Discrete Sliding Mode Control for a Class of WSN Based IoT with Time-delay and Nonlinear Characteristic

Ming Lyu[1], Jie Zhang[1], Lei Liu[1], Yuming Bo[1], Fengjun Xiao[2]

[1] School of Automaton, Nanjing University of Science & Technology, China
[2] School of Public Adminstration, Beihang University, China
lumtz@163.com, zhangjie_njust@163.com, liuleisdly@163.com, byming@njust.edu.cn, bhxfj@126.com

Abstract

This paper applies discrete sliding mode control to the WSN based IoT, to ensure its robust stability under different types of communication time-delay and nonlinear characteristic. Due to the limited sampling frequency, the system will approach a quasi-sliding surface in finite time. Linear matrix inequality (LMI) approach is used to prove the quadratically stability of the sliding surface and suitable Lyapunov function is constructed to obtain the appropriate control law. Two situation will be investigated in this paper, the system with signal time-delay and system with multiple time-delays. Furthermore, nonlinear characteristic of system has been taken into consideration. By employing non-singular transformation and reduced order processing, the proof of sliding surface stability and finite time approaching is simplified. Finally, two numerical simulations are employed to demonstrate the effectiveness and applicability of the proposed approach.

Keywords: Internet of things, Sliding mode control, Discrete-time system, Time-delay system, Nonlinear characteristic

1 Introduction

Recently, with the maturity of communication technology, sensor technology and other relative technology, WSN based IoT shows huge potential in many areas. Such as home appliance, logistics, environment monitoring, military systems, security tracking, smart grid and so on. Constituted by various tiny sensors or devices, WSN based IoT has superiority in aspects like easy to be deployed and extended, high fault tolerant [1-4].

However, many challenges still exist. Each node in WSN based IoT has limit power as well as that they seldom simultaneously meet the design goal [5]. Distributed filtering problem for sensor networks may cause the IoT unstable [6]. IoT is Constituted by several sensors, and the link between nodes may failure by various causes [7-8]. Some serious problems may even cause the whole system to crash [9-11].

Many researchers have done various encouraging work in the field of WSN based IoT. [12-13] are concerned with a new distributed H_∞ -consensus filtering problem over a finite-horizon for a class of sensor networks with multiple missing measurements and a class of polynomial nonlinear stochastic systems. [14] presents a decentralized event-triggered implementation, over sensor/actuator networks, of centralized nonlinear controllers. [15-16] classify the coverage problem from different angles, describe the evaluation metrics of coverage control algorithms, analyze the relationship between coverage and connectivity, compare typical simulation tools, and discuss research challenges and existing problems in this area. [17-18] use mobile sensor network in enabling an information-theoretic distributed control architecture to facilitate search and enabling a variety of new applications that rely on position information. [19] discusses the emerging application of device-free localization (DFL) using wireless sensor networks, which find people and objects in the environment in which the network is deployed, even in buildings and through walls. These networks are termed “RF sensor networks” and [20] presents an overview of principles and requirements for powering wireless sensors by radio-frequency (RF) energy harvesting or transport. [21] presents a novel vehicular clustering scheme integrating hierarchical clustering on the basis of classical routing algorithm. And [22] discusses the general principles of swarm intelligence and of its application to routing. In the aspect of environment protecting, [23] is concerned with the application of wireless sensor network (WSN) technology to long-duration and large-scale environmental monitoring. And [24] considers joint problems of control and communication in wireless sensor and actuator networks (WSANs) for building-environment control systems. The sliding mode control has strong robustness and its robustness will not be affected by parametric uncertainty, various perturbations and time-delay. It is very suitable for sensor network. The

*Corresponding Author: Jie Zhang; E-mail: zhangjie_njust@163.com
DOI: 10.3966/160792642019032002017

system under the sliding mode control needs to go through two stages to reach the steady state. In the first stage, the system moves from the initial state to the sliding surface, and will reach the sliding surface in finite time. In the second stage, The system state will shake back and forth near the sliding surface and finally be stable on the sliding surface. Once the system state keep stable on the sliding surface, it will not be affected by parameter and perturbation. In recent years, many new methods about how to choose sliding surface and obtain control law have been proposed too. [25] deals with mismatched uncertainties by designing a novel sliding surface based on the disturbance estimation and [26] puts forward another method which designing a continuous nonsingular terminal sliding mode control approach. [27-28] studies fault-tolerant control. [29-31] deals with problems such as Markovian jump systems. Most previous studies focused on continuous systems. However, sensor network which is controlled by computer is a typical discrete. So discrete-time sliding mode control should be more appropriate. Compared with continuous systems, sliding mode control in discrete situation has some difference. The main cause of the difference is finite sampling frequency. The time that the state reach the sliding surface can hardly be the sampling time, so it is difficult to make the state stable on the sliding surface. Instead, the system state will form a quasi-sliding surface around the ideal sliding surface. In recent years, many methods to deal with discrete sliding mode control have been proposed [32-35]. Based on a recursive analysis, [36] investigates he dynamical behaviors of discrete-time terminal sliding mode control systems based on Euler's discretization. [37-39] puts forward many other ways to design the sliding surface and control method.

This paper applies the discrete sliding mode control in the WSN based IoT to make the control protocol more suitable in practical situation. This paper mainly researches two types of WSN based IoT with different types of time-delay: signal time-delay and multiple time-delay, both of the two situation are under external nonlinear perturbation. To deal with the first situation, An LMI approach is used to solve appropriate parameter matrix and then demonstrating the stability of the system. Based on the first situation, the system with multiple time-delay uses equivalent order reduction to reduce the difficulty in proof of the stability. Suitable Lyapunov equation is constructed to prove the system will reach the steady state in finite time. The correctness of the two situation are proved by simulation.

The remainder of this paper is organized as follows. In section 2, the model of the systems are constructed by graph theory and main problems are put forward. In section 3, the proof process of signal and multiple time-delay is given. In section 4, the simulation numbers and result are provided to certify the property. Finally, in section 5, we conclude the whole paper briefly and raise some suppose of future research.

2 Problem Formulation

Concerning a type of WSN based IoT, information flow is transmitted among all sensors directly. Let $G=\{v,e\}$ be a directed graph of order n with the set of nodes $v=\{1,2,\cdots,n\}$, set of edges $e\subseteq v\times v$, edge $(i,j)\in e$ indicates that node j is able to receive information from node i. The adjacent matrix $A=[a_{ij}]\in R^{n\times n}$ of directed graph G is defined as follows: if information exchange exists between two nodes i and j, then a_{ij} denotes the throughput from i to j. At the same time, node i may also receive information from j, yet a_{ij} is not necessarily equal to a_{ji}. Taking into account that if the node is static, the connection structure inside the system is not varying with time. Then the model is given as below:

$$\begin{cases} x(k+1)=Ax(k)+A_d x(k-d) \\ \qquad\qquad +Bu(k)+g(x(k)) \\ x(k) \quad =\psi(k) \qquad k\in[-d,0] \end{cases} \tag{1}$$

where $x(k)\in R^n$ is system state, $u(k)\in R^m$ denotes system control input, $g(x(k))$ is nonlinear characteristic, A, A_d, and B are constant matrices with appropriate dimensions.

However, in practical system, WSN based IoT will change its structure by many reasons. The link between nodes may be changed or failure, and the movement of nodes may beyond communication range. These uncertain factors are reasons resulting system structure uncertainty. Then if each node only has information exchange with its adjacent node, the system model is described as below:

$$\begin{cases} x(k+1)=(A+\Delta A)x(k)+(A_d+\Delta A_d)x(k-d) \\ \qquad\qquad +Bu(k)+g(x(k)) \\ x(k) \quad =\psi(k) \qquad k\in[-d,0] \end{cases} \tag{2}$$

where $x(k)\in R^n$ denotes system state, $u(k)\in R^m$ is system control input, ΔA and ΔA_d are internal parameter perturbation arising from uncertain factors, $g(x(k))$ is nonlinear characteristic, A, A_d, and B are constant matrices with appropriate dimensions.

If a node in system is influenced by multiple time-delays, then the model is described as below:

$$\begin{cases} x(k+1)=(A+\Delta A)x(k)+\sum_{i=1}^{N}(A_d+\Delta A_{di})x(k-d_i) \\ \qquad\qquad +Bu(k)+g(x(k)) \\ x(k) \quad =\psi(k) \qquad k\in[-d,0] \end{cases} \tag{3}$$

where $x(k)\in R^n$ denotes system state, $u(k)\in R^m$ is system control input, ΔA and ΔA_d are internal parameter perturbation arising from uncertain factors, $g(x(k))$ is nonlinear characteristic, A, A_d, and B are constant matrices with appropriate dimensions, N is the number of agents in the system.

Considering the practical situation, only the system (2) and (3) will be investigated.

For the systems shown in (2) and (3), we will use the following assumptions:

Assumption 1. (A,B) is fully controllable and B is a column full rank.

Assumption 2. Time delay d_i is boundary, and

$$\| \underset{i\in N}{d_i} \| \le d .$$

Assumption 3. The perturbation parameter of the system satisfies

$$[\Delta A \;\; \Delta A_d] = EF(k)[H \;\; H_d]$$

Respectively, E, H and H_d, are known constant matrix, $F(k)$ is time-delay uncertain matrix, yet Lebesgue-measurable, and $F^T(k)F(k)\le I$.

Assumption 4. Suppose $g(x(k))$ is the nonlinear characteristic influenced by system state $x(k)$, then there is a positive constant ρ makes $g(x(k))^T g(x(k)) \le \rho x(k)^T x(k)$

The designing of sliding mode control usually conclude two parts. One part, designing a sliding surface and then certifying its quadratical stability. An other part is to solve the control law which can ensure the system state reaching the sliding surface in finite time.

Besides, some important lemmas are presented here.

Lemma 1. Let D, E, and $F(k)$ be real matrix of proper dimensions, and $F^T(k)F(k)\le I$, there exists a constant ε, which makes the following equation holds.

$$EF(k)H + H^T F^T(k)E^T \le \varepsilon^{-1}EE^T + \varepsilon H^T H$$

Lemma 2. Let a, b be real matrix of proper dimensions, there exists a constant matrix $X>0$, which makes the following equation hold.

$$ab + b^T a^T \le aXa^T + b^T X^{-1} b$$

Lemma 3. Given a symmetric matrix $S=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{bmatrix}$, where S_{11} is $r\times r$ dimensional, the following three conditions are equivalent:

(1) $S<0$;

(2) $S_{11}<0$, $S_{22}-S_{12}^T S_{11}^{-1} S_{12}<0$;

(3) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$;

Lemma 4. Let $x\in R^n$, $y\in R^n$ and matricx $Q>0$. Then, we have $2x^T Q_y \le x^T Q_x + y^T Q_y$.

3 Main Results

3.1 WSN Based IoT with Single Time-delay and Nonlinear Characteristic

In this section, we investigate the system with single time-delay and nonlinear characteristic.

