100 &lt; u_i \leq 110$</td>
<td>0.077</td>
<td>9.240</td>
</tr>
<tr>
<td>$110 &lt; u_i \leq 120$</td>
<td>0.284</td>
<td>34.080</td>
</tr>
<tr>
<td>$120 &lt; u_i$</td>
<td>0.492</td>
<td>59.040</td>
</tr>
</tbody>
</table>

4 Fuel-Efficient Route Plan App

For the design of fuel-efficient route plan app, the real-time traffic condition and the consumed fuel quantity of individual driving behavior are considered. However, the traffic condition may be influenced by other route plan apps (e.g., the shortest route plan app and the fast route plan app). Therefore, this study expresses the route plan as a game model to analyze the best responses of competitors to determine the fuel-efficient route plan. In this section, players in this game model are presented in Subsection 4.1, and the scenarios and candidate strategies of route plan are defined in Subsection 4.2. Finally, Subsection 4.3 shows the best response of each player.

4.1 Players

Three players who design and provide a route plan app join this game. The preferred strategy of each player is described as follows.

1) Player 1 selects the shortest route plan based on the lowest geo-distance. Player 1 plays as a traditional navigation system which does not consider the traffic condition to determine a route plan.

2) Player 2 selects the fastest route plan based on the lowest travel time. Player 2 plays as an Internet-based navigation system which does consider the traffic condition to determine a route plan.

3) Player 3 selects the fuel-efficient route plan app based on the traffic condition and fuel consumption estimation. Player 3 is proposed to plan the fuel-saving route in accordance with the traffic condition and individual behaviors.

4.2 Scenarios and Candidate Strategies

In this game, two routes (i.e., Route 1 and Route 2) from Node 1 to Node 2 are selected as candidate strategies for players (as Figure 2 shows). There are $Q$ vehicles distributed in these two routes, and $k_i$ vehicles are driven in Route $i$. The length of Route $i$ is defined as $d_i$ km, and the average speed of Route $i$ is defined as $u_i$ km/hr. The travel time $t_i$ can be measured in accordance with $d_i/u_i$ hr. Each player can develop the route plan according to their own preferred strategies. Table 2 summarizes notations in this game-theoretic model.

![Figure 2. Candidate strategies in the game model](image)

Table 2. Notations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>The number of total vehicle from Node 1 to Node 2 (unit: car)</td>
</tr>
<tr>
<td>$d_i$</td>
<td>The length of Route $i$ (unit: km)</td>
</tr>
<tr>
<td>$u_i$</td>
<td>The average speed of Route $i$ (unit: km/hr)</td>
</tr>
<tr>
<td>$k_i$</td>
<td>The number of vehicles in Route $i$ (unit: car)</td>
</tr>
<tr>
<td>$s_i$</td>
<td>The safe distance between two vehicles in Route $i$ (unit: m)</td>
</tr>
<tr>
<td>$t_i$</td>
<td>The travel time of Route $i$ (unit: hr)</td>
</tr>
<tr>
<td>$l$</td>
<td>The length of vehicle (unit: m)</td>
</tr>
<tr>
<td>$p_i$</td>
<td>The market share of Player $i$ (unit: %)</td>
</tr>
<tr>
<td>$g(u_i)$</td>
<td>The consumed fuel quantity of vehicle speed $u_i$ in each hour (unit: liter)</td>
</tr>
<tr>
<td>$f_i$</td>
<td>The consumed fuel quantity of Route $i$ (unit: liter)</td>
</tr>
</tbody>
</table>

4.2.1 Assumptions

The assumptions and limitations are given as follows for measuring the best response of each player.

• Player 1’s strategy is not influenced by traffic condition.

• Player 2's strategy can be influenced by traffic condition, so Player 2’s strategy is developed based on the best response of Player 1.

• Player 3’s strategy is developed based on the best response of Players 1 and 2. The game tree is showed in Figure 3.
The market share of Player 3 (i.e., $p_3$) is about zero.
The values of $Q_1$, $p_1$, $p_2$, $d_1$, and $d_2$ are predefined, and $d_1$ is longer than $d_2$.
Each vehicle can be driven with the aspirational vehicle speed with the adaptable safe distance in the recommended route.
The adaptable safe distance between each two vehicles in Route $i$ is assumed as $u_i/2$ m [18-19]. For instance, the adaptable safe distance is 50 m when the speed is 100 km/hr.

