Modeling Information Diffusion with the External Environment in Social Networks

Dingda Yang[1,4], Xiangwen Liao[2,4], Jingjing Wei[3,4], Guolong Chen[2,4], Xueqi Cheng[5]

[1] College of Physics and Information Engineering, Fuzhou University, China
[2] College of Mathematics and Computer Science, Fuzhou University, China
[3] College of Electronics and Information Science, Fujian Jiangxia University, China
[4] Fujian Provincial Key Laboratory of Network Computing and Intelligent Information Processing, Fuzhou University, China
[5] Institute of Computing Technology, Chinese Academy of Sciences, China
yangdingda20@163.com, liaoxw@fzu.edu.cn, weijj@fjjxu.edu.cn, cgl@fzu.edu.cn, cxq@ict.ac.cn

Abstract

This paper addresses the problem of modeling the information diffusion in social networks. We are motivated by the phenomenon that individuals can receive information from the external environment, which possibly causes the transition of individuals' states. Most of existing models are based on the assumption that information spread through social interactions between individuals in the interior network, but few attention is paid to the change of states due to the information obtaining from the external environment. This paper proposes an External Environment Involved Information Diffusion Model (EEIIDM) by integrating the effect of the external environment. Mechanisms of state transition are utilized to describe the information diffusion. Theoretical analyses of the model are carried out in both homogeneous and heterogeneous networks, demonstrating that all individuals are informed at the equilibrium when the external environment is introduced. Furthermore, simulation results show that the external environment makes the diffusion process less sensitive to the initial spreader.

Keywords: Social networks, Information diffusion, External environment

1 Introduction

Recently, information diffusion becomes a fundamental problem with the popularity of social networks. Information diffusion in social networks has a great impact on daily life. In presidential campaign, for example, electorates share their political beliefs and interact with others on politics [1]. Furthermore, understanding the mechanism of information diffusion can be utilized in the recognition and analysis of opinion leaders in public opinion [2]. Characterizing the process of the information diffusion that occurs in social networks is one of important issues [3]. This paper aims to model and analyze the process of information diffusion in social networks.

Researchers have modeled and analyzed information diffusion in social networks extensively. Numerous models have been improved in terms of propagation probability [4-5], diffusion process [6-7] and additional states [8-10]. Most of these models assume that information spread through social interactions between individuals in the interior network, ignoring the influence of the external environment. However, we analyzed the "Higgs Twitter" dataset (http://snap.stanford.edu/data/higgs-twitter.html) and found that only 61.38% retweets are retweeted from followees. The rest of retweets are possibly retweeted from the recommendation of Twitter system, the results returned by the search engine, hot topics on the Twitter, etc. These approaches can be regarded as the external environment. Recent studies have revealed that the external environment is an important source of information [11-14]. Different from these studies that focus on the detailed statistics and analyses of the effect of the external environment, we aim to develop a model to describe the process of the information diffusion with the external environment.

In this paper, an external environment involved information diffusion model (EEIIDM) is proposed to describe information diffusion in social networks, emphasizing the influence of the external environment. The process is governed by mechanisms of state transition. Then, the properties of the model are analyzed in homogeneous networks and heterogeneous networks respectively. The main conclusion reveals that all individuals are informed at equilibrium when the external environment is introduced. Theoretical analyses together with simulation results show that the external environment plays an important role in the information diffusion.

*Corresponding Author: Xiangwen Liao; E-mail: liaoxw@fzu.edu.cn
DOI: 10.3966/160792642019032002006

2 Related Work

Existing studies on modeling information diffusion can be roughly divided into two classes. The models in the first class are developed based on the topology of networks. The linear threshold model [15] and the independent cascade model [16] are the representative models. These models are usually utilized in the practical applications, such as maximizing the information diffusion [17-18], accelerating or restraining the spread of influence [19-21], etc. The other research direction focuses on the roles of individuals in the information diffusion. Researchers have proposed a variety of mathematical epidemic models according to the different diffusion processes, e.g., the SIR model and the SIS model [22]. There are also some works modeling the information diffusion utilizing evolutionary game theory [23-24]. These models are usually modeled to analyze the properties and phenomena of information diffusion.

The diffusion of information and the spreading of diseases are usually analyzed by using analogous models. Thus, epidemic models are utilized to analogize and formalize the information diffusion. Based on the epidemic models, information diffusion models have been investigated extensively from various aspects. A number of studies found that delicate factors can affect the propagation probability, such as social reinforcement [4], connective strength [5], value strength [25], and multiplex networks [26]. Moreover, researchers changed the diffusion process and made it more realistic by modifying state transition [6-7]. Besides, new states were proposed, such as exposed state [8], hibernating state [9] and known state [10]. Despite all these efforts, the influence of the external environment is still overlooked.

In many real-world social networks, individuals can receive information from the external environment, such mainstream media, search engines and recommendation engines. The external environment has an impact on the information diffusion. Experiments by Myers et al. indicated that 29% information is diffused due to the external environment [11]. Similarly, Li et al. have also measured that external environment affects nearly 50% to 70% of cascade nodes in Kaixin [13]. Ha et al. found that *BlogCast*, a high quality post on the main page, can cause information diffusion between the bloggers who have no explicit relationships [12]. Das et al. considered individuals' opinions are not only updated through interactions with neighbors, but also shaped by the external information sources [27]. Zhao et al. studied individuals' emotional transition in sentiment contagion influenced by the external information [28]. The most two relevant studies are [14] and [29]. However, the external influence in [14] is characterized by a fraction of media-agents, which play the same role with ordinary individuals. The latter study focused on the upper and lower bounds of spread time for specific networks. In this paper, we are concerned with the steady-state of the information diffusion.

