Discrete Sliding Mode Control for a Class of WSN Based IoT with Time-delay and Nonlinear Characteristic

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Abstract

This paper applies discrete sliding mode control to the WSN based IoT, to ensure its robust stability under different types of communication time-delay and nonlinear characteristic. Due to the limited sampling frequency, the system will approach a quasi-sliding surface in finite time. Linear matrix inequality (LMI) approach is used to prove the quadratically stability of the sliding surface and suitable Lyapunov function is constructed to obtain the appropriate control law. Two situation will be investigated in this paper, the system with signal time-delay and system with multiple time-delays. Furthermore, nonlinear characteristic of system has been taken into consideration. By employing non-singular transformation and reduced order processing, the proof of sliding surface stability and finite time approaching is simplified. Finally, two numerical simulations are employed to demonstrate the effectiveness and applicability of the proposed approach.

Keywords: Internet of things, Sliding mode control, Discrete-time system, Time-delay system, Nonlinear characteristic

1 Introduction

Recently, with the maturity of communication technology, sensor technology and other relative technology, WSN based IoT shows huge potential in many areas. Such as home appliance, logistics, environment monitoring, military systems, security tracking, smart grid and so on. Constituted by various tiny sensors or devices, WSN based IoT has superiority in aspects like easy to be deployed and extended, high fault tolerant [1-4].

However, many challenges still exist. Each node in WSN based IoT has limit power as well as that they seldom simultaneously meet the design goal [5]. Distributed filtering problem for sensor networks may cause the IoT unstable [6]. IoT is Constituted by several sensors, and the link between nodes may failure by various causes [7-8]. Some serious problems may even cause the whole system to crash [9-11].

Many researchers have done various encouraging work in the field of WSN based IoT. [12-13] are concerned with a new distributed $H_{\infty}$-consensus filtering problem over a finite-horizon for a class of sensor networks with multiple missing measurements and a class of polynomial nonlinear stochastic systems. [14] presents a decentralized event-triggered implementation, over sensor/actuator networks, of centralized nonlinear controllers. [15-16] classify the coverage problem from different angles, describe the evaluation metrics of coverage control algorithms, analyze the relationship between coverage and connectivity, compare typical simulation tools, and discuss research challenges and existing problems in this area. [17-18] use mobile sensor network in enabling an information-theoretic distributed control architecture to facilitate search and enabling a variety of new applications that rely on position information. [19] discusses the emerging application of device-free localization (DFL) using wireless sensor networks, which find people and objects in the environment in which the network is deployed, even in buildings and through walls. These networks are termed “RF sensor networks” and [20] presents an overview of principles and requirements for powering wireless sensors by radio-frequency (RF) energy harvesting or transport. [21] presents a novel vehicular clustering scheme integrating hierarchical clustering on the basis of classical routing algorithm. And [22] discusses the general principles of swarm intelligence and of its application to routing. In the aspect of environment protecting, [23] is concerned with the application of wireless sensor network (WSN) technology to long-duration and large-scale environmental monitoring. And [24] considers joint problems of control and communication in wireless sensor and actuator networks (WSANs) for building-environment control systems. The sliding mode control has strong robustness and its robustness will not be affected by parametric uncertainty, various perturbations and time-delay. It is very suitable for sensor network. The
Finally, in section 5, we conclude the whole paper briefly and raise some suppose of future research.

2 Problem Formulation

Concerning a type of WSN based IoT, information flow is transmitted among all sensors directly. Let \( G = \{v, e\} \) be a directed graph of order \( n \) with the set of nodes \( v = \{1, 2, \ldots, n\} \), set of edges \( e \subseteq v \times v \), edge \((i, j) \in e\) indicates that node \( j \) is able to receive information from node \( i \). The adjacent matrix \( A = [a_{ij}] \in \mathbb{R}^{n \times n} \) of directed graph \( G \) is defined as follows: if information exchange exists between two nodes \( i \) and \( j \), then \( a_{ij} \) denotes the throughput from \( i \) to \( j \). At the same time, node \( i \) may also receive information from \( j \), yet \( a_{ji} \) is not necessarily equal to \( a_{ij} \). Taking into account that if the node is static, the connection structure inside the system is not varying with time. Then the model is given as below:

\[
\begin{aligned}
  x(k+1) &= Ax(k) + A_j x(k-d) \\
  &\quad + Bu(k) + g(x(k)) \\
  x(k) &= \psi(k) \quad k \in [-d, 0]
\end{aligned}
\]  

where \( x(k) \in \mathbb{R}^n \) is system state, \( u(k) \in \mathbb{R}^m \) denotes system control input, \( g(x(k)) \) is nonlinear characteristic, \( A, A_j \), and \( B \) are constant matrices with appropriate dimensions.

However, in practical system, WSN based IoT will change its structure by many reasons. The link between nodes may be changed or failure, and the movement of nodes may beyond communication range. These uncertain factors are reasons resulting system structure uncertainty. Then if each node only has information exchange with its adjacent node, the system model is described as below:

\[
\begin{aligned}
  x(k+1) &= (A + \Delta A)x(k) + (A_j + \Delta A_j)x(k-d) \\
  &\quad + Bu(k) + g(x(k)) \\
  x(k) &= \psi(k) \quad k \in [-d, 0]
\end{aligned}
\]  

where \( x(k) \in \mathbb{R}^n \) denotes system state, \( u(k) \in \mathbb{R}^m \) is system control input, \( \Delta A \) and \( \Delta A_j \) are internal parameter perturbation arising from uncertain factors, \( g(x(k)) \) is nonlinear characteristic, \( A, A_j \), and \( B \) are constant matrices with appropriate dimensions.