4.2.2 Aspirational Vehicle Speed and Travel Time

For the calculation of aspirational vehicle speed and travel time, this study assumes that the value of safe distance (unit: m) is the half value of speed (unit: km) [18-19]. Therefore, the required safe distance of each vehicle is estimated in accordance with the vehicle length and the adaptable safe distance (shown in Equation (3)). Therefore, the number of vehicle in Route $i$ can be determined by Equation (4) according to the required safe distance of each vehicle. After the transposition of Equation (4), the aspirational vehicle speed can be calculated as $\frac{2000d_i}{k_i} - 2l$ by Equation (5). Furthermore, the length of Route $i$ can be considered to estimate the aspirational travel time by Equation (6).

$$u_i = \frac{2000 \times d_i - 2 \times l \times k_i}{k_i}$$ (5)

$$t_i = \frac{d_i}{u_i} = \frac{d_i}{2000 \times d_i - 2l}$$ (6)

4.2.3 The Cost Function of Each Player

The cost functions of strategies for players in this game are remarked as follows.
- The cost of Player 1’s Strategy 1 is $d_1$ in accordance with the length of Route 1.
- The cost of Player 1’s Strategy 2 is $d_2$ in accordance with the length of Route 2.
- The cost of Player 2’s Strategy 1 is $t_1$ in accordance with the travel time of Route 1.
- The cost of Player 2’s Strategy 2 is $t_2$ in accordance with the travel time of Route 2.
- The cost of Player 3’s Strategy 1 is $f_1$ which is defined as Equation (7). The fuel consumption of Route 1 is estimated as $f_1$.

$$f_1 = t_1 \times g(u_i)$$ (7)

- The cost of Player 3’s Strategy 2 is $f_2$ which is defined as Equation (8). The fuel consumption of Route 2 is estimated as $f_2$.

$$f_2 = t_2 \times g(u_2)$$ (8)

4.3 The Best Response of Each Player

The best responses of players are discussed in the follow subsections.

4.3.1 The Best Response of Player 1

The preferred strategy of Player 1 is the shortest route plan. Therefore, Strategy 2 will be selected when $d_1$ is longer than $d_2$. The navigation system built by Player 1 will recommend users to drive their vehicle through Route 2, so $p_1 \times Q$ vehicles will be driven in Route 2.

4.3.2 The Best Response of Player 2

The preferred strategy of Player 2 is the fast route plan. Player 2 develops a mix strategy in accordance with the ratio of $r$ for Strategy 1 and the ratio of $(1 - r)$ for Strategy 2. In the recommendation of Player 2’s app, $p_2 \times Q \times r$ vehicles will be driven in Route 1, and $p_2 \times Q \times (1 - r)$ vehicles will be driven in Route 2. Therefore, the objective function of Player 2 can be expressed as Equation (9) in accordance with game theory [20], and the total cost of Player 2 is defined as...
\[ \pi \text{ in Equation (9). The adaptable value of } r \text{ can be estimated by Equation (10) for the best response of Player 2. The proofs of Equation (10) are presented in Appendix A.} \]

### 4.3.3 The Best Response of Player 3

The preferred strategy of Player 3 is the most fuel-efficient route plan. The aspirational vehicle speed and travel time can be estimated in accordance with the adaptable value of \( r \) in Equation (10) based on the best responses of Player 1 and Player 2. Player 3 can adopt the estimated vehicle speeds (i.e., \( u_1 \) and \( u_2 \)) and travel time (i.e., \( t_1 \) and \( t_2 \)) into Equations (7) and (8) to calculate the costs of Strategy 1 and Strategy 2 for the development of the route plan.

### 5 Numerical Analyses

This section gives two case studies to illustrate the fuel consumption of the proposed route plan method which is compared with the shortest distance route and the shortest travel time route.