3 External Environment Involved Information Diffusion Model

When a piece of information is put out, it may diffuse widely to a large extent of the network due to the interactions between individuals. However, few epidemic models involve the influence of the external environment. Motivated by the experiments results of [11], the influence of the external environment needs to be considered. Thus, EEIIDM is proposed to address the information diffusion not only through the word-of-mouth effect between the individuals, but also with the influence of the external environment.

We consider a population consisting of N individuals as a network where nodes are individuals and edges are relationships of individuals. Analogously to the standard SIR model, each individual is in one of three states. *Susceptible* (S) individual indicates that it has not received information. *Infected* (I) individual denotes that it has received the information and decided to diffuse the information. *Recovered* (R) individual stands for that it has received the information, but has no interest in diffusing it. Initially, only one individual is infected as an initial spreader, and others are susceptible. In the subsequent time steps, information cascades occur in the network by certain mechanisms. A concise block diagram that illustrates individual's state transition is represented in Figure 1. Solid lines in dashed box indicate the internal influence and dashed lines donate the influence of the external environment.

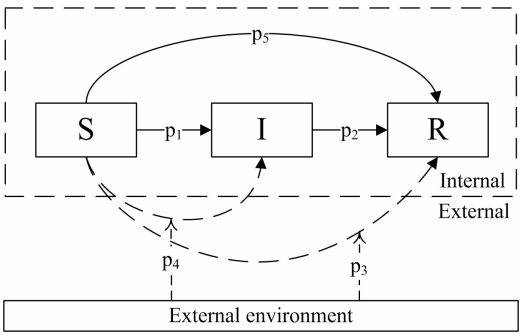


Figure 1. State transition diagram of the diffusion mechanisms

The mechanisms of EEIIDM can be summarized as follows. For a susceptible individual, he/she will become infected with probability p_1 when he/she contacts with an infected individual. But if he/she is not interested in diffusing information, he/she becomes a recovered individual with probability $(1-p_1)p_5$. At the same time, a part of individuals are influenced by

the external environment and receive external information in the current interaction. A susceptible individual in this group becomes infected with probability p_4. On the other hand, he/she may not be fond of it and becomes recovered with probability $(1-p_4)p_3$. For an infected individual, he/she loses diffusion ability and become recovered with probability p_2 spontaneously, namely recovering rate.

It is inappropriate that all individuals are influenced by the external environment simultaneously [14]. Therefore, we introduce a parameter $\theta \in [0,1]$ to take control of the proportion of individuals that the external environment affects within one time step. With the assumption that the external environment can affect a part of individuals during the process of information diffusion, susceptible individuals can be affected and become infected or recovered. Thus susceptible individuals will disappear gradually even if they do not have infected neighbors.

We illustrate a scenario in Figure 2. First of all, we focus on the information diffusion in the network. (a) Individual A creates a piece of news. B, C, D and E are susceptible at this time. (b) B reads the news but has no interest to diffuse it, so B becomes recovered. Meanwhile, C reads the news and diffuses it, so C becomes infected. (c) D receives the news from C and becomes infected state. (d) D ignores the news and enters the recovered state. E has no infected individuals in its neighbors, so that E cannot receive the information from other individuals in the network. The diffusion process terminates. (e) We assume three individuals can get the news from the external environment at this time step. Thus, E can become infected or recovered in current time.

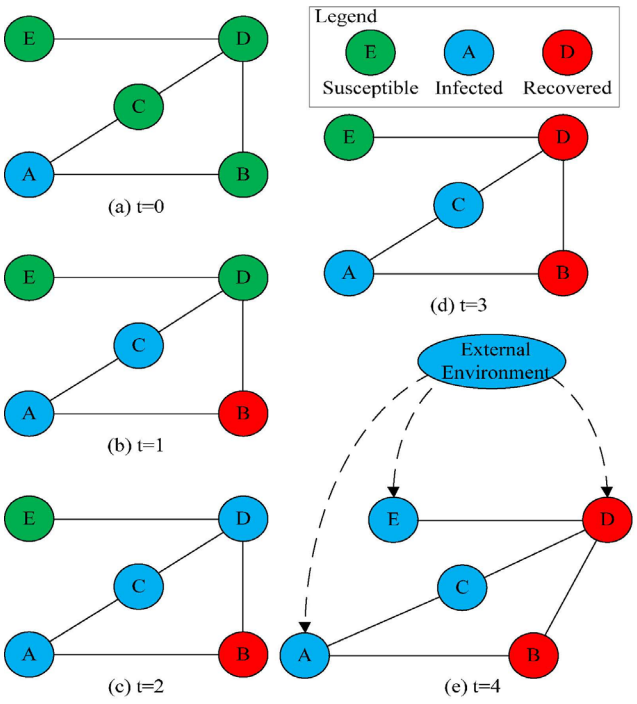


Figure 2. A diffusion example

4 Theoretical Analyses

In this section, the dynamical properties of the model are analyzed in homogeneous and heterogeneous networks respectively.

4.1 Steady-state Analyses in Homogeneous Networks

We characterize the individual degree with $\langle k \rangle$ for the sake of simplicity. We denote the densities of susceptible, infected and recovered individuals at time t as $S(t)$, $I(t)$ and $R(t)$, respectively. Using mean field methods, the model can be described as:

$$\begin{cases} \dfrac{dS(t)}{d(t)} = -p_1\langle k\rangle S(t)I(t) - ((1-p_1)p_3 + p_4)\theta S(t) \\ \qquad\qquad -(1-p_1)p_5\langle k\rangle S(t)I(t), & (a) \\ \dfrac{dI(t)}{d(t)} = p_1\langle k\rangle S(t)I(t) + p_4\theta S(t) - p_2 I(t), & (b) \\ \dfrac{dR(t)}{d(t)} = p_2 I(t) + (1-p_4)p_3\theta S(t) \\ \qquad\qquad +(1-p_1)p_5\langle k\rangle S(t)I(t). & (c) \end{cases} \quad \textbf{(1)}$$