For this system which meets assumption 1 to 4, we construct following sliding mode function:

$$\begin{aligned} &S(k) = Gx(k) + \sigma(k) \\ &\sigma(k+1) = Gx(k) - GAx(k) \\ &\qquad - GA_d x(k-d) + \sigma(k) \\ &\sigma(0) = -Gx(0) \end{aligned} \tag{4}$$

where $S(k)=[S_1(K), S_2(K), \cdots, S_m(K)]^T$, $G\in R^{m*n}$ and GB is nonsingular matrix. Then choosing $G=B^T P$.

According to the discrete sliding mode theory, while the system reach the sliding surface $S(k+1)=S(k)=0$, the ideal equivalent control law is

$$u = -(GB)^{-1}[G\Delta A x(k) + G\Delta A_d x(k-d) + Gg] \tag{5}$$

the equivalent control law (5) is only used to support the following proof and real control law will be given later. Substituting (5) into (2) the ideal dynamic sliding mode function is:

$$\begin{aligned} x(k+1) = &[A + \Delta A - B(GB)^{-1} G\Delta A]x(k) \\ &+ [A_d + \Delta A_d - B(GB)^{-1} G\Delta A_d]x(k-d) \\ &+ g(x(k+1)) - B(GB)^{-1} Gg(x(k)) \end{aligned} \tag{6}$$

Theorem 1. The system (2), the sliding surface is (4). If there existing positive matrix $P>0$ and $Q>0$ and constant $\varepsilon_j>0$ (i=1, 2, 3, 4, 5, 6) to make (7)-(9) exist, the system (2) is quadratically stable.

$$\begin{bmatrix} \Pi_1 & 0 & 2A^T PE & 0 \\ * & \Pi_2 & 0 & 2A_d^A PE \\ * & * & -\varepsilon_1 I & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} -B^T PB & B^T PB \\ * & -\varepsilon_3 I \end{bmatrix} < 0 \tag{8}$$

$$\begin{bmatrix} -P & PE \\ * & -\varepsilon_4 I \end{bmatrix} < 0 \tag{9}$$

Where * is acquired based on matrix symmetry, and

$$\Pi_1 = 4A^T PA + 4\varepsilon_1 H^T H + 3\varepsilon_3 H^T H + \\ + 5\varepsilon_4 H^T H - 2\rho I - P + Q$$

$$\Pi_2 = 4A_d^T PA_d + 4\varepsilon_2 H_d^T H_d + 4\varepsilon_3 H_d^T H_d + \\ + 4\varepsilon_4 H_d^T H_d + 2\rho I - Q$$

Choosing Lyapunov function as

$$V(k) = x^T Px(k) + \sum_{i=k-d}^{k-1} x^T(i) Qx(oi) \tag{10}$$

Substituting (6) into (10),

$$\begin{aligned} V(k) &= V(k+1) - V(k) \\ &= \Delta V_1(k) + \Delta V_2(k) \\ &+ \Delta V_3(k) + \Delta V_4(k) \end{aligned} \tag{11}$$

where

$$\begin{aligned} &\Delta V_1(k) = x^T(k)(A \\ &+\Delta A)^T P(A+\Delta A)x(k) \\ &-x^T(k)(A+\Delta A)^T \\ &PB(B^T PB)^{-1} B^T P\Delta A x(k) \\ &-x^T(k)\Delta A^T PB(B^T PB)^{-1} B^T \\ &P(A+\Delta A)x(k) \\ &+x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T P\Delta A x(k) \\ &-x^T(k)Px(k) + x^T(k)Qx(k) \end{aligned} \tag{12}$$

$$\begin{aligned} &\Delta V_2(k) = 2x^T(k)(A+\Delta A)^T P(A_d \\ &+\Delta A_d)x(k-d) \\ &-2x^T(k)(A+\Delta A)^T PB(B^T PB)^{-1} \\ &B^T P\Delta A_d x(k-d) \\ &-2x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T P\Delta A_d x(k-d) \\ &-2x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T P(A_d + \Delta A_d)x(k-d) \\ &+2x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T P\Delta A_d x(k-d) \end{aligned} \tag{13}$$

$$\begin{aligned} &\Delta V_3(k) = x^T(k)(k-d)(A_d \\ &+\Delta A_d)^T P(A_d + \Delta A_d)x(k-d) \\ &-2x^T(k)(A+\Delta A)^T PB(B^T PB)^{-1} \\ &-x^T(k-d)Qx(k-d) \\ &+x^T(k-d)\Delta A_d^T PB(B^T PB)^{-1} \\ &B^T P\Delta A_d x(k-d) \\ &-x^T(k-d)\Delta A^T PB(B^T PB)^{-1} B^T P \\ &(A_d + \Delta A_d)x(k-d) \\ &-x^T(k-d)(A_d + \Delta A_d)^T PB \\ &(B^T PB)^{-1} B^T P\Delta A_d x(k-d) \end{aligned} \tag{14}$$

$$\begin{aligned} &\Delta V_4(k) = x^T(k)(A+\Delta A)^T \\ &Pg(x(k+1)) \\ &+g^T(x(k+1))P(A+\Delta A)x(k) \\ &-x^T(k)(A+\Delta A)^T PB(B^T PB)^{-1} \\ &B^T Pg(x(k)) \\ &-g^T(x(k))PB(B^T PB)^{-1} \\ &B^T P(A+\Delta A)x(k) \\ &-x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T Pg(x(k+1)) \\ &-g^T(x+(k+))PB(B^T PB)^{-1} \\ &B^T P\Delta A x(k) \\ &+x^T(k)\Delta A^T PB(B^T PB)^{-1} \\ &B^T Pg(x(k)) \\ &+g^T(x(k))PB(B^T PB)^{-1} \\ &B^T P\Delta A x(k) \\ &+x^T(k-d)(A_d + \Delta A_d)^T Pg(x(k+1)) \\ &+g^T(x(k+1)P(A_d + \Delta A_d)x(k-d) \\ &-x^T(k-d)(A_d + \Delta A_d)^T Pg(x(k)) \\ &-g^T(x(k)P(A_d + \Delta A_d)x(k-d) \\ &-x^T(k-d)\Delta A_d^T PB(B^T PB)^{-1} \\ &B^T Pg(x(k+1)) \\ &-g^T(x(k+1))PB(B^T PT)^{-1} \\ &B^T P\Delta A_d x(k-d) \\ &+x^T(k-d)\Delta A_d^T PB(B^T PB)^{-1} \\ &B^T Pg(x(k)) \\ &+g^T(x(k))PB(B^T PB)^{-1} \\ &B^T P\Delta A_d x(k-d) \\ &-g^T(x(k+1))PB(B^T PB)^{-1} \\ &B^T Pg(x(k)) \\ &-g^T(x(k))PB(B^T PB)^{-1} \\ &B^T Pg(x(k+1)) \\ &+g^T(x(k+1))Pg(x(k+1)) \\ &+g^T(x(k))PB(B^T PB)^{-1} \\ &B^T Pg(x(k)) \end{aligned} \tag{15}$$

According to lemma 2, the second item and third item in $\Delta V_1(K)$ can be transformed to:

$$-x^T(k)(A+\Delta A)^T$$
$$PB(B^TPB)^{-1}B^TP\Delta Ax(k)$$
$$-x^T(k)\Delta A^TPB(B^TPB)^{-1}$$
$$B^TP(A+\Delta A)x(k)$$
$$\le x^T(k)(A+\Delta A)^TP(A+\Delta A)x(k)$$
$$+x^T(k)\Delta A^TPB(B^TPB)^{-1}B^TP\Delta Ax(k)$$

According to lemma 2, the first item in $\Delta V_2(K)$ can be transformed to:

$$2x^T(k)(A+\Delta A)^T$$
$$P(A_d+\Delta A_d)x(k-d)$$
$$\le x^T(k)(A+\Delta A)^TP(A+\Delta A)x(k)$$
$$+x^T(k-d)(A_d+\Delta A_d)^T$$
$$P(A_d+\Delta A_d)x(k-d)$$

And the second item can be transformed to:

$$-2x^T(k)x^T(A+\Delta A)^T$$
$$PB(B^TPB)^{-1}B^TP\Delta A_dx(k-d)$$
$$\le x^T(k-d)\Delta A_dxPB(B^TPB)^{-1}$$
$$B^TP\Delta A_dx(k-d)$$
$$+x^T(k)(A+\Delta A)^TP(A+\Delta A)x(k)$$

The third item can be transformed to:

$$-2x^T(k)\Delta A^TPB(B^TPB)^{-1}$$
$$B^TP(A_d+\Delta A_d)x(k-d)$$
$$\le x^T(k)\Delta A^TPB(B^TPB)^{-1}$$
$$B^TP\Delta Ax(k)+x^T(k-d)(A_d+\Delta A_d)^T$$
$$P(A+\Delta A_d)x(k-d)$$

The forth item can be transformed to:

$$2x^T(k)\Delta A^TPB(B^TPB)^{-1}$$
$$B^TP\Delta A_dx(k-d)$$
$$\le x^T(k)\Delta A^TP\Delta A_dx(k)$$
$$+x^T(k-d)\Delta A_d^TPB(B^TPB)^{-1}$$
$$B^TP\Delta A_dx(k-d)$$

And the second item and third item in $\Delta V_3(K)$ can be transformed to:

$$-x^T(k-d)x^T(k)(A_d+\Delta A_d)^T$$
$$PB(B^TPB)^{-1}B^TP\Delta A_dx(k-d)$$
$$-x^T(k-d)\Delta A_d^TPB(B^TPB)^{-1}$$
$$B^TP(A_d+\Delta A_d)x(k-d)$$
$$\le x^T(k-d)(A_d+\Delta A_d)^T$$
$$P(A_d+\Delta A_d)x(k-d)$$
$$+x^T(k-d)\Delta A_d^T$$
$$PB(B^TPB)^{-1}B^TP\Delta A_dx(k-d)$$

Then about $\Delta V_4(K)$, the first and second item:

$$x^T(k)(A+\Delta A)^TPg(x(k+1))$$
$$+g^T(x(k+1))P(A+\Delta A)x(k)$$
$$\le x^T(k)(A+\Delta A)^TP(A+\Delta A)x(k)$$
$$+g^T(x(k+1))P(x(k+1))$$

The third and forth item:

$$-x^T(k)(A+\Delta A)^TPB(B^TPB)^{-1}B^T$$
$$Pg(x(k))-g^T(x(k))PB(B^TPB)^{-1}$$
$$B^TP(A+\Delta A)x(k)$$
$$\le -x^T(k)(A+\Delta A)^TP(A+\Delta A)x(k)$$
$$-g^T(x(k))PB(B^TPB)^{-1}B^TPg(x(k))$$