#### 5.1 Case 1: Effects of \( d_1 \) on the \( c_1 \) and \( c_2 \)

For the purpose of demonstration, this study adopted some parameters as follows to present the game in Subsection 5.1: \( Q = 2,300 \) cars, \( d_2 = 10 \) km, \( l = 5 \) m, \( p_1 = 0.4 \), and \( p_2 = 0.6 \). Because the strategy of Player 1 was the shortest distance route plan, the best response of Player 1 was to recommend 920 users to drive their vehicles though Route 2. Furthermore, the value of \( r \) was determined by Equation (10) for the best response of Player 2 (shown in Table 3). For instance, the value of \( r \) was estimated as 0.89 while the route length of \( d_1 \) was 13 km. For the users of Player 2’s app, 1,226 vehicles were recommended to drive through Route 1, and 154 vehicles were recommended to drive through Route 2. Therefore, the vehicle speeds of Routes 1 and 2 were, respectively, 11.21 km/hr and 8.62 km/hr; the travel times of Routes 1 and 2 were, respectively, 1.16 and 1.16 hr. For the best response of Player 3, Table 1 was adopted as the parameter values for fuel consumption function \( g(u_i) \), while the consumed fuel quantities of Strategies 1 and 2 were, respectively, 4.59 and 2.78 liters which were calculated by Equations (11) and (12). Therefore, Route 2 was recommended to the users of Players 1’s and 3’s apps, and the consumed fuel quantity was estimated as 2.78 liters. Moreover, the average consumed fuel quantity of the route which was recommended by Player 2’s app was measured as 4.39 liters by Equation (13). Figure 4 shows the fuel consumption comparison of each player.

#### Table 3. Case 1: Experimental results

<table>
<thead>
<tr>
<th>( d_1 )</th>
<th>( r )</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.86</td>
<td>1180</td>
<td>1120</td>
<td>8.64</td>
<td>7.86</td>
<td>1.27</td>
<td>1.27</td>
<td>3.05</td>
<td>3.05</td>
</tr>
<tr>
<td>12</td>
<td>0.87</td>
<td>1205</td>
<td>1095</td>
<td>9.92</td>
<td>8.26</td>
<td>1.21</td>
<td>1.21</td>
<td>2.90</td>
<td>2.90</td>
</tr>
<tr>
<td>13</td>
<td>0.89</td>
<td>1226</td>
<td>1074</td>
<td>11.21</td>
<td>8.62</td>
<td>1.16</td>
<td>1.16</td>
<td>4.59</td>
<td>2.78</td>
</tr>
<tr>
<td>14</td>
<td>0.90</td>
<td>1244</td>
<td>1056</td>
<td>12.51</td>
<td>8.94</td>
<td>1.12</td>
<td>1.12</td>
<td>4.43</td>
<td>2.69</td>
</tr>
<tr>
<td>15</td>
<td>0.91</td>
<td>1259</td>
<td>1041</td>
<td>13.82</td>
<td>9.22</td>
<td>1.09</td>
<td>1.09</td>
<td>4.30</td>
<td>2.60</td>
</tr>
<tr>
<td>16</td>
<td>0.92</td>
<td>1273</td>
<td>1027</td>
<td>15.15</td>
<td>9.47</td>
<td>1.06</td>
<td>1.06</td>
<td>4.18</td>
<td>2.54</td>
</tr>
<tr>
<td>17</td>
<td>0.93</td>
<td>1284</td>
<td>1016</td>
<td>16.47</td>
<td>9.69</td>
<td>1.03</td>
<td>1.03</td>
<td>4.09</td>
<td>2.48</td>
</tr>
<tr>
<td>18</td>
<td>0.94</td>
<td>1295</td>
<td>1005</td>
<td>17.81</td>
<td>9.89</td>
<td>0.99</td>
<td>0.99</td>
<td>3.93</td>
<td>3.93</td>
</tr>
<tr>
<td>19</td>
<td>0.94</td>
<td>1304</td>
<td>996</td>
<td>19.15</td>
<td>10.08</td>
<td>0.98</td>
<td>0.98</td>
<td>3.87</td>
<td>3.87</td>
</tr>
<tr>
<td>20</td>
<td>0.95</td>
<td>1312</td>
<td>988</td>
<td>20.49</td>
<td>10.24</td>
<td>0.98</td>
<td>0.98</td>
<td>3.87</td>
<td>3.87</td>
</tr>
</tbody>
</table>
5.2 Case 2: Effects of $Q$ on the $c_1$ and $c_2$