The condition $S(t)+I(t)+R(t)=1$ must be satisfied. The system can be determined by

$$\begin{cases} \dfrac{dS(t)}{d(t)} = -p_1\langle k\rangle S(t)I(t) - ((1-p_4)p_3 + p_4)\theta S(t) \\ \qquad\qquad -(1-p_1)p_5\langle k\rangle S(t)I(t), & (a) \\ \dfrac{dI(t)}{d(t)} = p_1\langle k\rangle S(t)I(t) + p_4\theta S(t) - p_2 I(t). & (b) \end{cases} \quad \textbf{(2)}$$

We try to find the equilibrium of (2). The system becomes steady-state at last, namely $dS(t)/d(t)=0$, $dI(t)/d(t)=0$. By solving them, we can get the only existing situation $S(t)=I(t)=0$. So the equilibrium of system (2) is $P(0,0)$.

Theorem 1. The equilibrium $P(0,0)$ is local stable.

Proof. Let $X = -p_1\langle k\rangle S(t)I(t) - ((1-p_4)p_3 + p_4)\theta S(t) - (1-p_1)p_5\langle k\rangle S(t)I(t)$, $Y = p_1\langle k\rangle S(t)I(t) + p_4\theta S(t) - p_2 I(t)$, then, $\partial X/\partial S = -p_1\langle k\rangle I(t) - ((1-p_4)p_3 + p_4)\theta - (1-p_1)p_5\langle k\rangle I(t)$, $\partial X/\partial I = -p_1\langle k\rangle S(t) - (1-p_1)p_5\langle k\rangle S(t)$, $\partial Y/\partial S = p_1\langle k\rangle I(t) + p_4\theta$, $\partial Y/\partial I = p_1\langle k\rangle S(t) - p_2$. The Jacobian matrix of the equilibrium is

$$J(P) = J(0,0) = \begin{bmatrix} -((1-p_4)p_3 + p_4)\theta & 0 \\ p_4\theta & -p_2 \end{bmatrix}. \quad \textbf{(3)}$$

The roots of the characteristic polynomial are

$-\left((1-p_4)p_3+p_4\right)\theta$ and $-p_2$. Both of them have negative real parts. So the system has local stability at point $P(0,0)$ according to the Routh-Hurwitz stability criterion.

Theorem 2. The equilibrium $P(0,0)$ is globally asymptotically stable.

Proof. First, we make up a positive definite Lyapunov function $V(t)=S(t)+I(t)$, and

$$\begin{aligned}\frac{dV}{dt}&=\frac{dS(t)}{dt}+\frac{dI(t)}{dt}\\&=-p_1\langle k\rangle S(t)I(t)-\left((1-p_4)p_3+p_4\right)\theta S(t)\\&\quad-(1-p_1)p_5\langle k\rangle S(t)I(t)+p_1\langle k\rangle S(t)I(t)\\&\quad+p_4\theta S(t)-p_2I(t)\\&=-(1-p_4)p_3\theta S(t)-(1-p_1)p_5\langle k\rangle S(t)I(t)\\&\quad-p_2I(t)\\&\le 0.\end{aligned}\tag{4}$$

$V'(t)$ is less than or equal to 0, so the system has global asymptotic stability according to Lyapunov stability theory.

On the other hand, when we remove the influence of the external environment, susceptible individuals may not become extinct. The proof is omitted for brevity.

4.2 Final State Analyses in Heterogeneous Networks

We focus on the densities of different states at the stationary state. We classify individuals into $M+1$ categories according to their degrees, where M is the maximal degree. We denote the densities of susceptible, infected and recovered individuals with degree k at time t as $S(k,t)$, $I(k,t)$ and $R(k,t)$, respectively. The probability that a susceptible individual with degree k becomes infected in the time interval $[t,t+\Delta t]$ influenced by infected neighbors or the external environment is given by $P_{SI}'(k,t)$ or $P_{SI}''(k,t)$, respectively. $P(k'|k)$ means the conditional probability of a randomly chosen edge emanating from a node of degree k to a node of degree k', let $\Theta(k,t)=\sum_{k'}I(k',t)P(k'|k)$ denote the probability that an edge of a node of degree k points to an infected node at time t. Therefore, the probability that this susceptible individual becomes infected during $[t,t+\Delta t]$ after it meets an infected neighbor is $p_1\Delta t\Theta(k,t)$. Considering the individual has k neighbors, the probability $P_{SI}'(k,t)$ is given by

$$P_{SI}'(k,t)=1-\left[1-p_1\Delta t\Theta(k,t)\right]^k.\tag{5}$$

In the limit $\Delta t\to 0$, it is obvious that

$$P_{SI}'(k,t)=p_1k\Delta t\Theta(k,t).\tag{6}$$

The probability that an infected individual with degree k becomes recovered in the time interval $[t,t+\Delta t]$ is given by $P_{IR}'(k,t)$. Susceptible individuals can receive the external information and become infected with probability p_4. Infected individuals will stop diffusing and become recovered with probability p_2, thus

$$P_{SI}''(k,t)=p_4\Delta t,\tag{7}$$

$$P_{IR}'(k,t)=p_2\Delta t.\tag{8}$$

$I(k,t)$ decreases as infected individuals become recovered, and increases as susceptible individuals become infected influenced by infected neighbors, and increase as a part of individuals become infected influenced by the external environment. Therefore $I(k,t)$ changes as

$$\begin{aligned}I(k,t+\Delta t)=I(k,t)+S(k,t)P_{SI}'(k,t)\\+\theta S(k,t)P_{SI}''(k,t)-I(k,t)P_{IR}'(k,t).\end{aligned}\tag{9}$$