If a node in system is influenced by multiple time-delays, then the model is described as below:

\[
\begin{aligned}
  x(k+1) &= (A + \Delta A)x(k) + \sum_{i=1}^{N} (A_{ij} + \Delta A_{ij})x(k-d_i) \\
  &\quad + Bu(k) + g(x(k)) \\
  x(k) &= \psi(k) \quad k \in [-d, 0]
\end{aligned}
\]
where $x(k) \in \mathbb{R}^n$ denotes system state, $u(k) \in \mathbb{R}^m$ is system control input, $\Delta A$ and $\Delta A_j$ are internal parameter perturbation arising from uncertain factors, $g(x(k))$ is nonlinear characteristic, $A$, $A_j$, and $B$ are constant matrices with appropriate dimensions, $N$ is the number of agents in the system.

Considering the practical situation, only the system (2) and (3) will be investigated.

For the systems shown in (2) and (3), we will use the following assumptions:

**Assumption 1.** $(A, B)$ is fully controllable and $B$ is a column full rank.

**Assumption 2.** Time delay $d_i$ is boundary, and

$$\|d_i\| \leq d$$

**Assumption 3.** The perturbation parameter of the system satisfies

$$[\Delta A \Delta A_j] = EF(k)[H \ H_d]$$

Respectively, $E$, $H$ and $H_d$, are known constant matrix, $F(k)$ is time-delay uncertain matrix, yet Lebesgue-measurable, and $F^T(k)F(k) \leq I$.

**Assumption 4.** Suppose $g(x(k))$ is the nonlinear characteristic influenced by system state $x(k)$, then there is a positive constant $\rho$ makes

$$g(x(k))^T g(x(k)) \leq \rho x(k)^T x(k)$$

The designing of sliding mode control usually conclude two parts. One part, designing a sliding surface and then certifying its quadratical stability. An other part is to solve the control law which can ensure the system reach the sliding surface in finite time.

Besides, some important lemmas are presented here.

**Lemma 1.** Let $D, E$, and $F(k)$ be real matrix of proper dimensions, and $F^T(k)F(k) \leq I$, there exists a constant $\varepsilon$, which makes the following equation holds.

$$EF(k)H + H^T F^T(k)E^T \leq \varepsilon^{-1} EE^T + \varepsilon H^T H$$

**Lemma 2.** Let $a$, $b$ be real matrix of proper dimensions, there exists a constant matrix $X > 0$, which makes the following equation hold.

$$ab + b^T a^T \leq aXa^T + b^T X^{-1} b$$

**Lemma 3.** Given a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where $S_{11}$ is $r \times r$ dimensional, the following three conditions are equivalent:

1. $S < 0$;
2. $S_{11} < 0$, $S_{22} + S_{12}^T S_{11}^{-1} S_{12} < 0$;
3. $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$;

**Lemma 4.** Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ and matrix $Q > 0$.

Then, we have $2x^T Q_x \leq x^T Q_x + y^T Q_y$.

### 3 Main Results

#### 3.1 WSN Based IoT with Single Time-delay and Nonlinear Characteristic

In this section, we investigate the system with single time-delay and nonlinear characteristic.

For this system which meets assumption 1 to 4, we construct following sliding mode function:

\begin{equation}
S(k) = Gx(k) + \sigma(k)
\end{equation}

\begin{equation}
\sigma(k + 1) = Gx(k) - GAx(k) - G\Delta x(k) + \sigma(k)
\end{equation}

\begin{equation}
\sigma(0) = -Gx(0)
\end{equation}

where $S(k) = [S_1(K), S_2(K), \ldots, S_m(K)]^T$, $G \in \mathbb{R}^{m \times n}$ and GB is nonsingular matrix. Then choosing $G = B^T P$.

According to the discrete sliding mode theory, while the system reach the sliding surface $S(k + 1) = S(k) = 0$, the ideal equivalent control law is

$$u = -(GB)^{-1}[G\Delta x(k) + GB\Delta x(k-d) + Gg]$$

the equivalent control law (5) is only used to support the following proof and real control law will be given later. Substituting (5) into (2) the ideal dynamic sliding mode function is:

\begin{equation}
x(k + 1) = [A + \Delta A - B(GB)^{-1} G\Delta A] x(k) + [A_j + \Delta A_j - B(GB)^{-1} G\Delta A_j]x(k-d) + g(x(k + 1)) - B(GB)^{-1} Gg(x(k))
\end{equation}

**Theorem 1.** The system (2), the sliding surface is (4).

If there existing positive matrix $P > 0$ and $Q > 0$ and constant $\varepsilon > 0$ ($i=1, 2, 3, 4, 5, 6$) to make (7)-(9) exist, the system (2) is quadratically stable.

\begin{equation}
\begin{bmatrix}
\Pi_1 & 0 & 2A_1^T P & 0 \\
* & \Pi_2 & 0 & 2A_2^T P \\
* & * & -\varepsilon_1 I & 0 \\
* & * & * & -\varepsilon_2 I
\end{bmatrix} < 0
\end{equation}

\begin{equation}
\begin{bmatrix}
-B^T P B & B^T P B \\
* & -\varepsilon_3 I
\end{bmatrix} < 0
\end{equation}

\begin{equation}
\begin{bmatrix}
-P & PE \\
* & -\varepsilon_4 I
\end{bmatrix} < 0
\end{equation}

Where * is acquired based on matrix symmetry, and
\[ \Pi_1 = 4A^TPA + 4\varepsilon_c H^TH + 3\varepsilon_c H^T H + 5\varepsilon_c H^T H - 2\rho I - P + Q \]
\[ \Pi_2 = 4A_d^TPA_d + 4\varepsilon_c H^T H_d + 4\varepsilon_c H^T H_d + 2\rho I - Q \]

Choosing Lyapunov function as

\[ V(k) = x^TPx(k) + \sum_{i=1}^{k-1} x^T(i)Qx(i) \] (10)