According to lemma 4 The fifth and sixth item:

$$-x^T(k)\Delta A^TPB(B^TPB)^{-1}B^TPg(x(k+1))$$
$$-g^T(x(k+1))PB(B^TPB)^{-1}B^TP\Delta Ax(k)$$
$$\le -x^T(k)\Delta A^TPB(B^TPB)^{-1}B^T$$
$$PPB(B^TPB)^{-1}B^TP\Delta Ax(k)$$
$$-g^T(x(k+1))g(x(k+1))$$

The seventh and eighth item:

$$x^T(k)\Delta A^TPB(B^TPB)^{-1}B^TPg(x(k))$$
$$+g^T(x(k))PB(B^TPB)^{-1}B^TP\Delta Ax(k)$$
$$\le x^T(k)\Delta A^TPB(B^TPB)^{-1}B^T$$
$$PPB(B^TPB)^{-1}B^TP\Delta Ax(k)$$
$$+g^T(x(k))g(x(k))$$

As first to eighth items, the ninth to sixteenth item:

$$x^T(k-d)(A_d+\Delta A_d)^TPg(x(k+1))$$
$$+g^T(x(k+1))P(A_d+\Delta A_d)x(k-d)$$
$$\le -x^T(k-d)(A_d+\Delta A_d)^T$$
$$P(A_d-\Delta A_d)x(k-d)$$
$$+g^T(x(k+1))Pg(x(k+1))$$

$$-x^T(k-d)(A_d+\Delta A_d)^T$$
$$PB(B^TPB)^{-1}B^TPg(x(k))$$
$$-g^T(x(k))PB(B^TPB)^{-1}B^TP(A_d$$
$$+\Delta A_d)x(k-d)$$
$$\le x^T(k-d)(A_d$$
$$+\Delta A_d)^TP(A_d$$
$$+\Delta A_d)x(k-d)$$
$$+g^T(x(k))PB(B^TPB)^{-1}B^TPg(x(k))$$

$$
\begin{aligned}
&-x^T(k-d)\Delta A_d^T \\
&PB(B^TPB)^{-1}B^TPg(x(k+1)) \\
&-g^T(x(k+1))PB(B^TPB)^{-1} \\
&B^TP\Delta A_d x(k-d) \\
&\le -x^T(k-d)\Delta A_d^T \\
&PB(B^TPB)^{-1}B^T \\
&PPB(B^TPB)^{-1}B^TP\Delta A_d x(k-d) \\
&-g^T(x(k+1))g(x(k+1))
\end{aligned}
$$

$$
\begin{aligned}
&x^T(k-d)\Delta A_d^T \\
&PB(B^TPB)^{-1}B^TPg(x(k)) \\
&+g^T(x(k))PB(B^TPB)^{-1} \\
&B^TP\Delta A_d x(k-d) \\
&\le x^T(k-d)\Delta A_d^T PB(B^TPB)^{-1}B^T \\
&PPB(B^TPB)^{-1}B^TP\Delta A_d x(k-d) \\
&+g^T(x(k))g(x(k))
\end{aligned}
$$

And according to lemma 2, the seventeenth and eighteenth item:

$$
\begin{aligned}
&-g^T(x(k+1))PB(B^TPB)^{-1} \\
&B^TPg(x(k)) \\
&-g^T(x(k))PB(B^TPB)^{-1} \\
&B^TPg(x(k+1)) \\
&-g^T(x(k+1))Pg(x(k+1)) \\
&\le x^T(k-d)\Delta A_d^T PB(B^TPB)^{-1}B^T \\
&PPB(B^TPB)^{-1}B^TP\Delta A_d x(k-d) \\
&-g^T(x(k))PB(B^TPB)^{-1}B^TPg(x(k))
\end{aligned}
$$

Besides, there existing ε_i (i=1, 2, 3, 4), using lemma 1:

$$
\begin{aligned}
&x^T(k)(A+\Delta A)^T P(A+\Delta A)x(k) \\
&= x^T(k)A^TPAx(k) \\
&+x^T(k)A^TP\Delta Ax(k) \\
&+x^T(k)\Delta A^TPAx(k) \\
&+x^T(k)\Delta A^TP\Delta Ax(k) \\
&\le x^T(k)[A^TPA+\varepsilon_1^{-1}A^T \\
&PEE^TPA+\varepsilon_1 H^TH \\
&+\Delta A^TP\Delta A]x(k)
\end{aligned}
$$

$$
\begin{aligned}
&x^T(k-d)(A+\Delta A_d)^T \\
&P(A_d+\Delta A_d)x(k-d) \\
&= x^T(k-d)A_d^TPA_dx(k-d) \\
&+x^T(k-d)A_d^TP\Delta A_dx(k-d) \\
&+x^T(k-d)\Delta A_d^TPA_dx(k-d) \\
&+x^T(k-d)\Delta A_d^TP\Delta A_dx(k-d) \\
&\le x^T(k-d)[\Delta A_d^TPA_d \\
&+\varepsilon_1^{-1}A_d^TPEE^TPA_d \\
&+\varepsilon_2 H_d^TH_d \\
&+\Delta A_d^TP\Delta A_d]x(k-d)
\end{aligned}
$$

Substituting the result above into (11), then:

$$
\Delta V(k) \le [x^T(k) \quad x^T(k-d)M]\begin{bmatrix} x(k) \\ x(k-d) \end{bmatrix} \tag{16}
$$

Where,

$$
M = \begin{bmatrix} \Pi_3 & 0 \\ 0 & \Pi_4 \end{bmatrix} \tag{17}
$$

$$
\begin{aligned}
\Pi_3 &= 4AP^TA+\varepsilon_1^{-1}A^TPEE^TPA \\
&\quad +4\varepsilon_1 H^TH+5\Delta A^TP\Delta A \\
&\quad +3\Delta A^TPB(B^TPB)^{-1}B^TP\Delta A \\
&\quad -2\rho I-P+Q \\
\Pi_4 &= 4A_dP^TA_d+4\varepsilon_2^{-1}A_d^TPEE^TPA_d \\
&\quad +4\varepsilon_2 H_d^TH_d+4\Delta A_d^TP\Delta A_d \\
&\quad +4\Delta A_d^TPB(B^TPB)^{-1}B^TP\Delta A_d \\
&\quad +2\rho I-Q
\end{aligned} \tag{18}
$$

While $M<0$, $\Delta V(K)<0$. According to lemma 3, $M<0$ equals to:

$$
\begin{bmatrix}
\tilde{\Pi}_5 & 0 & 2A^TPE & \sqrt{3}\Delta A^TPB & 0 & \sqrt{5}\Delta A^T & 0 & 0 \\
0 & \tilde{\Pi}_6 & 0 & 0 & 2\Delta A_d^TPB & 0 & 2\Delta A_d^T & 2\Delta A_d^TPE \\
2E^TPA & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\
\sqrt{3}B^TP\Delta A & 0 & 0 & -B^TPB & 0 & 0 & 0 & 0 \\
0 & 2B^TP\Delta A_d & 0 & 0 & -B^TPB & 0 & 0 & 0 \\
\sqrt{5}\Delta A & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2\Delta A_d & 0 & 0 & 0 & -P^{-1} & -P^{-1} & 0 \\
0 & 2E^TPA_d & & & 0 & 0 & 0 & -\varepsilon_2 I
\end{bmatrix} < 0 \tag{19}
$$

Where,

$$\begin{aligned}\Pi_5 &= 4A^T PA + 4\varepsilon_1 H^T + 2\rho I - P + Q \\ \Pi_6 &= 4A_d^T PA_d + 4\varepsilon_2 H_d^T H_d + 2\rho I - Q\end{aligned} \tag{20}$$

And, (19) can be transformed into:

$$\begin{bmatrix} \tilde{\Pi}_5 & 0 & 2A^T PE & 0 & 0 & \sqrt{5}\Delta A^T & 0 & 0 \\ 0 & \tilde{\Pi}_6 & 0 & 0 & 0 & 0 & 2\Delta A_d^T & 2\Delta A_d^T PE \\ 2E^T PA & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -B^T PB & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -B^T PB & 0 & 0 & 0 \\ \sqrt{5}\Delta A & 0 & 0 & 0 & 0 & -P^{-1} & 0 & 0 \\ 0 & 2\Delta A_d & 0 & 0 & 0 & 0 & -P^{-1} & 0 \\ 0 & 2E^T PA_d & & & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} + \tag{21}$$

$$W_1^T \begin{bmatrix} F^T & 0 \\ 0 & F^T \end{bmatrix} Y_1^T + Y_1 \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix} W_1 < 0$$

Where,

$$\begin{aligned} W_1 &= \begin{bmatrix} \sqrt{5}H & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2H_d & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ Y_1 &= \begin{bmatrix} 0 & 0 & E^T PB & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E^T PB & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{22}$$

According to lemma 1,

$$W_1^T F^T Y_1^T + Y_1 F W_1 \le \varepsilon_3^{-1} Y_1 Y_1^T + \varepsilon_3 W_1^T W_1$$

So, (21) can be turned into (23) and (24),

$$\begin{bmatrix} \tilde{\Pi}_7 & 0 & 2A^T PE & \sqrt{5}\Delta A^T & 0 & 0 \\ 0 & \Pi_8 & 0 & 0 & 2\Delta A_d^T & 2A_d^T PE \\ 2E^T PA & 0 & -\varepsilon_1 I & 0 & 0 & 0 \\ \sqrt{5}\Delta A & 0 & 0 & -P^{-1} & 0 & 0 \\ 0 & 2\Delta A_d & 0 & 0 & -P^{-1} & 0 \\ 0 & 2E^T PA_d & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} -B^T PB & B^T PE \\ E^T PB & -\varepsilon_3 I \end{bmatrix} < 0 \tag{24}$$

Where,

$$\begin{aligned} \Pi_7 &= 4A^T PA + 4\varepsilon_1 H^T H \\ &\quad + 3\varepsilon_3 H^T H + 2\rho I - P + Q \\ \Pi_8 &= 4A_d^T PA_d + 4\varepsilon_2 H_d^T H_d \\ &\quad + 4\varepsilon_3 H_d^T H + 2\rho I - Q \end{aligned} \tag{25}$$

Repeating the step form (21) to (25), (23) can be transformed into

$$\begin{bmatrix} \Pi_1 & 0 & 2A^T PE & 0 \\ 0 & \Pi_2 & 0 & 2A_d^T PE \\ 2E^T PA & 0 & -\varepsilon_1 I & 0 \\ 0 & 2E^T PA_d & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} -P & PE \\ E^T P & -\varepsilon_4 I \end{bmatrix} < 0 \tag{27}$$

So, the proof of Theorem 1 is complete. Next part will give the process of constructing of the control law.