The transition rate of $I(k,t)$ is given by

$$\begin{aligned}\frac{\partial I(k,t)}{\partial t}&=\lim_{\Delta t\to 0}\frac{I(k,t+\Delta t)-I(k,t)}{\Delta t}\\&=ptkp_1S(k,t)\Theta(k,t)+p_4\theta S(k,t)-p_2I(k,t).\end{aligned}\tag{10}$$

Similarly, we can get the corresponding change rate of $S(k,t)$ and $R(k,t)$. So the nonlinear dynamical equations in network with arbitrary k degree correlations are

$$\begin{cases}\dfrac{\partial S(k,t)}{\partial(t)}=-kp_1S(k,t)\Theta(k,t)-((1-p_4)p_3+p_4)\theta S(k,t)\\\qquad\qquad-k(1-p_1)p_5S(k,t)\Theta(k,t), & (a)\\[2ex]\dfrac{\partial I(k,t)}{\partial(t)}=kp_1S(k,t)\Theta(k,t)+p_4\theta S(k,t)-p_2I(k,t), & (b)\\[2ex]\dfrac{\partial R(k,t)}{\partial(t)}=p_2I(k,t)+(1-p_4)p_3\theta S(k,t)\\\qquad\qquad+k(1-p_1)p_5S(k,t)\Theta(k,t). & (c)\end{cases}\tag{11}$$

Theorem 3. The final scale of recovered individuals is $R(k,\infty)=1$ with the influence of the external environment.

Proof. In the initial stages, there is only one infected node, so we assume $S(k,0)=1$. Integrating both sides of (11)(a), after some elementary manipulations we can get

$$\begin{aligned}S(k,t)&=S(k,0)\cdot e^{-\int_0^t k(p_1+(1-p_1)p_5)\Theta(k,t)dt}\cdot e^{-\int_0^t((1-p_4)p_3+p_4)\theta dt}\\&=e^{-\int_0^t k(p_1+(1-p_1)p_5)\Theta(k,t)dt}\cdot e^{-((1-p_4)p_3+p_4)\theta t}.\end{aligned}\tag{12}$$

When the system tends to the stationary state, the definite integral $\int_0^t k(p_1+(1-p_1)p_5)\Theta(k,t)dt$ is a nonnegative number, so the first term of (12) is less than 1. The second term $e^{-((1-p_4)p_3+p_4)\theta t}$ tends to 0 at last. $S(k,t)$ is the product term of them, its value tends to 0 at the stationary state, that is to say $S(k,\infty)=0$.

From the perspective of infected nodes, $\partial I(k,t)/\partial(t)$ is equal to 0 when t is large enough. We can get from (11)(b) that $I(k,\infty)=0$, in other words, all the nodes in the network are recovered nodes in the end, i.e., $R(k,\infty)=1$.

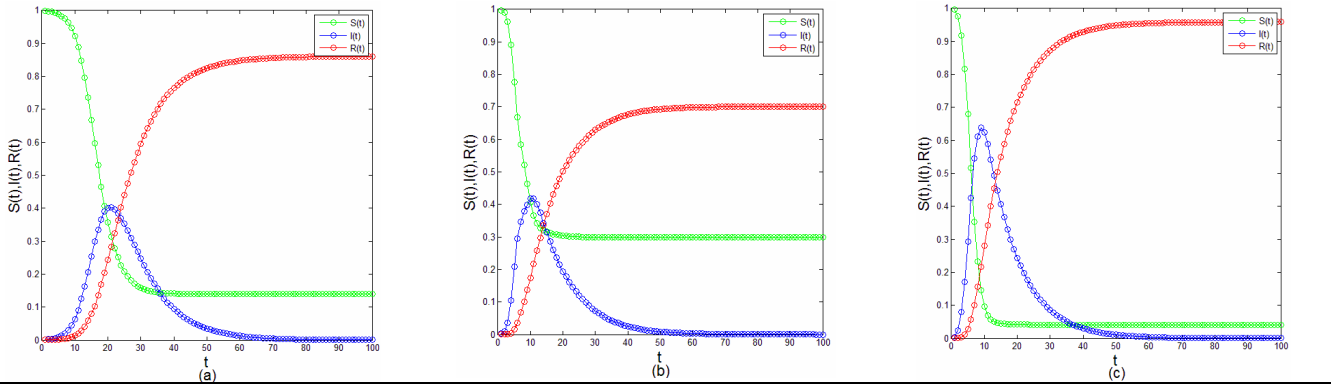
On the other hand, without the influence of the external environment, e.g., $p_3=p_4=0.0$, we can get the function $S(k,t)=e^{-\int_0^t k(p_1+(1-p_1)p_5)\Theta(k,t)dt}$, so $S(k,\infty)\neq 0$.

5 Experiments

In this section, we investigate the properties of EEIIDM on synthetic small-world and scale-free networks, and real-world Facebook network (http://snap.stanford.edu/data/egonets-Facebook.html). The small-world network has 1000 nodes, and the number of linked neighbors is 2 and replacement probability is 0.5. The scale-free network has 1000 nodes and the average degree is 4. The Facebook network has 4039 nodes and 88234 edges. We focus on the qualitative analysis, so the parameters of the model are set empirically as: $p_1=0.20$, $p_2=0.10$, $p_3=0.05$, $p_4=0.10$, $p_5=0.10$, $\theta=0.50$ and the number of time steps is 100. Time step is increased by 1 after N individuals update their states. We conduct each experiment 1000 times and take the average to obtain approximate results.