Substituting (6) into (10),

\[ V(k) = V(k + 1) - V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) \]

where

\[ \Delta V_1(k) = x^T(k)(A + \Delta A)^TP(A + \Delta A)x(k) \]
\[ \Delta V_2(k) = x^T(k)(A + \Delta A)^TPB(B^TPB)^{-1}B^TP \Delta A x(k) \]
\[ \Delta V_3(k) = x^T(k)P\Delta A x(k) + x^T(k)Qx(k) \]
\[ \Delta V_4(k) = x^T(k)(A + \Delta A)^TPB(B^TPB)^{-1}B^TP \]

According to lemma 2, the second item and third item in \( \Delta V_1(K) \) can be transformed to:
\[-x^T(k)(A + \Delta A)^T\]
\[-x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]
\[-x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[-x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[-x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

According to lemma 2, the first item in $\Delta V_2(K)$ can be transformed to:

\[2x^T(k)(A + \Delta A)^T P(A_j + \Delta A_j)x(k - d)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

And the second item can be transformed to:

\[-2x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k - d)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

The third item can be transformed to:

\[-2x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k - d)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

The forth item can be transformed to:

\[2x^T(k)(A + \Delta A)^T P(A_j + \Delta A_j)x(k - d)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

And the second item and third item in $\Delta V_3(K)$ can be transformed to:

\[-x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]
\[-x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P \Delta A x(k)\]

Then about $\Delta V_3(K)$, the first and second item:

\[x^T(k)(A + \Delta A)^T P g(x(k + 1))\]
\[+ g^T(x(k + 1)) P(A + \Delta A)x(k)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ g^T(x(k + 1)) P(x(k + 1))\]

The third and forth item:

\[-x^T(k)(A + \Delta A)^T PB(B^T PB)^{-1} B^T P g(x(k))\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ g^T(x(k + 1)) P(x(k + 1))\]

According to lemma 4 The fifth and sixth item:

\[x^T(k)(A + \Delta A)^T P B(B^T PB)^{-1} B^T P g(x(k))\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ g^T(x(k + 1)) P(x(k + 1))\]

The seventh and eighth item:

\[x^T(k)(A + \Delta A)^T P B(B^T PB)^{-1} B^T P g(x(k))\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ g^T(x(k + 1)) P(x(k + 1))\]

As first to eighth items, the ninth to sixteenth item:

\[x^T(k)(A + \Delta A)^T P g(x(k + 1))\]
\[+ g^T(x(k + 1)) P(A + \Delta A)x(k)\]
\[\leq x^T(k)(A + \Delta A)^T P(A + \Delta A)x(k)\]
\[+ g^T(x(k + 1)) P(x(k + 1))\]
\[-x^T (k-d) \Delta A_j^T \]
\[PB(B^T PB)^{-1} B^T P g(x(k+1)) \]
\[-g^T (x(k+1))PB(B^T PB)^{-1} \]
\[B^T P \Delta A_j x(k-d) \leq -x^T (k-d) \Delta A_j^T \]
\[PB(B^T PB)^{-1} B^T \]
\[PPB(B^T PB)^{-1} B^T P \Delta A_j x(k-d) \]
\[-g^T (x(k+1)) g(x(k+1)) \]
\[x^T (k-d) \Delta A_j^T \]
\[PB(B^T PB)^{-1} B^T P g(x(k)) \]
\[+g^T (x(k))PB(B^T PB)^{-1} \]
\[B^T P \Delta A_j x(k-d) \leq x^T (k-d) \Delta A_j^T PB(B^T PB)^{-1} B^T \]
\[PPB(B^T PB)^{-1} B^T P \Delta A_j x(k-d) \]
\[+g^T (x(k)) g(x(k+1)) \]

And according to lemma 2, the seventeenth and eighteenth item:
\[-g^T (x(k+1))PB(B^T PB)^{-1} \]
\[B^T P g(x(k)) \]
\[-g^T (x(k))PB(B^T PB)^{-1} \]
\[B^T P g(x(k+1)) \]
\[-g^T (x(k+1)) P g(x(k+1)) \]
\[\leq x^T (k-d) \Delta A_j^T PB(B^T PB)^{-1} B^T \]
\[PPB(B^T PB)^{-1} B^T P \Delta A_j x(k-d) \]
\[-g^T (x(k))PB(B^T PB)^{-1} B^T P g(x(k)) \]

Besides, there existing \( \varepsilon_i \) (i=1, 2, 3, 4), using lemma 1:
\[x^T (k)(A + \Delta A)^T P (A + \Delta A)x(k) \]
\[= x^T (k) A^T P A x(k) \]
\[+ x^T (k) A^T P A x(k) \]
\[+ x^T (k) A^T P A x(k) \]
\[+ x^T (k) \Delta A^T P A x(k) \]
\[\leq x^T (k) [A^T P A + \varepsilon_i^{-1} A] \]
\[PEE^T P A + \varepsilon_i H^T H \]
\[+ \Delta A^T P \Delta A x(k) \]
\[x^T (k-d) (A + \Delta A_j)^T \]
\[P (A_j + \Delta A_j)x(k-d) \]
\[= x^T (k-d) A^T P A x(k-d) \]
\[+ x^T (k-d) A^T P A x(k-d) \]
\[+ x^T (k-d) A^T P A x(k-d) \]
\[+ x^T (k-d) A^T P A x(k-d) \]

+\[x^T (k-d) \Delta A_j^T P A_j x(k-d) \]
\[+ x^T (k-d) \Delta A_j^T P A_j x(k-d) \]
\[\leq x^T (k-d) [\Delta A_j^T P A_j \]
\[+ \varepsilon_i H_j^T H_d \]
\[+ \Delta A_j^T P \Delta A_j] x(k-d) \]

Substituting the result above into (11), then:
\[\Delta V(k) \leq [x^T (k) x^T (k-d) M \]
\[x(k)] \tag{16} \]