Theorem 2. For a system like (2) which meets the assumption (1) - (5), the sliding surface is shown as (4). The system can reach the stable state under the control law (28) no matter the initial state.

$$u(k) = \begin{cases} -\zeta\rho(k)\dfrac{S(k)}{\| S(k) \|} & , \| S(k) \| \ne 0 \\ 0 & , \| S(k) \| = 0 \end{cases} \tag{28}$$

Where, $\rho(k) = \rho_0 + \eta \| S(k) \|$, $\rho_0(k) = \| RE \| \| Hx(k) \| + \| RE \| \| H_d x(k-d) \|$, $R = (B^T PB)^{-1} B^T P$, $\eta = \dfrac{\rho_0(1-\zeta) + \| R \| \| g \|}{\rho_0 \zeta^2 \| B^T PB \|}$, $\zeta > 2$. Then the system state can enter the quasi-sliding surface Θ within finite time, and then keep stable in it.

$$\Theta = \| S(k) \| \le \Gamma,$$

$$\Gamma = \sqrt{\frac{\rho_0^2 [\| B^T PB \| (1 + \frac{1}{2}\zeta^2) + \frac{1}{2} \| PB \|^2] + \frac{1}{2}\varepsilon \| x(k) \|^2}{\zeta\eta \frac{1}{2} \| B^T PB \| \zeta^2 \eta^2}} \tag{29}$$

Choosing the Lyapunov function:

$$V(k)=\frac{1}{2}S^T(k)(B^TPB)^{-1}S(k) \tag{30}$$

Taking (4) into $\Delta V(k)$,

$$\begin{aligned} V(k)&=\frac{1}{2}S^T(k+1)(B^TPB)^{-1}S(k+1)\\ &\quad-\frac{1}{2}S^T(k)(B^TPB)^{-1}S(k)\\ &=S^T(k)(B^TPB)^{-1}\Delta S(k)\\ &\quad+\frac{1}{2}\Delta S^T(k)(B^TPB)^{-1}\Delta S(k) \end{aligned} \tag{31}$$

Where, $\Delta S(k)=S(k+1)-S(k)$. Substituting (4) and (28) in it, then:

$$\begin{aligned} \Delta S(k)&=B^T[\Delta A(k)+\Delta A_d(k-d)\\ &\quad-B\zeta\rho(k)\frac{S(k)}{\|S(k)\|}+g(x(k))] \end{aligned} \tag{32}$$

Taking (32) into (31)

$$\begin{aligned} &S^T(k)(B^TPB)^{-1}\Delta S(k)\\ &=S^T(k)R[EFHx(k)\\ &+EFH_dx(k-d)]-\zeta\rho(k)\|S(k)\|\\ &+S^T(k)Rg(x(k))\\ &\le\|S^T\|[\|RE\|\|Hx(k)\|\\ &+\|RE\|\|H_dx(k-d)\|]\\ &-\zeta(\rho_0+\eta\|S(k)\|)\|S(k)\|\\ &+\|S(k)\|\|R\|\|g\|\\ &=[\rho_0(1-\zeta)+\|R\|\|g\|]\|S(k)\|\\ &-\zeta\eta\|S(k)\|^2 \end{aligned} \tag{33}$$

$$\begin{aligned} &\frac{1}{2}\Delta S^T(k)(B^TPT)^{-1}\Delta S^T\\ &=\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)]E^TPB(B^TPB)^{-1}B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)B^T\rho(K)\zeta\frac{S^T(k)}{\|S(k)\|}\\ &-\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)]^TE^TPB\zeta\rho\frac{S^T(k)}{\|S(k)\|}\\ &-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\rho(k)\zeta B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &+\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)^TE^TPB(B^TPB)^{-1}B^TPg(x(k))\\ &+\frac{1}{2}g^T(x(k))PB(B^TPB)^{-1}B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)B^TPg[x(k))\\ &-\frac{1}{2}g^T(x(k))PB\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\ &+\frac{1}{2}g^T(x(k))PB(B^TPB)^{-1}B^TPg(x(k)) \end{aligned} \tag{34}$$

According to lemma 2,

$$\begin{aligned} &-\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)]^TE^TPB\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\ &-\frac{1}{2}\frac{S(k)}{\|S(k)\|}\rho(k)\zeta B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &\le\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)]^TE^TPB(B^TPB)^{-1}B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\rho(k)\zeta B^TPB\zeta\rho\frac{S(k)}{\|S(k)\|} \end{aligned}$$

$$\begin{aligned} &+\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)]^TE^TPB(B^TPB)^{-1}B^TPg(x(k))\\ &+\frac{1}{2}g^T(x(k))PB(B^TPB)^{-1}B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &\le-\frac{1}{2}[FH_dx(k-d)\\ &+FHx(k)^TE^TPB(B^TPB)^{-1}\\ &B^TPPB(B^TPB)^{-1}B^TPE[FHx(k)\\ &+FH_dx(k-d)]\\ &-\frac{1}{2}g^T(x(k))g(x(k)) \end{aligned}$$

$$\begin{aligned} &-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)B^TPg(x(k))\\ &-\frac{1}{2}g^T(x(k))PB\zeta\rho(k)\frac{S(k)}{\|S(k)\|} \end{aligned}$$

$$\leq -\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)(B^T PB)$$
$$\rho(k)\zeta\frac{S(k)}{\|S(k)\|}$$
$$-\frac{1}{2}g^T(x(k))PB(B^T PB)^{-1}B^T Pg(x(k))$$

Then (34) can be transformed into:

$$\begin{aligned}
&\frac{1}{2}\Delta S^T(k)(B^T PB)^{-1}\Delta S(k) \\
&=[FH_d x(k-d) \\
&+FHx(k)]E^T PB(B^T PB)^{-1}B^T PE[FHx(k) \\
&+FH_d x(k-d)] \\
&+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)B^T PB\rho(k)^T\zeta\frac{S(k)}{\|S(k)\|} \\
&+\frac{1}{2}[FH_d x(k-d) \\
&+FHx(k)]^T E^T PB(B^T PB)^{-1} \\
&B^T PPB(B^T PB)^{-1}B^T PE[FHx(k) \\
&+FH_d x(k-d)] \\
&+\frac{1}{2}\rho x^T(k)x(k) \\
&=\rho_0^2\|B^T PB\|+\frac{1}{2}\rho^2(k)\zeta^2\|B^T PB\| \\
&+\frac{1}{2}\varepsilon\|x(k)\|^2+\frac{1}{2}\rho_0^2\|PB\|^2
\end{aligned} \tag{35}$$

Substituting (35) and (33) into (31),

$$\begin{aligned}
\Delta V(k) &\leq [\rho_0(1-\zeta) \\
&+\|R\|\|g\|]\|S(k)\| \\
&-\zeta\eta\|S(k)\|^2 \\
&+\rho_0^2\|B^T PB\| \\
&+\frac{1}{2}\rho^2(k)\zeta^2\|B^T PB\| \\
&+\frac{1}{2}\varepsilon\|x(k)\|^2 \\
&+\frac{1}{2}\rho^2\|PB\| \\
&=\frac{1}{2}\|B^T PB\|\zeta^2\rho_0^2 \\
&+\rho_0^2\|B^T PB\| \\
&+\frac{1}{2}\rho_0^2\|PB\| \\
&+\frac{1}{2}\varepsilon\|x(k)\|^2 \\
&+\|S(k)\|^2(\frac{1}{2}\|B^T PB\| \\
&\zeta^2\eta^2-\zeta\eta)
\end{aligned} \tag{36}$$

So, it is easily to gain that while $\|S(k)\|>\Gamma$, $\Delta V(k)<0$. Therefore, the trajectory of the system converges to quasi-sliding surface Θ.

Remark 1. Based on Theorem 1 and Theorem 2, with the effect of sliding mode control, system (2) is quadratically stable and will reach the stable state in finite time.

3.2 Uncertain System with Multiple Time-delay

The structure of WSN based IoT can be unstable. In this section, uncertain system with multiple time-delay will be discussed, which is shown in (3). A nonsingular matrix T can be chosen such that $TB=\begin{bmatrix}0_{(n-m)\times m}\\ B_m\end{bmatrix}$, where B_m is nonsingular with $rank(B_m)=m$. By previous research $T=\begin{bmatrix}U_2^T\\ U_1^T\end{bmatrix}$, where $U_1\in R^{n\times m}$ and $U_2\in R^{n\times(n-m)}$ are two unitary matrices resulting from singular value decomposition of matrix B, then:

$$B=[U_1\quad U_2]\begin{bmatrix}\Sigma\\ 0_{(n-m)\times m}\end{bmatrix}V^T$$

where $\Sigma\in R^{m\times m}$ is a symmetric positive-definite matrix, $V\in R^{m\times m}$ is a unitary matrix, by the state transformation $y=Tx$, and (2) has the regular form

$$\begin{cases}
y(k+1)=(\overline{A}+\Delta\overline{A})y(k) \\
\qquad +\sum_{i=1}^{N}(\overline{A_d}+\Delta\overline{A_d})y_i(k-d_i) \\
\qquad +\begin{bmatrix}0_{(n-m)\times m}\\ B_m\end{bmatrix}u(k)+Tg(x(k)) \\
y(k)=\Phi(k)
\end{cases} \tag{37}$$

where $\overline{A}=TAT^{-1}$, $\overline{A_d}=TA_dT^{-1}$, $\Delta\overline{A}=T\Delta AT^{-1}$, $\Delta\overline{A_d}=T\Delta A_dT^{-1}$ and $\Phi(k)=T\Psi(k)$, in addition we have

$$\begin{cases} y_1(k+1) = (\overline{A}_{11} + \Delta\overline{A}_{11}) y_1(k) \\ \quad + \sum_{i=1}^{N} (\overline{A}_{d11} + \Delta\overline{A}_{d11}) y_{i1}(k-d_i) \\ \quad + (\overline{A}_{11} + \Delta\overline{A}_{12}) y_2(k) \\ \quad + \sum_{i=1}^{N} (\overline{A}_{d12} + \Delta\overline{A}_{d12}) y_{i2}(k-d_i) + g_1 \\ y_2(k+1) = (\overline{A}_{21} + \Delta\overline{A}_{21}) y_1(k) \\ \quad + \sum_{i=1}^{N} (\overline{A}_{d21} + \Delta\overline{A}_{d21}) y_{i1}(k-d_i) \\ \quad + (\overline{A}_{22} + \Delta\overline{A}_{22}) y_2(k-d_i) \\ \quad + \sum_{i=1}^{N} (\overline{A}_{d22} + \Delta\overline{A}_{d22}) y_{i2}(k-d_i) \\ \quad + B_m u(k) \\ y_1(k) = \Phi_1(k) \\ y_2(k) = \Phi_2(k) \end{cases} \tag{38}$$

where $y_1 \in R^{(n-m)}$, $y_2 \in R^m$, $B_m = \Sigma V^T$, $\overline{A_{11}} = U_2^T A U_2$, $\overline{A_{12}} = U_2^T A U_1$, $\overline{A_{d11}} = U_2^T A_d U_2$, $\overline{A_{d12}} = U_2^T A_d U_1$, $\overline{A_{11}} = U_2^T GD(k) H U_2$, $\Delta\overline{A_{11}} = U_2^T GD(k) H_d U_2$, $\Delta\overline{A_{12}} = U_2^T GD(k) H_d U_1$, $\Delta\overline{A_{d12}} = U_2^T GD(k) H_d U_1$, $g_1 = U_2^T g(x(k))$, $g_2 = U_1^T g(x(k))$, $\Phi_1(k) \in R^{(n-m)}$ and $\Phi_2(k) \in R^m$ are the sub-blocks of $\Phi(k)$.