5.1 The Comparison of EEIIDM and State-of-the Art

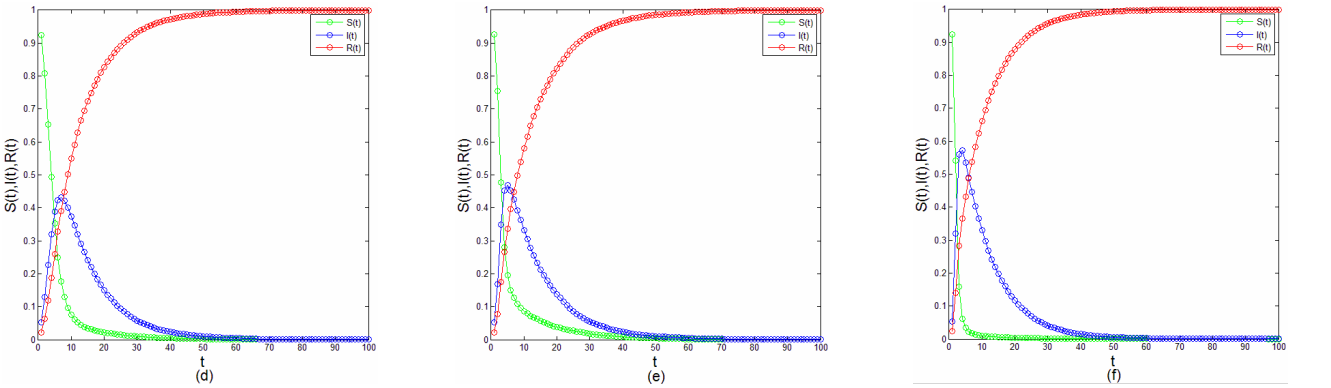
The model (referred to Basic model) proposed in [14] is the most relevant model in modeling the external influence. It is also the state-of-the-art to the best of our knowledge. The results of comparison bewteen Basic model and EEIIDM are shown in Figure 3.



(a) are the results of Basic model in small-world, scale-free and Facebook networks

(b) are the results of Basic model in small-world, scale-free and Facebook networks

(c) are the results of Basic model in small-world, scale-free and Facebook networks



(d) are the results of EEIIDM in small-world, scale-free and Facebook networks

(e) are the results of EEIIDM in small-world, scale-free and Facebook networks

(f) are the results of EEIIDM in small-world, scale-free and Facebook networks

Figure 3. The dynamics of $S(t)$, $I(t)$ and $R(t)$

The general trends of densities are similar in two models. The density of susceptible individuals diminishes continuously until it reaches the stability. The density of infected individuals grows rapidly at the beginning, but decreases rapidly after reaching a peak until it reaches the balance. The density of recovered individuals goes up quickly at first and thereafter declines until reaching the balance.

However, there is a remarkable difference between Basic model and EEIIDM. For Basic model, many individuals are susceptible at the end of diffusion, which implies that many individuals have not received the information. On the contrary, the density of susceptible individuals approximates to 0 and the density of recovered individuals approximates to 1 ultimately, which means that almost all individuals are informed. This phenomenon is consistent with the theoretical analyses. In the situation that the external environment can affect individuals constantly, individuals can reach the information easily. In a word, our proposed model is more reasonable.

5.2 The Influence of Different Initial Spreader

In this section, the influence of different initial spreaders on information diffusion is investigated in Facebook network. Influential spreaders are important in network analysis [30]. Spreaders with more influence lead to spreading faster and wider in networks. Therefore, we choose different influential node as initial spreaders, verifying whether initial spreader affects the process of information diffusion. The degree centrality depicts the node from its local connectivity. The PageRank centrality takes advantage of the global information. The k-core centrality identifies the nodes' position from the core and periphery of the network [31]. They are utilized to reflect the influence of a node from a different perspective and the results are shown in Figure 4(a), Figure 4(b), Figure 4(c).

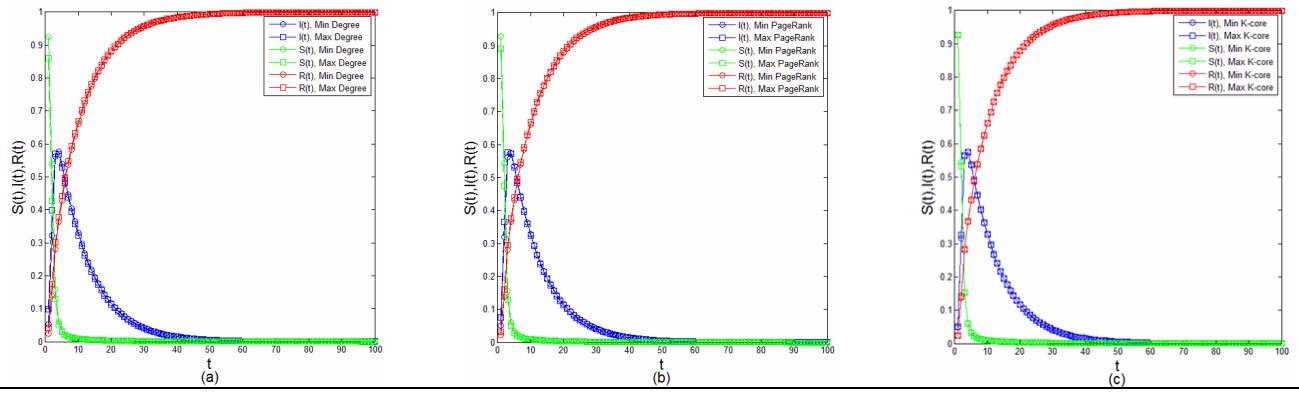


Figure 4. The dynamics of $S(t)$, $I(t)$ and $R(t)$ with the external influence for different initial spreaders