Where,
\[M = \begin{bmatrix} \Pi_3 & 0 \\ 0 & \Pi_4 \end{bmatrix} \tag{17} \]

\[\Pi_3 = 4 A P^T A + \varepsilon_i^{-1} A^T P E E^T P A \]
\[+ 4 \varepsilon_i H^T H + 5 \Delta A^T P \Delta A \]
\[+ 3 \Delta A^T P B(B^T PB)^{-1} B^T P \Delta A \]
\[-2 \rho I + P + Q \]
\[\Pi_4 = 4 A_j P^T A_j + 4 \varepsilon_i^{-1} A_j^T P E E^T P A_j \]
\[+ 4 \varepsilon_i H_j^T H_d + 4 \Delta A_j^T P \Delta A_j \]
\[+ 4 \Delta A_j^T P B(B^T PB)^{-1} B^T P \Delta A_j \]
\[+ 2 \rho I - Q \]

While \( M < 0 \), \( \Delta V(k) < 0 \). According to lemma 3, \( M < 0 \) equals to:
\[
\begin{bmatrix}
\Pi_4 & 0 & 2 A^T PE & \sqrt{3} \Delta A^T PB \\
0 & \Pi_3 & 0 & 0 \\
2 E^T PA & 0 & -\varepsilon_i I & 0 \\
\sqrt{5} B^T P \Delta A & 0 & 0 & -B^T PB \\
0 & 2B^T P \Delta A & 0 & 0 \\
\sqrt{5} \Delta A & 0 & 0 & 0 \\
0 & 2 \Delta A & 0 & 0 \\
0 & 2 E^T PA & 0 & 0 \\
2 \Delta A^T PB & 0 & 2 \Delta A_j & 2 \Delta A_j^T PE \\
0 & 0 & 0 & 0 \\
-B^T PB & 0 & 0 & 0 \\
0 & -P & -P & 0 \\
0 & 0 & 0 & -\varepsilon_i I \\
\end{bmatrix} < 0 \tag{19}
\]
Where,
\[
\Pi_s = 4A^T PA + 4\varepsilon_i H^T I + 2\rho I - P + Q
\]
\[
\Pi_s = 4A^T PA_s + 4\varepsilon_i H^T H_s + 2\rho I - Q
\]

And, (19) can be transformed into:
\[
\begin{bmatrix}
\Pi_3 & 0 & 2A^T PE & 0 \\
0 & \Pi_6 & 0 & 0 \\
2E^T PA & 0 & -\varepsilon_i I & 0 \\
0 & 0 & 0 & -B^T PB \\
0 & 0 & 0 & 0 \\
\sqrt{5}\Delta A & 0 & 0 & 0 \\
0 & 2\Delta A_d & 0 & 0 \\
0 & 2E^T PA_j & 0 & 0 \\
0 & \sqrt{5}\Delta A^T & 0 & 0 \\
0 & 0 & 2\Delta A_d & 2\Delta A_j PE \\
0 & 0 & 0 & 0 \\
-B^T PB & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & -P^{-1} & 0 & 0 \\
0 & 0 & -P^{-1} & 0 \\
0 & 0 & 0 & -\varepsilon_i I \\
W_1^T \begin{bmatrix} F^T \\ 0 \end{bmatrix} Y^T_i + Y_i \begin{bmatrix} F \\ 0 \end{bmatrix} W_1 < 0
\end{bmatrix}
\]

Where,
\[
W_1 = \begin{bmatrix} SH & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2H_j & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E^T PB & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E^T PB & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

According to lemma 1,
\[
W_1^T F^T Y^T_i + Y_i F W_1 \leq \varepsilon_i Y_i^T + \varepsilon_i W_1^T W_1
\]

So, (21) can be turned into (23) and (24),
\[
\begin{bmatrix}
\Pi_3 & 0 & 2A^T PE & 0 \\
0 & \Pi_6 & 0 & 0 \\
2E^T PA & 0 & -\varepsilon_i I & 0 \\
\sqrt{5}\Delta A & 0 & 0 & 0 \\
0 & 2\Delta A_d & 0 & 0 \\
0 & 2E^T PA_j & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\Pi_1 & 0 & 2A^T PE & 0 \\
0 & \Pi_2 & 0 & 2A^T_j PE \\
2E^T PA & 0 & -\varepsilon_i I & 0 \\
0 & 2E^T PA_j & 0 & -\varepsilon_i I \\
\end{bmatrix} < 0
\]

Where,
\[
\Pi_3 = 4A^T PA + 4\varepsilon_i H^T I + 2\rho I - P + Q
\]
\[
\Pi_s = 4A^T PA_s + 4\varepsilon_i H^T H_s + 2\rho I - Q
\]

Repeating the step form (21) to (25), (23) can be transformed into
\[
\begin{bmatrix}
\Pi_1 & 0 & 2A^T PE & 0 \\
0 & \Pi_2 & 0 & 2A^T_j PE \\
2E^T PA & 0 & -\varepsilon_i I & 0 \\
0 & 2E^T PA_j & 0 & -\varepsilon_i I \\
\end{bmatrix} < 0
\]

So, the proof of Theorem 1 is complete. Next part will give the process of constructing of the control law.

Theorem 2. For a system like (2) which meets the assumption (1) - (5), the sliding surface is shown as (4).