It is obvious that the first equation of (38) represents the dynamics of sliding motion (37), so the sliding surface is designed as

$$\begin{aligned} & S(k) = G(k) + \sigma(k) \\ & \sigma(k+1) = Gx(k) - GAx(k) \\ & \qquad GA_d x(k-d) + \sigma(k) \\ & \sigma(0) = -Gx(0) \end{aligned} \tag{39}$$

where $Gx = Cy_1 + y_2 = [CI]y$, $C \in R^{m \times (n-m)}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, substituting $y_2 = -Cy_1$ to the first equation of (38), then the sliding motion is

$$\begin{cases} y_1(k+1) = (\overline{A}_{11} + \Delta\overline{A}_{11}) \\ \quad - \overline{A}_{12} C \\ \quad - \Delta\overline{A}_{12} C) y_1(k) \\ \quad + \sum_{i=1}^{N} (\overline{A}_{d11} + \Delta\overline{A}_{d11} - barA_{d12} C \\ \quad - \Delta\overline{A}_{d12} C) y_{i1}(k-d_i) + g_1 \\ y_1(k) = \Phi_1(k) \end{cases} \tag{40}$$

Letting $\tilde{A}_{11} = \overline{A}_{11} - \overline{A}_{12} C$, $\Delta\tilde{A}_{11} = \Delta\overline{A}_{11} - \Delta\overline{A}_{12} C = U_2^T GD(k) H(U_2 - U_1 C)$, $\tilde{A}_{d11} = \overline{A}_{d11} - \overline{A}_{d12} C$, $\Delta\tilde{A}_{d11} = \Delta\overline{A}_{d11} - \Delta\overline{A}_{d12} C = U_2^T GD(k) H_d (U_2 - U_1 C)$.

$$\begin{cases} y_1(k+1) = (\tilde{A}_{11} + \Delta\tilde{A}_{11}) y_1(k) \\ \quad + \sum_{i=1}^{N} (\tilde{A}_{d11} \\ \quad + \Delta\tilde{A}_{d11}) y_{i1}(k-d_i) + g_1 \\ y_1(k) = \Phi_1(k) \end{cases} \tag{41}$$

Theorem 3. The system (2), the sliding surface is (4). If there existing positive matrix $P > 0$ and $Q > 0$ and constant $\varepsilon_i > 0$ (i=1, 2, 3, 4), $\lambda > 0$ to make (42), (44) exist, the system (3) is quadratically stable.

$$\begin{bmatrix} \Pi_1 & 0 & \tilde{A}_{11}^T PE & 0 & \lambda\sqrt{\rho} I \\ * & \Pi_2 & 0 & \sqrt{3}A_{d11}^T PE & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\lambda \end{bmatrix} \tag{42}$$

$$\begin{bmatrix} -P & PE \\ * & -\varepsilon_3 I \end{bmatrix} < 0 \tag{43}$$

$$U^T P U \le \lambda I \tag{44}$$

Where * is acquired based on matrix symmetry, and

$$\begin{aligned} \Pi_1 &= \tilde{A}_{11}^T P \tilde{A}_{11} + \varepsilon_{11} H^T H \\ &\quad + \varepsilon_3 H^T H - P + NQ \\ \Pi_2 &= 3\tilde{A}_{d11}^T P \tilde{A}_{d11} + 3\varepsilon_2 H_d^T H_d \\ &\quad + 3\varepsilon_3 H_d^T H_d - NQ \end{aligned}$$

Choosing Lyapunov function as

$$V(k) = x^T(k) P x(k) + \sum_{i=k-d}^{k-1} x^T(i) Q x(i) \tag{45}$$

substituting (42) into (45),

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \Delta V_1(k) + \Delta V_2(k) \\ &\quad + \Delta V_3(k) + \Delta V_4(k) \end{aligned} \tag{46}$$

where

$$\begin{aligned} \Delta V_1(k) &= y_1^T(k)(\tilde{A}_{11} + \Delta\tilde{A}_{11})^T \\ &\quad P(\tilde{A}_{11} + \Delta\tilde{A}_{11})^T y_1(k) \end{aligned} \tag{47}$$

$$\begin{aligned} \Delta V_2(k) &= y_1^T(k)(\tilde{A}_{11} + \Delta\tilde{A}_{11})^T \\ &\quad P(\tilde{A}_{d11} + \Delta\tilde{A}_{d11}) y_{i1}(k-d_i) \\ &\quad + y_{i1}^T(k-d_i)(\tilde{A}_{d11} + \Delta\tilde{A}_{d11})^T \\ &\quad P(\tilde{A}_{11} + \Delta\tilde{A}_{11}) y_1(k) \end{aligned} \tag{48}$$

$$\Delta V_3(k) = y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})y_{i1}(k-d_i) \quad \textbf{(49)}$$

$$\begin{aligned}\Delta V_4(k) = {} & g^T U_2 P U_2^T g \\ & + g^T U_2 P(\tilde{A}_{11}+\Delta\tilde{A}_{11})y_1(k) \\ & + y_1^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T P U_2^T g \\ & + y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P U_2^T g \\ & + g^T U_2 P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})y_{i1}(k-d_i)\end{aligned} \quad \textbf{(50)}$$

According to lemma 2,

$$\begin{aligned} & y_1^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T P(\tilde{A}_{d11} + \Delta\tilde{A}_{d11})y_{i1}(k-d_i) \\ & + y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P(\tilde{A}_{11}+\Delta\tilde{A}_{11})y_1(k) \\ \le {} & y_1^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T P(\tilde{A}_{11}+\Delta\tilde{A}_{11})y_1(k) \\ & + y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})y_{i1}(k-d_i)\end{aligned}$$

$$\begin{aligned} & g^T U_2 P(\tilde{A}_{11}+\Delta\tilde{A}_{11})y_1(k) \\ & + y_1^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11}) P U_2^T g \\ \le {} & -y_1^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T P(\tilde{A}_{11}-\Delta\tilde{A}_{11})y_1(k) \\ & + g^T U_2 P U_2^T g\end{aligned}$$

$$\begin{aligned} & y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P U_2^T g \\ & + g^T U_2 P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})y_{i1}(k-d_i) \\ \le {} & y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P(\tilde{A}_{d11} + \Delta\tilde{A}_{d11})y_{i1}(k-d_i) + g^T U_2 P_2^T g\end{aligned}$$

Besides, there existing ε_i (i=1, 2), using lemma 1:

$$\begin{aligned} & y_i^T(k)(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T P(\tilde{A}_{11}+\Delta\tilde{A}_{11})y_1(k) \\ \le {} & y_1^T(k)[\tilde{A}_{11}^T P\tilde{A}_{11} + \varepsilon_1^{-1}\tilde{A}_{11}^T PEE^T P\tilde{A}_{11} \\ & + \varepsilon_1 H^T H + \Delta\tilde{A}_{11}^T P\Delta\tilde{A}_{11}]y_1(k)\end{aligned}$$

$$\begin{aligned} & y_{i1}^T(k-d_i)(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T P U_2^T g \\ & + g^T U_2 P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})y_{i1}(k-d_i) \\ \le {} & y_{i1}^T(k-d_i)[\tilde{A}_{d11}P\tilde{A}_{d11} + \varepsilon_1^{-1}\tilde{A}_{d11}^T PEE^T P\tilde{A}_{d11} \\ & + \varepsilon_2 H_d^T H_d + \Delta\tilde{A}_{d11}^T P\Delta\tilde{A}_{d11}]y_{i1}(k-d_i)\end{aligned}$$

Substituting the result above into (46), then:

$$\Delta A(k) \le \begin{bmatrix} y_1^T(k) \\ y_1^T(k-d_1) \\ \vdots \\ y_{N1}^T(k-d_N) \end{bmatrix} M \begin{bmatrix} y_1(k) \\ y_1(k-d_1) \\ \vdots \\ y_{N1}(k-d_N) \end{bmatrix} \quad \textbf{(51)}$$

Where,

$$M\begin{bmatrix} \Pi_3 & 0 \\ 0 & \Pi_4 \end{bmatrix} \quad \textbf{(52)}$$

$$\begin{aligned}\Pi_3 = {} & 1\tilde{A}_{11}P\tilde{A}_{11} \\ & + 1\varepsilon_1^{-1}\tilde{A}_{11}^T PEE^T P\tilde{A}_{11} \\ & + 1\varepsilon_1^{-1}H^T H \\ & + 1\Delta\tilde{A}_{11}^T P\Delta\tilde{A}_{11} \\ & - P + NQ + \rho\lambda I \\ \Pi_4 = {} & 3\tilde{A}_{d11}P\tilde{A}_{d11} \\ & + 3\varepsilon_2^{-1}\tilde{A}_{d11}^T PEE^T P\tilde{A}_{d11} \\ & + 3\varepsilon_2 H_d^T H_d \\ & + 3\Delta\tilde{A}_{d11}^T P\Delta\tilde{A}_{d11} - NQ\end{aligned} \quad \textbf{(53)}$$

While $M<0$, $\Delta V(k)<0$. According to lemma 3, $M<0$ equals to:

$$\begin{bmatrix} \tilde{\Pi}_5 & 0 & \tilde{A}_{11}^T PE & \Delta\tilde{A}_{11}^T P & 0 & 0 & \sqrt{\rho}I \\ 0 & \tilde{\Pi}_6 & 0 & 0 & \sqrt{3}\Delta\tilde{A}_{d11}^T P & \sqrt{3}A_{d11}^T PE & 0 \\ E^T\tilde{A}_{11} & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ P\Delta\tilde{A}_{11} & 0 & 0 & P^{-1} & 0 & 0 & 0 \\ 0 & \sqrt{3}P\Delta\tilde{A}_{d11} & 0 & 0 & -P^{-1} & 0 & 0 \\ 0 & \sqrt{3}E^T P\tilde{A}_{d11} & 0 & 0 & 0 & -\varepsilon_2 I & 0 \\ \sqrt{\rho}I & 0 & 0 & 0 & 0 & 0 & -\lambda^{-1} \end{bmatrix} < 0 \quad \textbf{(54)}$$

Where,

$$\begin{aligned}\Pi_5 &= \tilde{A}_{11}P\tilde{A}_{11} + \varepsilon_1 H^T H - P + NQ \\ \Pi_6 &= 3\tilde{A}_{d11}P\tilde{A}_{d11} + 3\varepsilon_2 H_d^T H_d - NQ\end{aligned} \quad \textbf{(55)}$$