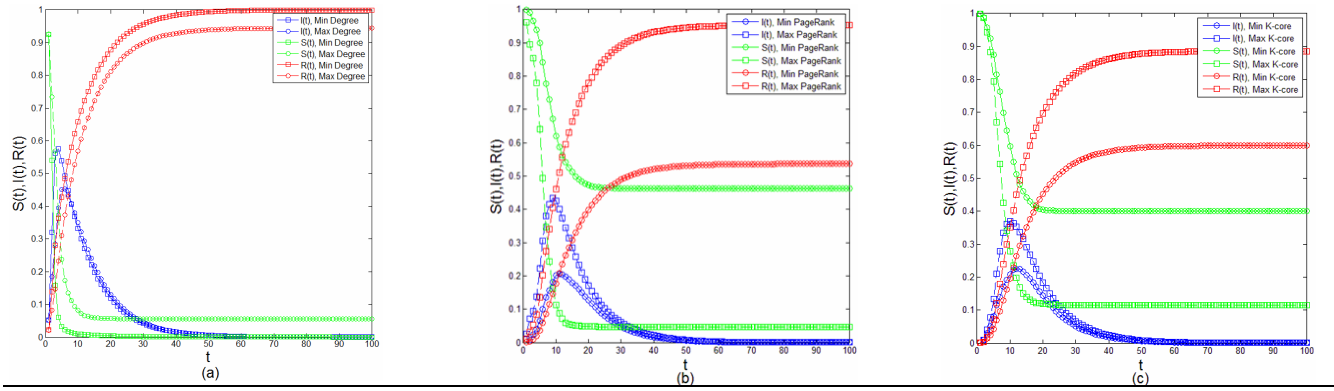


Figure 5. The dynamics of $S(t)$, $I(t)$ and $R(t)$ without the external influence for different initial spreaders

With the influence of the external environment, the densities of recovered individuals tend to 1.0 in the end. However, different initial spreader has little effect on the process of information diffusion. This result seems to contradict with previous studies that an initial spreader with higher centrality index leads to spreading faster and wider in closed world [32].

We perform the experiment again without the external environment. The results are shown in Figure 5. We find that comparing to minimum centrality, if the initial spreader has the maximum centrality, the number of infected and recovered increases faster in the beginning and the density is also greater at the final steady-state. The results also indicate the significant effect of the initial spreader: a higher centrality of initial spreader leads to a higher peak of the information diffusion. This expected result reflects the reality that if information is from an influential spreader in a closed world, information diffuses faster and wider when the impact of different influential

spreaders is small in an open environment.

Furthermore, we also investigate the required time for equilibrium with (Case 1) or without (Case 2) the external environment. Table 1 shows the average duration of the diffusion process with different initial spreaders. With the external environment, duration of the diffusion process has little difference. It further verifies that the external environment makes the diffusion insensitive to the initial spreader. On the other hand, duration is less in the conditions that initial spreader is set on minimum centrality node without the external environment. Influential initial spreader leads to wider spreading, so it gives a persistent and lively discussion in the network. There is a remarkable difference between two cases when initial spreader is set on minimum centrality. But the duration has little difference for the initial spreader with maximum centrality except Degree. This indicates the external environment promotes diffusion process for non-influence spreader.

Table1. The average duration for different initial spreaders in Case 1 and Case 2

Centrality	Degree		PageRank		K-core	
	Min	Max	Min	Max	Min	Max
Case 1	84.93	82.90	83.70	80.07	86.41	86.97
Case 2	84.25	84.93	52.54	83.36	57.33	80.09

5.3 The Influence of the External Environment

We further investigate the influence of the external environment in Facebook network. Extensive solutions are carried out by varying the parameters of the external environment. When θ is fixed, individuals who can receive external information are influenced by p_3 and p_4 together. To understand the influence of the external environment, we increase these parameters with the same ratios. Without loss of generality, we make p_4 twice as large as p_3 and let p_3 range from 0.00 to 0.20. In a special case, i.e., $p_3 = 0.00$, $p_4 = 0.00$, there is no consideration of the external environment, the model becomes the SIR model with internal recovering rate. The other parameters are the same as above.

The solid-dot line in Figure 6 is different from any other cases that there are still some susceptible individuals in the network when they reach the stability. We checked all these individuals and found that their neighbors are all susceptible or recovered individuals, so they can never be infected in a closed world. In addition, the diffusion process with strong influence of the external environment proceeds faster than that with weak influence of the external environment. But regardless of the degree of external influence, the density of recovered individuals tends to one gradually. As a consequence, there is no doubt that the external environment can influence the information diffusion at some degree.

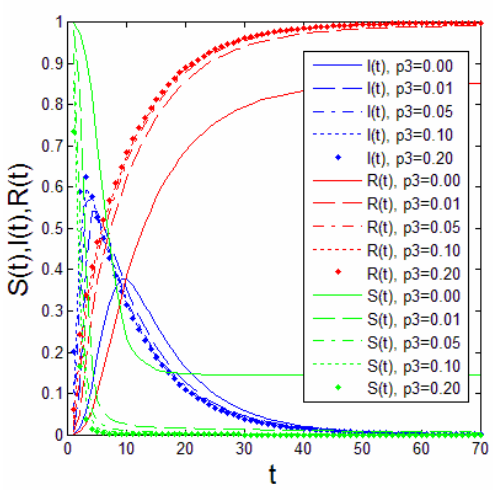


Figure 6. The dynamics of $S(t)$, $I(t)$ and $R(t)$ for different values of p_3

No matter how the initial state is, susceptible and infected individuals will disappear gradually, and almost all the individuals in the network turn into recovered with the influence of the external environment. Diminishing influence of interior network and the external environment is conducive to the inhibition of network information. In general, the simulation results are congruent with the theoretical analysis in Section 4.