The system can reach the stable state under the control law (28) no matter the initial state.

\[
u(k) = \begin{cases} -\zeta(k) \frac{S(k)}{\|S(k)\|} , & \|S(k)\| \neq 0 \\ 0 & \|S(k)\| = 0 \end{cases}
\]

Where,
\[
\rho(k) = \rho_0 + \eta \|S(k)\| , \rho_0(k) = \|RE\| Hx(k) + \|RE\| Hx(k - d) , R = (B^T PB)^{-1} B^T P , \eta = \rho_0(1 - \zeta) + \|R\| g , \rho_0 \zeta \|B^T PB\| , \zeta > 2
\]

Then the system state can enter the quasi-sliding surface \( \Theta \) within finite time, and then keep stable in it.

\[
\Theta = \|S(k)\| \leq \Gamma
\]

\[
\Gamma = \sqrt{\rho_0 \left[ \|B^T PB\| \left( \frac{1}{2} + \frac{1}{2} \|P\|^2 \right)^2 + \frac{1}{2} \|x(k)\|^2 \right]}
\]
Choosing the Lyapunov function:

\[ V(k) = \frac{1}{2} S^T(k)(B^T PB)^{-1} S(k) \]  

(30)

Taking (4) into \( \Delta V(k) \),

\[ V(k) = \frac{1}{2} S^T(k + 1)(B^T PB)^{-1} S(k + 1) \]

\[ - \frac{1}{2} S^T(k)(B^T PB)^{-1} S(k) \]

\[ = S^T(k)(B^T PB)^{-1} \Delta S(k) \]

\[ + \frac{1}{2} \Delta S^T(k)(B^T PB)^{-1} \Delta S(k) \]

(31)

Where, \( \Delta S(k) = S(k + 1) - S(k) \). Substituting (4) and (28) in it, then:

\[ \Delta S(k) = B^T [\Delta A(k) + \Delta A_d(k - d)] \]

\[ - B \xi \rho(k) \frac{S(k)}{\| S(k) \|} + g(x(k))] \]

(32)

Taking (32) into (31)

\[ S^T(k)(B^T PB)^{-1} \Delta S(k) \]

\[ = S^T(k) R[EFHx(k) \]

\[ + EFHx(k - d) - \xi \rho(k) \| S(k) \| \]

\[ + S^T(k) Rg(x(k)) \]

\[ \leq \| S^T \| \| R \| Hx(k) \| \]

\[ + \| RE \| \| H_d x(k - d) \| \]

\[ - \xi (\rho_0 + \eta \| S(k) \|) \| S(k) \| \]

\[ + \| S(k) \| \| R \| \| g \| \]

\[ = [\rho_0 (1 - \xi) + \| R \| \| g \| \| S(k) \| \]

\[ - \xi \eta \| S(k) \|] \]

\[ \leq S^T(k)(B^T PB)^{-1} \Delta S^T \]

\[ \leq \frac{1}{2} [FH_d x(k - d)] \]

\[ + FHx(k)^T E^T PB \xi \rho(k) \frac{S(k)}{\| S(k) \|} \]

\[ + \frac{1}{2} S^T(k) \rho(k)B^T \rho(k) \frac{S^T(k)}{\| S(k) \|} \]

\[ - \frac{1}{2} [FH_d x(k - d)] \]

\[ + FHx(k)^T E^T PB \xi \rho(k) \frac{S^T(k)}{\| S(k) \|} \]

\[ - \frac{1}{2} S^T(k) \rho(k)B^T \rho(k) \]

\[ \leq \frac{1}{2} g^T(x(k))PB \xi \rho(k) \frac{S(k)}{\| S(k) \|} \]

\[ + FH_d x(k - d)] \]

(33)

According to lemma 2,

\[ \leq - \frac{1}{2} [FH_d x(k - d)] \]

\[ + FHx(k)^T E^T PB \xi \rho(k) \frac{S(k)}{\| S(k) \|} \]

\[ - \frac{1}{2} S^T(k) \rho(k)B^T \rho(k) \frac{S^T(k)}{\| S(k) \|} \]

\[ + \frac{1}{2} [FH_d x(k - d)] \]

\[ + FHx(k)^T E^T PB \xi \rho(k) \frac{S(k)}{\| S(k) \|} \]

\[ - \frac{1}{2} S^T(k) \rho(k)B^T \rho(k) \]

\[ \leq \frac{1}{2} g^T(x(k))PB \xi \rho(k) \frac{S(k)}{\| S(k) \|} \]

\[ + FH_d x(k - d)] \]

(34)
\[
\begin{align*}
\Delta V(k) \leq & \left[ \rho_0 (1 - \zeta) + \| R \| g \| S(k) \| - \zeta \eta \| S(k) \|^2\right. \\
& + \rho_0^2 \| B^T PB \| + \frac{1}{2} \rho_0^2 \| B^T PB \| \\
& + \frac{1}{2} \zeta^2 \| x(k) \|^2 + \rho_0^2 \| PB \| \\
& + \frac{1}{2} \zeta \| x(k) \|^2 + \rho_0^2 \| PB \| \\
& + \| S(k) \|^2 \left( \frac{1}{2} \| B^T PB \| \right. \\
& + \zeta \eta - \zeta \eta
\end{align*}
\]

So, it is easily to gain that while \( \| S(k) \| > \Gamma \), \( \Delta V(k) < 0 \). Therefore, the trajectory of the system converges to quasi-sliding surface \( \Theta \).

**Remark 1.** Based on Theorem 1 and Theorem 2, with the effect of sliding mode control, system (2) is quadratically stable and will reach the stable state in finite time.

### 3.2 Uncertain System with Multiple Time-delay

The structure of WSN based IoT can be unstable. In this section, uncertain system with multiple time-delay will be discussed, which is shown in (3). A nonsingular matrix \( T \) can be chosen such that \( TB = \begin{bmatrix} (0_{(n-m)\times m}) & B_m \end{bmatrix} \), where \( B_m \) is nonsingular with \( \text{rank}(B_m) = m \). By previous research \( T = \begin{bmatrix} U_1^T & U_2^T \end{bmatrix} \), where \( U_1 \in R^{n \times (n-m)} \) and \( U_2 \in R^{m \times (n-m)} \) are two unitary matrices resulting from singular value decomposition of matrix \( B \), then:

\[
B = [U_1 \quad U_2] \begin{bmatrix} \Sigma & 0_{(n-m)\times m} \end{bmatrix} V^T
\]

where \( \Sigma \in R^{n \times m} \) is a symmetric positive-definite matrix, \( V \in R^{n \times m} \) is a unitary matrix, by the state transformation \( y = Tx \), and (2) has the regular form