And, (54) can be transformed into:

$$\begin{bmatrix} \tilde{\Pi}_5 & 0 & \tilde{A}_{11}^T PE & 0 & 0 & 0 & \sqrt{\rho} I \\ 0 & \tilde{\Pi}_6 & 0 & 0 & 0 & \sqrt{3} A_{d11}^T PE & 0 \\ E^T \tilde{A}_{11} & 0 & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -P^{-1} & 0 & 0 \\ 0 & \sqrt{3} E^T P \tilde{A}_{d11} & 0 & 0 & 0 & -\varepsilon_2 I & 0 \\ \sqrt{\rho} I & 0 & 0 & 0 & 0 & 0 & -\lambda^{-1} \end{bmatrix} + \tag{56}$$

$$W_1^T \begin{bmatrix} F^T & 0 \\ 0 & F^T \end{bmatrix} Y_1^T + Y_1 \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix} W_1 < 0$$

Where,

$$W_1 = \begin{bmatrix} H & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{3} H_d & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$Y_1 = \begin{bmatrix} 0 & 0 & 0 & E^T P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E^T P & 0 & 0 \end{bmatrix} \tag{57}$$

According to lemma 1,

$$W_1^T F^T Y_1^T + Y_1 F W_1 \le \varepsilon_4^{-1} Y_1 Y_1^T + \varepsilon_3 W_1^T W_1$$

So, (54) can be turned into (58) and (59),

$$\begin{bmatrix} \Pi_1 & 0 & \tilde{A}_{11}^T PE & 0 & \sqrt{\rho} I \\ 0 & \Pi_2 & 0 & \sqrt{3} A_{d11}^T PE & 0 \\ E^T P \tilde{A}_{11} & 0 & -\varepsilon_1 I & 0 & 0 \\ 0 & \sqrt{3} E^T P \tilde{A}_{d11} & 0 & -\varepsilon_2 I & 0 \\ \sqrt{\rho} I & 0 & 0 & 0 & -\lambda^{-1} \end{bmatrix} \tag{58}$$

$$\begin{bmatrix} -P & PE \\ E^T P & -\varepsilon_3 I \end{bmatrix} < 0 \tag{59}$$

Multiply $diag\{I\ I\ I\ I\ \lambda\}$ on both side of (58), then

$$\begin{bmatrix} \Pi_1 & 0 & \tilde{A}_{11}^T PE & 0 & \lambda\sqrt{\rho} I \\ 0 & \Pi_2 & 0 & \sqrt{3} A_{d11}^T PE & 0 \\ E^T P \tilde{A}_{11} & 0 & -\varepsilon_1 I & 0 & 0 \\ 0 & \sqrt{3} E^T P \tilde{A}_{d11} & 0 & -\varepsilon_2 I & 0 \\ \lambda\sqrt{\rho} I & 0 & 0 & 0 & -\lambda \end{bmatrix}$$

So, the proof of Theorem 3 is complete. Next part will give the process of constructing of the control law.

Theorem 4. For a system like (3) which meets the assumption (1) - (5), the sliding surface is shown as (4). The system can reach the stable state under the control law (39) no matter the initial state.

$$u(k) = \begin{cases} -\zeta\rho(k) \dfrac{S(k)}{\| S(k) \|} & , \| S(k) \| \ne 0 \\ 0 & , \| S(k) \| = 0 \end{cases} \tag{60}$$

Where, $\rho(k) = \rho_0 + \eta \| S(k) \|$, $\rho_0(k) = \| RE \| \| Hx(k) \| + N \| RE \| \| H_d x(k-d) \|$, $R = (GB)^{-1} G$, $\eta = \dfrac{\rho_0(1-\zeta) + \| r \| \| g \|}{2\rho_0(\frac{1}{2} + \| GB \| \zeta^2)}$, $\zeta > 2$. Then the system state can enter the quasi-sliding surface Θ within finite time, and then keep stable in it.

$$\Theta = \| S(k) \| \le \Gamma,$$
$$\Gamma = \sqrt{-\frac{\rho_0^2 [\frac{1}{2} \| GB \| + \zeta^2 (\frac{1}{2} + \| GB \|)] + \frac{1}{2} \varepsilon \| x(k) \|^2}{(\frac{1}{2} + \| GB \|) \zeta^2 \eta^2 - \zeta\eta}} \tag{61}$$

Choosing the Lyapunov function:

$$V(k) = \frac{1}{2} S^T(k) (GB)^{-1} S(k) \tag{62}$$

Taking (39) into $\Delta V(k)$,

$$\begin{aligned} V(k) &= \frac{1}{2} S^T(k+1) (GB)^{-1} S(k+1) \\ &\quad - \frac{1}{2} S^T(k) (GB)^{-1} S(k) \\ &= S^T (GB)^{-1} \Delta S(k) \\ &\quad + \frac{1}{2} \Delta S^T(k) (GB)^{-1} \Delta S(k) \end{aligned} \tag{63}$$

Where, $\Delta S(k) = S(k+1) - S(k)$. Substituting (39) and (60) in it, then:

$$\begin{aligned} \Delta S(k) &= G\Delta A x(k) + \sum_{i=1}^{N} G \Delta A_d x(k-d_i) \\ &\quad - Gb\zeta\rho(k) + \frac{S(k)}{\| S(k) \|} + Gg(x(k)) \end{aligned} \tag{64}$$

Taking (64) into (63)

$$\begin{aligned} & S^T(k) (GB)^{-1} \Delta S(k) \\ &= S^T(k) R [EFHx(k) + NEFH_d x(k-d_i)] \\ &\quad - \zeta\rho(k) \| S(k) \| + S^T(k) R g(x(k)) \\ &\le \| S(k) \| [[\| RE \| \| Hx(k) \|] \\ &\quad + N \| RE \| \| H_d x(k-d_i) \|] \\ &\quad - \zeta(\rho_0 + \eta \| S(k) \|) \| S(k) \| \\ &\quad + \| S(k) \| \| R \| \| g \| \\ &= [\rho_0 (1-\zeta) + \| R \| \| g \|] \| S(k) \| \\ &\quad - \zeta\eta \| S(k) \|^2 \end{aligned} \tag{65}$$

$$
\begin{aligned}
&\frac{1}{2}\Delta S^T(k)(GB)^{-1}\Delta S(k)\\
&=\frac{1}{2}[NFH_d x(k-d_i)\\
&+FHx(k)]E^TG^T(GB)^{-1}GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)GB\rho(k)\zeta\frac{S(k)}{\|S(k)\|}\\
&-\frac{1}{2}[NHFH_d x(k-d_i)\\
&+FHx(k)]^T E^TG^T\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\
&-\frac{1}{2}\frac{S(k)}{\|S(k)\|}\rho(k)\zeta GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&+\frac{1}{2}[NFH_d x(k-d_i)\\
&+FHx(k)]^T E^TG^T(GB)^{-1}Gg(x(k))\\
&+\frac{1}{2}g^T(x(k))G^T(GB)^{-1}GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)Gg(x(k))\\
&-\frac{1}{2}g^T(x(k))G^T\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\
&-\frac{1}{2}g^T(x(k))G^TGB^{-1}Gg(x(k))
\end{aligned}
\tag{66}
$$

According to lemma 2,

$$
\begin{aligned}
&-\frac{1}{2}[NFX_d x(k-d_i)+FHx(k)]^T\\
&E^TG^T\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\
&-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\rho(k)\zeta\\
\le\ & GE[FH_d x(k-d)+FHx(k)]^T\\
&E^TG^T(GB)^{-1}GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\rho(k)\zeta GB\zeta\rho(k)\frac{S(k)}{\|S(k)\|}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1}{2}[NFH_d x(k-d_i)\\
&+FHx(k)]^T E^TG^T(GB)^{-1}Gg(x(k))\\
&+\frac{1}{2}g^T(x(k))G^T(GB)^{-1}-GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
\le\ & -\frac{1}{2}[NFH_d x(k-d_i)\\
&+NHx(k)]^T E^TG^T(GB)^{-1}GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&-\frac{1}{2}g^T(x(k))G^T(GB)^{-1}Gg(x(k))
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(g)Gg(x(k))\\
&-\frac{1}{2}g^T(x(k))G^T\zeta\rho(k)\frac{S(k)}{\|S(k)\|}\\
\le\ & +\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)\rho(k)\zeta\frac{S^T(k)}{\|S(k)\|}\\
&+\frac{1}{2}g^T(x(k))g(x(k))
\end{aligned}
$$

Then (66) can be transformed into:

$$
\begin{aligned}
&\frac{1}{2}\Delta S^T(GB)^{-1}\Delta S(k)\\
\le\ & \frac{1}{2}[NFH_d x(k-d_i)]\\
&+FHx(k)]E^TG^T(GB)^{-1}GE[FHx(k)\\
&+NFH_d x(k-d_i)]\\
&+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(g)GB\rho(k)\zeta\frac{S(k)}{\|S(k)\|}\\
&+\frac{1}{2}\frac{S^T(k)}{\|S(k)\|}\zeta\rho(k)\rho(k)\zeta\frac{S(k)}{\|S(k)\|}\\
&+\frac{1}{2}\rho x^T(k)x(k)\\
&=\frac{1}{2}\rho_0^2\|GB\|+\rho^2(k)\zeta^2\|GB\|\\
&+\frac{1}{2}\varepsilon\|x(k)\|^2+\frac{1}{2}\rho^2(k)\zeta^2
\end{aligned}
\tag{67}
$$

Substituting (67) and (65) into (63),

$$
\begin{aligned}
\Delta V(k)\le\ & [\rho_0(1-\zeta)+\|R\|\|g\|]\|S(k)\|\\
&-\zeta\eta\|S(k)\|^2\\
&+\frac{1}{2}\rho_0^2\|GB\|+\rho^2(k)\zeta^2\|GB\|\\
&+\frac{1}{2}\varepsilon\|x(k)\|^2+\frac{1}{2}\rho^2(k)\zeta^2\\
&=\rho_0^2\zeta^2\|GB\|+\frac{1}{2}\varepsilon\|x(k)\|^2+\\
&+\rho_0^2\zeta^2(\frac{1}{2}+\|GB\|)\\
&+(\frac{1}{2}+\|GB\|)\zeta^2\eta^2\|S(k)\|^2\\
&-\zeta\eta^2\|S(k)\|
\end{aligned}
\tag{68}
$$

So, it is easily to gain that while $\| S(k) \| > \Gamma$, $\Delta V(k) < 0$. Therefore, the trajectory of the system converges to quasi-sliding surface Θ.

Remark 2. Based on Theorem 3 and Theorem 4, with the effect of sliding mode control, system (3) is quadratically stable and will reach stable state in finite time.