Figure 7 shows the maximum density of infected individuals $I_{\max}$ with different θ from 0.0 to 1.0 during the diffusion process. $I_{\max}$ increases smoothly with the increasing of θ from 0.1 to 1.0. This phenomenon accords to common sense that the more individuals are influenced by the external environment in each time step, the more individuals take part in diffusing. But $I_{\max}$ has a dramatic increment between $\theta = 0.0$ and $\theta = 0.1$. The inset of the figure is the local feature when θ is from 0.0 to 0.1 with interval of 0.01. $I_{\max}$ has a distinct difference in the cases weather θ is zero or not. $\theta = 0.0$ means that none individual is influenced by the external environment. As a consequence, the external environment plays a significant role in diffusion.

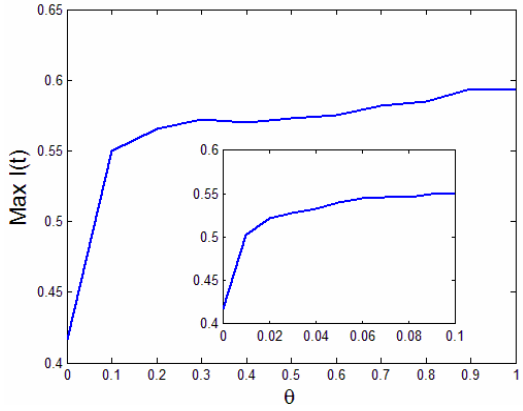


Figure 7. $I_{\max}$ with different θ

6 Conclusion

In this paper, we proposed EEIIDM model, in which individuals can receive information via their neighbors and the external environment. In the model, interactions of information are preceded according to mechanisms of state transition. The theoretical analyses of the model are carried out in both homogeneous and heterogeneous networks. The analytical and the simulations results show that EEIIDM is less sensitive to the initial spreader because of the influence of the external environment. An initial spreader with higher centrality does not result in a faster and wider diffusion any more. We also investigate difference influence of the external environment. The results indicate that the increasing of the external influence leads to promotion of the diffusion process. However, susceptible and infected individuals die out when the system reaches the stability, regardless of the initial conditions and degrees of influence.

This work may shed new light on understanding the mechanism of informtion diffusion in social networks. We will explore more features, e.g., social reinforcement [4], social trust [33].

Acknowledgments

This research project was supported by the National Natural Science Foundation of China (Nos. 61772135, U1605251), the Open Project of Key Laboratory of Network Data Science & Technology of Chinese Academy of Sciences (Nos. CASNDST201606, CASNDST201708) and the Directors Project Fund of Key Laboratory of Trustworthy Distributed Computing and Service (BUPT)Ministry of Education(No. 2017KF01).

References

[1] S. Wattal, D. Schuff, M. Mandviwalla, C. B. Williams, Web 2.0 and Politics: The 2008 U.S. Presidential Election and an E-Politics Research Agenda, *Mis Quarterly*, Vol. 34, No. 4, pp. 669-688, December, 2010.

[2] A. Ellero, G. Fasano, A. Sorato, Stochastic Model of Agent Interaction with Opinion Leaders, *Physical Review E*, Vol. 87, No. 4, pp. 042806, April, 2013.

[3] P. S. Van Eck, W. Jager, P. S. Leeflang, Opinion Leaders' Role in Innovation Diffusion: A Simulation Study, *Journal of Product Innovation Management*, Vol. 28, No. 2, pp. 187-203, February, 2011.

[4] L. Lü, D.-B. Chen, T. Zhou, The Small World Yields the Most Effective Information Spreading, *New Journal of Physics*, Vol. 13, No. 12, pp. 123005, December, 2011.

[5] F. Roshani, Y. Naimi, Effects of Degree-biased Transmission Rate and Nonlinear Infectivity on Rumor Spreading in Complex Social Networks, *Physical Review E*, Vol. 85, No. 3, pp. 036109, March, 2012.

[6] B. Zhang, X. Guan, M. J. Khan, Y. Zhou, A Time-varying Propagation Model of Hot Topic on BBS Sites and Blog Networks, *Information Sciences*, Vol. 187, pp. 15-32, March, 2012.

[7] H.-F. Zhang, F. Xiong, Y. Liu, H.-C. Chao, Modeling and Analysis of Information Dissemination in Online Social Networks, *Journal of Internet Technology*, Vol. 16, No. 1, pp. 1-10, January, 2015.

[8] A. Abta, A. Kaddar, H. T. Alaoui, Global Stability for Delay SIR and SEIR Epidemic Models with Saturated Incidence Rates, *Electronic Journal of Differential Equations*, Vol. 2012, No. 23, pp. 1-13, February, 2012.

[9] L. Zhao, J. Wang, Y. Chen, Q. Wang, J. Cheng, H. Cui, SIHR Rumor Spreading Model in Social Networks, *Physica A: Statistical Mechanics and its Applications*, Vol. 391, No. 7, pp. 2444-2453, April, 2012.

[10] D. Yang, X. Liao, H. Shen, X. Cheng, G. Chen, Modeling the Reemergence of Information Diffusion in Social Network, *Physica A: Statistical Mechanics and its Applications*, Vol. 490, pp. 1493-1500, January, 2018.

[11] S. A. Myers, C. Zhu, J. Leskovec, Information Diffusion and External Influence in Networks, *Proceedings of the 18th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, Beijing, China, 2012, pp. 33-41.

[12] J. Ha, S.-W. Kim, C. Faloutsos, S. Park, An Analysis on Information Diffusion through BlogCast in a Blogosphere, *Information Sciences*, Vol. 290, pp. 45-62, January, 2015.

[13] J. Li, J. Xiong, X. Wang, Measuring the External Influence in Information Diffusion, *2015 16th IEEE International Conference on Mobile Data Management*, Pittsburgh, PA, 2015, pp. 92-97.

[14] C. Liu, X.-X. Zhan, Z.-K. Zhang, G.-Q. Sun, P. M. Hui, How Events Determine Spreading Patterns: Information Transmission via Internal and External Influences on Social Networks, *New Journal of Physics*, Vol. 17, No. 11, pp. 113045, November, 2015.