\[
\begin{align*}
y(k + 1) = & (\bar{A} + \Delta \bar{A}) y(k) + \sum_{i=1}^{N} (\bar{A}_i + \Delta \bar{A}_i) y_i(k - d_i) + \begin{bmatrix} 0_{(n-m)\times m} & B_m \end{bmatrix} \mu(k) + Tg(x(k)) \\
y(k) = & \Phi(k)
\end{align*}
\]

where \( \bar{A} = TAT^{-1}, \quad \bar{A}_i = T_{A_i} T^{-1}, \quad \Delta \bar{A} = T \Delta A T^{-1}, \quad \Delta \bar{A}_i = T \Delta A_i T^{-1} \) and \( \Phi(k) = T \Psi(k) \), in addition we have
\[
y_{i}(k+1) = (\overline{A}_{i1} + \Delta \overline{A}_{i1})y_{i}(k) + \sum_{j=1}^{N}(\overline{A}_{j1} + \Delta \overline{A}_{j1})y_{j}(k-d_{j}) \\
+ (\overline{A}_{i1} + \Delta \overline{A}_{i1})y_{i}(k-d_{i}) + g_{i} \\
\Delta \overline{A}_{i1} - \Delta \overline{A}_{j1}C = U_{2}^{T}GD(k)H_{j}(U_{2} - U_{j})C.
\]

(41)

**Theorem 3.** The system (2), the sliding surface is (4). If there existing positive matrix \(P > 0\) and \(Q > 0\) and constant \(\varepsilon_{j} > 0\) (i=1, 2, 3, 4), \(\lambda > 0\) to make (42), (44) exist, the system (3) is quadratically stable.

\[
\begin{bmatrix}
\Pi_{1} & 0 & \overline{A}_{i1}^{T}PE & 0 \\
* & \Pi_{2} & 0 & \sqrt{\lambda\varepsilon_{j}}PE \\
* & * & -\varepsilon_{j}I & 0 \\
* & * & * & -\varepsilon_{j}I
\end{bmatrix} < 0
\]

(42)

(43)

(44)

Where \(*\) is acquired based on matrix symmetry, and

\[
\begin{align*}
\Pi_{1} & = \overline{A}_{i1}^{T}P_{1}\overline{A}_{i1} + \varepsilon_{j}H_{j}^{T}H \\
& + \varepsilon_{j}H_{j}^{T}P - P + \varepsilon_{j}H_{j}^{T}Q \\
\Pi_{2} & = 3\overline{A}_{j1}^{T}P_{2}\overline{A}_{j1} + 3\varepsilon_{j}H_{j}^{T}H_{j} \\
& + 3\varepsilon_{j}H_{j}^{T}P_{2} + \varepsilon_{j}H_{j}^{T}Q
\end{align*}
\]

Choosing Lyapunov function as

\[
V(k) = x^{T}(k)Px(k) + \sum_{i=1}^{N}x^{T}(i)Qx(i)
\]

(45)

substituting (42) into (45),

\[
\Delta V_{1}(k) = V(k+1) - V(k) \\
= \Delta V_{1}(k) + \Delta V_{2}(k) \\
+ \Delta V_{3}(k) + \Delta V_{4}(k)
\]

(46)

where

\[
\Delta V_{1}(k) = y_{i}^{T}(k)(\overline{A}_{i1} + \Delta \overline{A}_{i1})y_{i}(k) \\
\Delta V_{2}(k) = y_{i}^{T}(k)(\overline{A}_{j1} + \Delta \overline{A}_{j1})y_{i}(k-d_{j}) + g_{i}
\]

(47)

(48)
\[
\Delta V_3(k) = y^T_i(k)(k - d_j)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k)
\]
\[
\Delta V_4(k) = g^T U_j P \tilde{A}_{d_{i1}} y_i(k) + g^T U_j P(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k) + y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k)
\]

According to lemma 2,
\[
y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k) + y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k)
\]
\[
\leq - y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k) + g^T U_j P \tilde{A}_{d_{i1}} y_i(k) + g^T U_j P \tilde{A}_{d_{i1}} y_i(k) + g^T U_j P \tilde{A}_{d_{i1}} y_i(k)
\]

Besides, there existing \( \varepsilon_i(i = 1, 2) \), using lemma 1:
\[
y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k)
\]
\[
\leq y^T_i(k)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k) + \varepsilon_i \tilde{A}_{d_{i1}}^T P \tilde{A}_{d_{i1}} + \varepsilon_i H^T H + \Delta \tilde{A}_{d_{i1}}^T P \Delta \tilde{A}_{d_{i1}} y_i(k)
\]
\[
y^T_i(k)(k - d_j)(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k) + g^T U_j P(\tilde{A}_{d_{i1}} + \Delta \tilde{A}_{d_{i1}}) y_i(k)
\]

Substituting the result above into (46), then:
\[
\Delta \mathcal{A}(k) \leq \begin{bmatrix}
\bar{y}^T_i(k) \\
y^T_i(k)(k - d_j) \\
\vdots \\
y^T_{y_1}(k)(k - d_n)
\end{bmatrix} M \begin{bmatrix}
y_i(k) \\
y_i(k - d_j) \\
\vdots \\
y_i(k - d_n)
\end{bmatrix}
\]

Where,
\[
M = \begin{bmatrix}
\Pi_3 & 0 \\
0 & \Pi_4
\end{bmatrix}
\]

\[
\Pi_3 = 1 \tilde{A}_{d_{i1}} P \tilde{A}_{d_{i1}} + 1 \varepsilon_i \tilde{A}_{d_{i1}}^T P \tilde{A}_{d_{i1}} + \varepsilon_i H^T H + 1 \Delta \tilde{A}_{d_{i1}} P \Delta \tilde{A}_{d_{i1}} - P + NQ + \rho \lambda I
\]

\[
\Pi_4 = 3 \tilde{A}_{d_{i1}}^T P \tilde{A}_{d_{i1}} + 3 \varepsilon_i \tilde{A}_{d_{i1}}^T P \tilde{A}_{d_{i1}} + 3 \varepsilon_i H^T H + 3 \Delta \tilde{A}_{d_{i1}} P \Delta \tilde{A}_{d_{i1}} - NQ
\]