4 Numerical Simulations

In this section, all the theorems above will be tested. Consider system (2), where

$$A = \begin{bmatrix} 0.05 & 0 & 0.06 \\ 0.01 & 0.05 & 0.08 \\ 0.05 & 0.23 & 0.01 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0.05 & 0 & 0.15 \\ 0.3 & 0 & 0.05 \\ 0.1 & 0.17 & 0.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.01 \end{bmatrix}, E = \begin{bmatrix} 0.05 \\ 0.15 \\ 0.05 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.3 \\ 0.15 \\ 0.1 \end{bmatrix}, H_d = \begin{bmatrix} 0.03 \\ 0 \\ 0.1 \end{bmatrix}^T,$$

$$g(x(k)) = \begin{bmatrix} \dfrac{0.08 * x1}{(2 * x2 * x2) + 1} \\ 0.1 * x2 * \sin(x3) \\ 0.1 * x3 * \cos * (x3) \end{bmatrix},$$

$$F(k) = \sin(k)$$

The initial states are: $x(0) = [-0.2 \; -0.2 \; 0.2]^T$, $d = 1$. Based on Theorem 1, we have

$$P = \begin{bmatrix} 22.7248 & 6.0182 & 10.8938 \\ 6.0182 & 19.0537 & 6.2682 \\ 10.8938 & 6.2682 & 29.9641 \end{bmatrix},$$

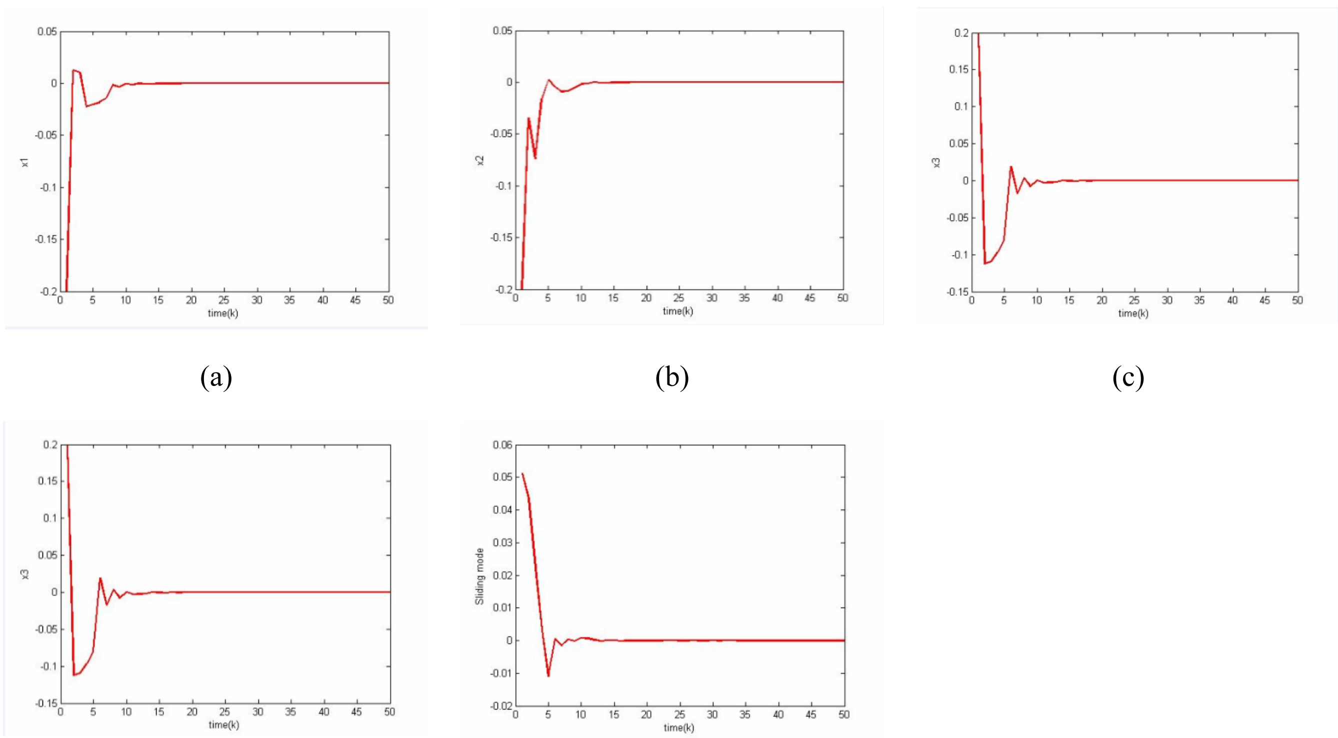
$$Q = \begin{bmatrix} 16.4676 & 7.0749 & 5.7473 \\ 3.2684 & 2.3397 & 2.3397 \\ 5.7473 & 0.0627 & 9.4726 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.1089 & 0.0627 & 0.2996 \end{bmatrix}$$

$$\varepsilon_1 = 1.6358, \varepsilon_2 = 9.3111,$$

$$\varepsilon_3 = 1.1291, \varepsilon_4 = 1.2714,$$

Then the simulation result is shown in Figure 1.



(a) (b) (c)

(d) (e)

Figure 1. States (x1, x2, x3), control input and sliding mode

In fact, it is hard to acquire ideal sliding mode. From the figure, we can see that the practical sliding mode is converged within a region around the ideal sliding surface.

Consider system (3), where

$$A = \begin{bmatrix} 0.5 & -0.23 \\ 0.03 & -0.02 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0.05 & 0.03 \\ 0.22 & 0.13 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, T = \begin{bmatrix} 0 & 0.8 \\ 0.8 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.02 \\ 0.05 \end{bmatrix}^T, H_d = \begin{bmatrix} 0.03 \\ 0.05 \end{bmatrix}^T,$$

$$E = \begin{bmatrix} 0.01 \\ 0.5 \end{bmatrix}^T, F(k) = \sin(k)$$

$$g(x(k)) = \begin{bmatrix} 0.08 * x1/((2 * x2 * x2) + 1) \\ 0.1 * x2 * \cos(x2) \end{bmatrix}$$

The initial states are: $x(0) = [-0.2 - 0.2]^T$, $d_i \in [0\ 1]$, $N = 4$, $\rho = 0.2$. Based on Theorem 1, we have

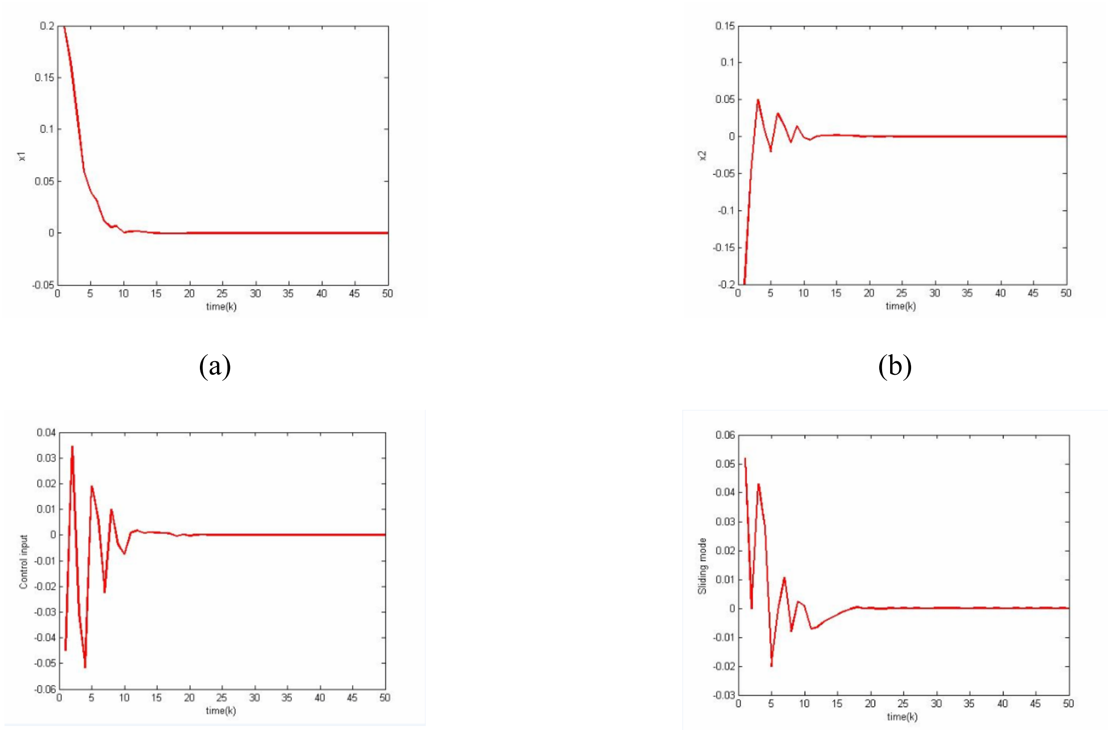
$$P = \begin{bmatrix} 150.0054 & 2.53772.5377 & 38.0364 \end{bmatrix},$$

$$Q = \begin{bmatrix} 17.2764 & 0.5824 \\ 0.5824 & 1.8132 \end{bmatrix}$$

$$\varepsilon_1 = 97.0120, \varepsilon_2 = 85.0112,$$

$$\varepsilon_3 = 86.7580, \lambda = 138.0773, c = 1.13.$$

The simulation results are shown in Figure 2.



(a) (b)

(c) (d)

Figure 2. States (x1, x2), control input and sliding mode

From the control law, although the design method is simplified, for systems requiring fast response, control parameters Φ should be enlarged.

5 Conclusion

In this paper, we investigate the sliding surface and control law of WSN based IoT with different types of time-delay and external nonlinear perturbation. LMI is used in both of the two situation for acquiring appropriate parameters of sliding mode controller to ensure the system quadratic stability. By nonsingular transformation and equivalent reduced order system, the complexity of parameter in multiple time-delay is reduced. It's clear that the system is eventually stabilized within an area called quasi-sliding surface. However there are also some aspects needed to be improved such as the problem of chattering, shorter reaching time. All the problems are needed further study.

Acknowledgments

This work was supported by the Natural Science Foundation of Jiangsu Province of China (Grant No. BK 20180467) and Research Start-up Funds of Nanjing University of Science and Technology.

References

[1] J. Gubbi, R. Buyya, S. Marusic, M. Palaniswami, Internet of Things (IoT): A Vision, Architectural Elements, and Future Directions, *Future Generation Computer Systems*, Vol. 29, No. 7, pp. 1645-1660, September, 2013.

[2] A. Zanella, N. Bui, A. Castellani, L. Vangelista, M. Zorzi, Internet of Things for Smart Cities, *IEEE Internet of Things Journal*, Vol. 1, No. 1, pp. 22-32, February, 2014.

[3] C. Perera, A. Zaslavsky, P. Christen, D. Georgakopoulos, Context Aware Computing for The Internet of Things: A Survey, *IEEE Communications Surveys & Tutorials*, Vol. 16, No. 1, pp. 414-454, First Quarter, 2014.