For the purpose of demonstration, this study adopted some parameters as follows to present the game in Subsection 5.2: $d_2 = 15$ km, $d_2 = 10$ km, $l = 5$ m, $p_1 = 0.4$, and $p_2 = 0.6$. For instance, the value of $r$ was estimated as 0.87 while the value of $Q$ was 1400 cars. Because the strategy of Player 1 was the shortest distance route plan, the best response of Player 1 was to recommend 560 users to drive their vehicles through Route 1. Furthermore, the value of $r$ was determined by Equation (10) for the best response of Player 2 (as Table 4 shows). For the users of Player 2’s app, 741 vehicles were recommended to be driven through Route 1, and 99 vehicles were recommended to be driven through Route 2. Therefore, the vehicle speeds of Routes 1 and 2 were 30.50 km/hr and 20.33 km/hr, respectively; the travel times of Routes 1 and 2 were 0.49 hr and 0.49 hr, respectively. For the best response of Player 3, Table 1 was adopted for parameter values of the fuel consumption function $g(u)$, and the consumed fuel quantities of Strategies 1 and 2 were 0.77 liters and 3.25 liters which were calculated by Equations (7) and (8). Therefore, Route 1 was recommended to the users of Player 3’s app, and the consumed fuel quantity was estimated as 0.77 liters. However, Route 2 was recommended to the users of Player 1’s app, and the consumed fuel quantity was estimated as 3.25 liters. Moreover, the average consumed fuel quantity of the route recommended by Player 2’s app was measured as 1.06 liters. Figure 5 shows the fuel consumption comparison of each player.

6 Conclusions and Future Work

This study adopts a FCEM to measure the consumed fuel quantity of each vehicle speed interval (i.e., a cost function) in accordance with each individual’s optimal behavior. Furthermore, a mobile app is designed to consider the best responses of other route plan apps (e.g., the shortest route plan app and the fast route plan app) and plan a fuel-efficient route according to the consumed fuel quantity. The numerical analysis results showed that the proposed fuel-efficient route plan app can support fuel-saving for logistics industries.

In the future, the route plan app can be implemented in accordance with the proposed method for reducing fuel consumption. Furthermore, the complex road network including several routes (i.e., multiple strategies) can be considered and selected by players. Furthermore, the market share of Player 3 can be increased to influence Player 2’s strategy.
Acknowledgments

The research was funded by Fuzhou University, grant number 510730/XRC-18075.

References


Biographies

Chi-Lun Lo is working toward a Ph.D. degree in the Department of Transportation and Logistics Management of National Chiao Tung University (NCTU), Taiwan. He also serves as a researcher for the Business Solutions Laboratory in the Telecommunication Laboratories, Chunghwa Telecom Co. Ltd., Taiwan, R.O.C.
Chi-Hua Chen is a distinguished professor (a Minjiang Scholar and a Qishan Scholar) at the College of Mathematics and Computer Science of Fuzhou University in China. He received a Ph.D. degree from the Department of Information Management and Finance of NCTU in 2013. His recent research interests include Internet of things and deep learning.

Jin-Li Hu is currently a professor at Institute of Business and Management, NCTU, Taiwan. He received Ph.D in Economics, SUNY at Stony Brook, USA. His research interest includes applied game theory, efficiency and productivity, and energy policy.

Kuen-Rong Lo received a Ph.D. degree in electronic engineering from the NCTU, Taiwan, in 2000. He currently is an IoT (Internet of Things) Laboratory Managing Director in the Telecommunication Laboratories, Chunghwa Telecom Co. Ltd., responsible for the ITS (Intelligent Transport Systems) technologies and related service systems.

Hsun-Jung Cho received a Ph.D. degree in urban and regional planning (major in Transportation Networks) from the University of Pennsylvania, Philadelphia, in 1989. Following 1989, he joined the Department of Transportation Engineering and Management (TEM), NCTU, Hsinchu, Taiwan. His research interests include sensitivity analysis of network equilibrium, network design, traffic simulation, intelligent transportation systems, and urban planning.

Appendix A. Partial Differential Equation Proof

The partial differential equation proof of Equation (10) is expressed as Equation A(1).

\[
\begin{align*}
\frac{\partial \pi}{\partial r} &= \partial \left( p_t \times Q \times t_t + \left( p_t \times Q \times (1-r) + (1-p_t) \times Q \times t_t \right) \right) \\
&= \frac{\partial \left( p_t \times Q \times t_t \right)}{\partial r} + \frac{\partial \left( p_t \times Q \times (1-r) \times t_t + (1-p_t) \times Q \times t_t \right)}{\partial r} \\
&= \frac{p_t \times Q \times (1-t_t)}{\partial r} \\
&= p_t \times Q \times (t_t - t_t)
\end{align*}
\]