While \( M < 0 \), \( \Delta V(k) < 0 \). According to lemma 3, \( M < 0 \) equals to:
\[
\begin{bmatrix}
0 & -P & \Delta \tilde{A}_{d_{i1}}^T P \\
E^T \tilde{A}_{d_{i1}} & 0 & -\varepsilon_i I \\
0 & \sqrt{3} P \tilde{A}_{d_{i1}} & 0 \\
\sqrt{3} E^T P \tilde{A}_{d_{i1}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Where,
\[
\Pi_3 = \tilde{A}_{d_{i1}} P \tilde{A}_{d_{i1}} + \varepsilon_i H^T H - P + NQ
\]

\[
\Pi_4 = 3 \tilde{A}_{d_{i1}} P \tilde{A}_{d_{i1}} + 3 \varepsilon_i H^T H - NQ
\]

And, (54) can be transformed into:
Theorem 4. For a system like (3) which meets the assumption (1) - (5), the sliding surface is shown as (4). The system can reach the stable state under the control law (39) no matter the initial state.

\[
\begin{align*}
\Pi & = \begin{bmatrix} P & 0 & 0 & 0 \\ 0 & \Pi_1 & 0 & 0 \\ \tilde{A}_i^T P A_i & 0 & 0 & 0 \\ \sqrt{P} & 0 & 0 & 0 \\ P \rho & 0 & 0 & 0 \\ \sqrt{P} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sqrt{P} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \end{bmatrix}, \\
\end{align*}
\]

Choosing the Lyapunov function:

\[
V(k) = \frac{1}{2} S^T(k)(GB)^{-1} S(k)
\]

Taking (39) into \( \Delta V(k) \),

\[
\begin{align*}
\Delta V(k) &= \frac{1}{2} S^T(k+1)(GB)^{-1} S(k+1) \\
&- \frac{1}{2} S^T(k)(GB)^{-1} S(k) \\
&= S^T(k+1)(GB)^{-1} S(k+1) \\
&+ \frac{1}{2} \Delta S^T(k)(GB)^{-1} \Delta S(k)
\end{align*}
\]

Where, \( \Delta S(k) = S(k+1) - S(k) \). Substituting (39) and (60) in it, then:

\[
\begin{align*}
\Delta S(k) &= G \Delta A x(k) + \sum_{i=1}^{N} G \Delta A_i x(k - d_i) \\
&- G b_\tilde{r} \rho(k) + \frac{S(k)}{||S(k)||} + G g(x(k))
\end{align*}
\]

Taking (64) into (63)

\[
S^T(k)(GB)^{-1} \Delta S(k)
\]

\[
= S^T(k) R [E F H x(k) + N E F H_j x(k - d_j)] \\
- \tilde{\rho}(k) ||S(k)|| + S^T(k) R g(x(k)) \\
\leq ||S(k)|| ||| E F H ||| H_j x(k - d_j) || \\
+ N ||| E F H ||| H_j x(k - d_j) || \\
- \tilde{\rho}(k) + \eta ||S(k)|| ||S(k)|| \\
+ ||S(k)|| ||R|| ||g|| ||S(k)|| \\
= [\rho_0(1 - \tilde{\rho} + \eta ||R|| ||g|| ||S(k)||) \\
- \tilde{\rho} \eta ||S(k)||]^2
\]
\[
\frac{1}{2} \Delta S^T(k)(GB)^{-1} \Delta S(k)
\]
\[
= \frac{1}{2} \left[ NFH_x x(k) - d_t \right] + F H(x(k)) E^T G^T(GB)^{-1} GE[FH_x(k) + NFH_x x(k) - d_t)]
\]
\[
+ \frac{1}{2} \left[ NFH_x x(k) - d_t \right]
\]
\[
+ F H(x(k)) E^T G^T(GB)^{-1} G (x(k))
\]
\[
+ \frac{1}{2} g^T(x(k)) G^T(GB)^{-1} G(g(x(k))
\]
\[
\leq \frac{1}{2} \left[ NFH_x x(k) - d_t \right] + NH(x(k)) E^T G^T(GB)^{-1} G [F H_x(k) + NFH_x x(k) - d_t)]
\]
\[
+ \frac{1}{2} g^T(x(k)) G^T(GB)^{-1} G (g(x(k))
\]
\[
= \frac{1}{2} \left[ NFH_x x(k) - d_t \right] + F H(x(k)) E^T G^T(GB)^{-1} G [F H_x(k) + NFH_x x(k) - d_t)]
\]
\[
+ \frac{1}{2} g^T(x(k)) G^T(GB)^{-1} G(g(x(k))
\]
\[
+ \frac{1}{2} \left[ NFH_x x(k) - d_t \right]
\]
\[
+ F H(x(k)) E^T G^T(GB)^{-1} G (x(k))
\]
\[
+ \frac{1}{2} g^T(x(k)) G^T(GB)^{-1} G(g(x(k))
\]
\[
+ \frac{1}{2} \left[ NFH_x x(k) - d_t \right]
\]

Then (66) can be transformed into:
\[
\frac{1}{2} \Delta S^T(GB)^{-1} \Delta S(k)
\]
\[
\leq \frac{1}{2} \left[ NFH_x x(k) - d_t \right] + F H(x(k)) E^T G^T(GB)^{-1} G [F H_x(k) + NFH_x x(k) - d_t)]
\]
\[
+ \frac{1}{2} g^T(x(k)) G^T(GB)^{-1} G(g(x(k))
\]
\[
+ \frac{1}{2} \left[ NFH_x x(k) - d_t \right]
\]

Substituting (67) and (65) into (63),
\[
\Delta V(k) \leq \left[ \rho^0(1 - \zeta)+ \| R \| | g \| \right] | S(k) || - \zeta \eta | S(k) ||
\]
\[
+ \frac{1}{2} \rho^0 \| GB \| + \rho^2(k) \zeta^2 \| GB \|
\]
\[
+ \frac{1}{2} \varepsilon \| x(k) \| + \frac{1}{2} \rho^2(k) \zeta^2
\]
\[
= \rho^0 \zeta^2 \| GB \| + \frac{1}{2} \varepsilon \| x(k) \| + \frac{1}{2} \rho^2(k) \zeta^2
\]

(68)
So, it is easily to gain that while \( \| S(k) \| > \Gamma \), \( \Delta V(k) < 0 \). Therefore, the trajectory of the system converges to quasi-sliding surface \( \Theta \).