[4] A. Al-Fuqaha, M. Guizani, M. Mohammadi, M. Aledhari, M. Ayyash, Internet of Things: A Survey on Enabling Technologies, Protocols, and Applications, *IEEE Communications Surveys & Tutorials*, Vol. 17, No. 4, pp. 2347-2376, June, 2015.

[5] Z. Yan, P. Zhang, A. V. Vasilakos, A Survey on Trust Management for Internet of Things, *Journal of Network and Computer Applications*, Vol. 42, pp. 120-134, June, 2014.

[6] H. Dong, Z. Wang, H. Gao, Distributed H_∞ Filtering for a Class of Markovian Jump Nonlinear Time-Delay Systems Over Lossy Sensor Networks, *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 10, pp. 4665-4672, October, 2013.

[7] S. Kar, J. M. F. Moura, Distributed Consensus Algorithms in Sensor Networks: Quantized Data and Random Link Failures, *IEEE Transactions on Signal Processing*, Vol. 58, No. 3, pp. 1383-1400, March, 2010.

[8] Z. Sheng, S. Yang, Y. Yu, A. V. Vasilakos, J. A. Mccann, K. K. Leung, A Survey on the IETF Protocol Suite for the Internet of Things: Standards, Challenges, and Opportunities, *IEEE Wireless Communications*, Vol. 20, No. 6, pp. 91-98, December, 2013.

[9] N. A. Pantazis, S. A. Nikolidakis, D. D. Vergados, Energy-Efficient Routing Protocols in Wireless Sensor Networks: A Survey, *IEEE Communications Surveys & Tutorials*, Vol. 15, No. 2, pp. 551-591, January, 2013.

[10] J. A. Stankovic, Research Directions for the Internet of Things, *IEEE Internet of Things Journal*, Vol.1, No. 1, pp. 3-9, February, 2014.

[11] H. Dong, Z. Wang, H. Gao, Distributed Filtering for a Class of Time-Varying Systems Over Sensor Networks With Quantization Errors and Successive Packet Dropouts, *IEEE Transactions on Signal Processing*, Vol. 60, No. 6, pp. 3164-3173, June, 2012.

[12] B. Shen, Z. Wang, Y. S. Hung, Distributed H_∞-Consensus Filtering in Sensor Networks with Multiple Missing Measurements: The Finite-horizon Case, *Automatica*, Vol. 46, No. 10, pp. 1682-1688, October, 2010.

[13] B. Shen, Z. Wang, Y. S. Hung, G. Chesi, Distributed Hinfty Filtering for Polynomial Nonlinear Stochastic Systems in Sensor Networks, *IEEE Transactions on Industrial Electronics*, Vol. 58, No. 5, pp. 1971-1979, May, 2011.

[14] M. Mazo, P. Tabuada, Decentralized Event-Triggered Control Over Wireless Sensor/Actuator Networks, *IEEE Transactions on Automatic Control*, Vol. 56, No. 10, pp. 2456-2461, October, 2011.

[15] B. Wang, Coverage Problems in Sensor Networks: A Survey, *Acm Computing Surveys*, Vol. 43, No. 4, pp. 1-53, September, 2011.

[16] C. Zhu, C. Zheng, L. Shu, G. Han, A Survey on Coverage and Connectivity Issues in Wireless Sensor Networks, *Journal of Network and Computer Applications*, Vol. 35, No. 2, pp. 619-632, March, 2012.

[17] G. M. Hoffmann, C. J. Tomlin, Mobile Sensor Network Control Using Mutual Information Methods and Particle Filters, *IEEE Transactions on Automatic Control*, Vol. 55, No. 1, pp. 32-47, January, 2010.

[18] M. Z. Win, A. Conti, S. Mazuelas, Y. Shen, W. M. Gifford, D. Dardari, M. Chiani, Network Localization and Navigation via Cooperation, *IEEE Communications Magazine*, Vol. 49, No. 5, pp. 56-62, May, 2011.

[19] N. Patwari, J. Wilson, RF Sensor Networks for Device-Free Localization: Measurements, Models, and Algorithms, *Proceedings of the IEEE*, Vol. 98, No. 11, pp. 1961-1973, November, 2010.

[20] H. J. Visser, R. J. M. Vullers, RF Energy Harvesting and Transport for Wireless Sensor Network Applications: Principles and Requirements, *Proceedings of the IEEE*, Vol. 101, No. 6, pp. 1410-1423, June, 2013.

[21] Y. Liu, N. Xiong, Y. Zhao, A. V. Vasilakos, J. Gao, Y. Jia, Multi-layer Clustering Routing Algorithm for Wireless Vehicular Sensor Networks, *IET Communications*, Vol. 4, No. 7, pp. 810-816, April, 2010.

[22] M. Saleem, G. A. Di Caro, M. Farooq, Swarm Intelligence Based Routing Protocol for Wireless Sensor Networks: Survey and Future Directions, *Information Sciences*, Vol. 181, No. 20, pp. 4579-4624, October, 2011.

[23] P. Corke, T. Wark, R. Jurdak, W. Hu, P. Valencia, D. Moore, Environmental Wireless Sensor Networks, *Proceedings of the IEEE*, Vol. 98, No. 11, pp. 1903-1917, November, 2010.

[24] X. Cao, J. Chen, Y. Xiao, Y. Sun, Building-Environment Control With Wireless Sensor and Actuator Networks: Centralized Versus Distributed, *IEEE Transactions on Industrial Electronics*, Vol. 57, No. 11, pp. 3596-3605, November, 2010.

[25] J. Yang, S. Li, X. Yu, Sliding-Mode Control for Systems With Mismatched Uncertainties via a Disturbance Observer, *IEEE Transactions on Industrial Electronics*, Vol. 60, No. 1, pp. 160-169, January, 2013.

[26] J. Yang, S. Li, J. Su, X. Yu, Continuous Nonsingular Terminal Sliding Mode Control for Systems with Mismatched Disturbances, *Automatica*, Vol. 49, No. 7, pp. 2287-2291, July, 2013.

[27] H. Li, H. Gao, P. Shi, X. Zhao, Fault-tolerant Control of Markovian Jump Stochastic Systems via the Augmented Sliding Mode Observer Approach, *Automatica*, Vol. 50, No. 7, pp. 1825-1834, July, 2014.

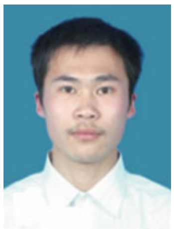
[28] M. Liu, P. Shi, Sensor Fault Estimation and Tolerant Control for It? Stochastic Systems with a Descriptor Sliding Mode Approach, *Automatica*, Vol. 49, No. 5, pp. 1242-1250, May,

2013.
[29] Bei Chen, Yugang Niu, Yuanyuan Zou, Adaptive Sliding Mode Control for Stochastic Markovian Jumping Systems with Actuator Degradation, *Automatica*, Vol. 49, No. 6, pp. 1748-1754, June, 2013.
[30] H. Li, P. Shi, D. Yao, L. Wu, Observer-based Adaptive Sliding Mode Control for Nonlinear Markovian Jump Systems, *Automatica*, Vol. 46, pp. 133-142, February, 2016.
[31] H. Zhang, J. Wang, Y. Shi, Robust Sliding-mode Control for Markovian Jump Systems Subject to Intermittent Observations and Partially Known Transition Probabilities, *Systems & Control Letters*, Vol. 62, No. 12, pp. 1114-1124, December, 2013.
[32] Y. Lin, Y. Shi, R. Burton, Modeling and Robust Discrete-Time Sliding-Mode Control Design for a Fluid Power Electrohydraulic Actuator (EHA) System, *IEEE/ASME Transactions on Mechatronics*, Vol. 18, No. 1, pp. 1-10, February, 2013.
[33] Q. Xu, Adaptive Discrete-Time Sliding Mode Impedance Control of a Piezoelectric Microgripper, *IEEE Transactions on Robotics*, Vol. 29, No. 3, pp. 663-673, June, 2013.
[34] S. Qu, X. Xia, J. Zhang, Dynamics of Discrete-Time Sliding-Mode-Control Uncertain Systems With a Disturbance Compensator, *IEEE Transactions on Industrial Electronics*, Vol. 61, No. 7, pp. 3502-3510, July, 2014.
[35] T. Bernardes, V. F. Montagner, H. A. Gründling, H. Pinheiro, Discrete-Time Sliding Mode Observer for Sensorless Vector Control of Permanent Magnet Synchronous Machine, *IEEE Transactions on Industrial Electronics*, Vol. 61, No. 4, pp. 1679-1691, April, 2014.
[36] S. Li, H. Du, X. Yu, Discrete-Time Terminal Sliding Mode Control Systems Based on Euler's Discretization, *IEEE Transactions on Automatic Control*, Vol. 59, No. 2, pp. 546-552, February, 2014.
[37] A. A. Khandekar, G. M. Malwatkar, B. M. Patre, Discrete Sliding Mode Control for Robust Tracking of Higher Order Delay Time Systems with Experimental Application, *Isa Transactions*, Vol. 52, No. 1, pp. 36-44, January, 2013.
[38] X. Yu, C. Wu, F. Liu, L. Wu, Sliding Mode Control of Discrete-time Switched Systems with Time-delay, *Journal of the Franklin Institute*, Vol. 350, No. 1, pp. 19-33, February, 2013.
[39] J.-M. Chang, H.-C. Chao, J.-L. Chen, C.-F. Lai, An Efficient Service Discovery system for Dual-Stack Cloud File Service, *IEEE Systems Journal*, Vol. 6, No. 4, pp. 584-592, December, 2012.

Biographies

Ming Lyu received her Ph.D. degree in Control Theory and Control Engineering in 2007, all from Nanjing University of Science and Technology, Nanjing, China. She is currently an associate research fellow in the School of Automation, Nanjing University of Science and Technology, Nanjing, China.

Jie Zhang received his Ph.D. degree in Control Theory and Control Engineering in 2011, from Nanjing University of Science and Technology, Nanjing, China. He is currently an associate research fellow in the School of Automation, Nanjing University of Science and Technology, Nanjing, China.



Lei Liu is currently working toward the Ph.D. degree in control science and engineering at the School of Automation, Nanjing University of Science and Technology, Nanjing, China. His current research interests include stochastic filtering, nonlinear stochastic systems, multi-agent system, variable structure control and networked control systems.

Yuming Bo received his Ph.D. degree from the Nanjing University of Science and Technology, China. He worked as an assistant professor, associate professor and professor, respectively, in School of Automation in Nanjing University of Science and Technology, China.

Fengjun Xiao received his B.S. degree in Economics from the BeiHang University in 2009. He received his Master degree in Technology Policy in 2014. He has been a Doctoral Candidate under the supervision of Prof. Chengzhi Li and his current research interests include network security and emergency management.