[15] D. J. Watts, A Simple Model of Global Cascades on Random Networks, *Proceedings of the National Academy of Sciences*, Vol. 99, No. 9, pp. 5766-5771, April, 2002.

[16] J. Goldenberg, B. Libai, E. Muller, Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-mouth, *Marketing Letters*, Vol. 12, No. 3, pp. 211-223, August, 2001.

[17] D. Kempe, J. Kleinberg, É. Tardos, Maximizing the Spread of Influence through a Social Network, *Proceedings of the ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, Washington, DC, 2003, pp. 137-146.

[18] K. Kandhway, J. Kuri, Using Node Centrality and Optimal Control to Maximize Information Diffusion in Social Networks, *IEEE Transactions on Systems Man & Cybernetics Systems*, Vol. 47, No. 7, pp. 1099-1110, July, 2017.

[19] F. Xiong, Z. Y. Li, Effective Methods of Restraining Diffusion in Terms of Epidemic Dynamics, *Scientific Reports*, Vol. 7, No. 1, pp. July, 2017.

[20] Z. Lu, Y. Wen, G. Cao, Information Diffusion in Mobile Social Networks: The Speed Perspective, *IEEE INFOCOM 2014 - IEEE Conference on Computer Communications*, Toronto, ON, Canada, 2014, pp. 1932-1940.

[21] J. Wang, C. Jiang, Z. Han, T. Q. S. Quek, Y. Ren, Private Information Diffusion Control in Cyber Physical Systems: A Game Theory Perspective, *2017 26th International Conference on Computer Communication and Networks (ICCCN)*, Vancouver, BC, Canada, 2017, pp. 1-10.

[22] W. O. Kermack, A. G. McKendrick, A Contribution to the Mathematical Theory of Epidemics, *Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences*, Vol. 115, No. 772, pp. 700-721, October, 1927.

[23] C. Jiang, Y. Chen, K. J. R. Liu, Evolutionary Dynamics of Information Diffusion Over Social Networks, *IEEE Transactions on Signal Processing*, Vol. 62, No. 17, pp. 4573-4586, September, 2014.

[24] C. Jiang, Y. Chen, K. J. R. Liu, Graphical Evolutionary Game for Information Diffusion Over Social Networks, *IEEE Journal of Selected Topics in Signal Processing*, Vol. 8, No. 4, pp. 524-536, August, 2014.

[25] J. Wang, C. Jiang, T. Q. S. Quek, X. Wang, Y. Ren, The Value Strength Aided Information Diffusion in Socially-Aware Mobile Networks, *IEEE Access*, Vol. 4, pp. 3907-3919, August, 2016.

[26] S. Gomez, A. Diaz-Guilera, J. Gomez-Gardenes, C. J. Perez-Vicente, Y. Moreno, A. Arenas, Diffusion Dynamics on Multiplex Networks, *Physical Review Letters*, Vol. 110, No. 2, pp. 028701, January, 2013.

[27] R. Das, J. Kamruzzaman, G. Karmakar, Consistency Driven Opinion Formation Modelling in Presence of External Sources, *2015 International Joint Conference on Neural Networks (IJCNN)*, Killarney, Ireland, 2015, pp. 1-8.

[28] L. Zhao, J. Wang, R. Huang, H. Cui, X. Qiu, X. Wang, Sentiment Contagion in Complex Networks, *Physica A: Statistical Mechanics and Its Applications*, Vol. 394, pp. 17-23, January, 2014.

[29] S. Banerjee, A. Gopalan, A. K. Das, S. Shakkottai, Epidemic Spreading with External Agents, *IEEE Transactions on Information Theory*, Vol. 60, No. 7, pp. 4125-4138, July, 2014.

[30] D. Chen, L. Lü, M.-S. Shang, Y.-C. Zhang, T. Zhou, Identifying Influential Nodes in Complex Networks, *Physica A: Statistical Mechanics and Its Applications*, Vol. 391, No. 4, pp. 1777-1787, February, 2012.

[31] M. Kitsak, L. K. Gallos, S. Havlin, F. Liljeros, L. Muchnik, H. E. Stanley, H. A. Makse, Identification of Influential Spreaders in Complex Networks, *Nature Physics*, Vol. 6, No. 11, pp. 888-893, November, 2010.

[32] T. Zhu, B. Wang, B. Wu, C. Zhu, Maximizing the Spread of Influence Ranking in Social Networks, *Information Sciences*, Vol. 278, pp. 535-544, September, 2014.

[33] F. Xiong, Y. Liu, J. Cheng, Modeling and Predicting Opinion Formation with Trust Propagation in Online Social Networks, *Communications in Nonlinear Science & Numerical Simulation*, Vol. 44, pp. 513-524, March, 2017.

Biographies

Dingda Yang is currently a Ph.D. candidate at Fuzhou University. His research interests include social networks, data mining and information diffusion.

Xiangwen Liao is currently an Associated Professor at Fuzhou University. He received his Ph.D. degree from the Institute of Computer Sciences, CAS in 2009. His research interests include natural language processing, information retrieval, and social networks.

Jingjing Wei is currently a lecturer at Fujian Jiangxia University. She received her Ph.D. degree from Fuzhou University in 2017. Her research interests include natural language processing, information retrieval, and social networks.

Guolong Chen is currently a Professor at Fuzhou University. He received his Ph.D. degree from Xi'an Jiaotong University in 2002. His main research interests include computing intelligence, big data analysis, and very large scale integration.

Xueqi Cheng is currently a Professor at the Institute of Computing Technology, CAS. He received his Ph.D. degree from the Institute of Computing Technology, CAS in 2006. His main research interests include bid data analysis, web search, and data mining.