**Remark 2.** Based on Theorem 3 and Theorem 4, with the effect of sliding mode control, system (3) is quadratically stable and will reach stable state in finite time.

### 4 Numerical Simulations

In this section, all the theorems above will be tested. Consider system (2), where

\[
A = \begin{bmatrix} 0.05 & 0 & 0.06 \\ 0.01 & 0.05 & 0.08 \\ 0.05 & 0.23 & 0.01 \end{bmatrix},
\]

\[
A_d = \begin{bmatrix} 0.3 & 0 & 0.05 \\ 0.1 & 0.17 & 0.1 \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ 0.05 \end{bmatrix},
\]

\[
E = \begin{bmatrix} 0.05 \\ 0.15 \\ 0.05 \end{bmatrix},
\]

\[
H = \begin{bmatrix} 0.3 \\ 0.15 \\ 0.1 \end{bmatrix},
\]

\[
H_d = \begin{bmatrix} 0.03 \\ 0.1 \end{bmatrix}.
\]

The initial states are: \( x(0) = [0.2 \ 0.2 \ 0.2] \), \( d = 1 \). Based on Theorem 1, we have

\[
\]

\[
Q = \begin{bmatrix} 16.4676 & 7.0749 & 5.7473 \\ 3.2684 & 2.3397 & 2.3397 \\ 5.7473 & 0.0627 & 9.4726 \end{bmatrix},
\]

\[
G = \begin{bmatrix} 0.1089 & 0.0627 & 0.2996 \\ 0.1 \end{bmatrix}.
\]

\[
\epsilon_1 = 1.6358, \epsilon_2 = 9.3111, \epsilon_3 = 1.1291, \epsilon_4 = 1.2714.
\]

Then the simulation result is shown in Figure 1.

\[
g(x(k)) = \frac{0.08 * x_1}{(2 * x_2 * x_2 + 1)} - \frac{0.1 * x_2 * \sin(x_3)}{x_3},
\]

\[
F(k) = \sin(k)
\]

In fact, it is hard to acquire ideal sliding mode. From the figure, we can see that the practical sliding mode is converged within a region around the ideal sliding surface.

**Figure 1.** States \((x_1, x_2, x_3)\), control input and sliding mode
Consider system (3), where

\[
A = \begin{bmatrix}
0.5 & -0.23 \\
0.03 & -0.02 \\
0.05 & 0.03 \\
0.22 & 0.13 \\
0.05 & 0.03 \\
0.22 & 0.13
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0.02 \\
0.05 \\
0.01 \\
0.5 \\
0.02 \\
0.05
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
0.02^T \\
0.05^T \\
0.01^T \\
0.5^T
\end{bmatrix},
\]

\[
H_f = \begin{bmatrix}
0.03^T \\
0.05^T
\end{bmatrix},
\]

\[
E = \begin{bmatrix}
0.01^T \\
0.5^T
\end{bmatrix},
\]

\[
F(k) = \sin(k)
\]

\[
g(x(k)) = \begin{bmatrix}
0.08* x_1 / ((2*x_2*x_2+1)) \\
0.1* x_2 * \cos(x_2)
\end{bmatrix}
\]

The initial states are: \( x(0) = [-0.2 - 0.2]^T \), \( d \in [0 1] \), \( N = 4 \), \( \rho = 0.2 \). Based on Theorem 1, we have

\[
P = \begin{bmatrix}
150.0054 & 2.53772 & 5.37772 & 38.0364 \\
2.53772 & 0.5824 & 0.5824 & 1.8132 \\
5.37772 & 0.5824 & 1.8132 & 38.0364 \\
38.0364 & 1.8132 & 38.0364 & 150.0054
\end{bmatrix},
\]

\[
Q = \begin{bmatrix}
17.2764 & 0.5824 & 0.5824 & 1.8132 \\
0.5824 & 1.8132 & 1.8132 & 38.0364 \\
0.5824 & 1.8132 & 1.8132 & 38.0364 \\
1.8132 & 38.0364 & 38.0364 & 150.0054
\end{bmatrix},
\]

\[
\epsilon_1 = 97.0120, \epsilon_2 = 85.0112,
\]

\[
\epsilon_3 = 86.7580, \lambda = 138.0773, e = 1.13.
\]

The simulation results are shown in Figure 2.

![Figure 2](image)

(a) (b) (c) (d)

**Figure 2.** States (x1, x2), control input and sliding mode

From the control law, although the design method is simplified, for systems requiring fast response, control parameters \( \Phi \) should be enlarged.

## 5 Conclusion

In this paper, we investigate the sliding surface and control law of WSN based IoT with different types of time-delay and external nonlinear perturbation. LMI is used in both of the two situation for acquiring appropriate parameters of sliding mode controller to ensure the system quadratic stability. By nonsingular transformation and equivalent reduced order system, the complexity of parameter in multiple time-delay is reduced. It’s clear that the system is eventually stabilized within an area called quasi-sliding surface. However there are also some aspects needed to be improved such as the problem of chattering, shorter reaching time. All the problems are needed further study.

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Discrete Sliding Mode Control for a Class of WSN Based IoT with Time-delay and Nonlinear Characteristic


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