Reliability Model and Algorithms of High-Proportion Nodes in Wireless Sensor Networks

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Abstract

It typically identifies the working and state capabilities of a wireless sensor network by assessing the ratio of reliable sensor nodes and the size of the diameter in a wireless sensor network, reflecting local performance in wireless sensor networks. In this paper, a mathematical model of network reliability with diameter constraint D and node-proportion constraint λ is proposed to meet the performance-evaluation requirements of WSN. The computation of the proposed reliability can be simplified by removing irrelevant subgraphs and irrelevant spanning trees. In particular, a subgraph is irrelevant if it contains two nodes whose distance is greater than D. Thus, a reduction algorithm is designed based on the irrelevant subgraphs. The proposed algorithm is improved by removing irrelevant subgraphs and irrelevant trees. The example illustrates that 27 subgraphs are deleted, and 72.97% subgraphs are not included in the computation of the reliability. More examples are carried out to verify the conclusion and show the effectiveness and efficiency of the algorithm.

Keywords: Network reliability, Spanning tree, Diameter constraint, Subgraph, Quality of service

1 Introduction

Wireless sensor networks (WSN) are spatially distributed autonomous sensors to monitor physical or environmental conditions, such as temperature, sound, pressure, etc. and to cooperatively pass their data through the network to a main location. Network reliability is an important factor to evaluate network performance and has been utilized in many real-world applications such as cyber-physical system [1], data center networks [1], wireless networks [2-4], etc.. Especially, for WSN, F. Engmann et al [5]. proposed an algorithm to minimize the energy consumed by sensor nodes communicating over multi-hop links to improve WSN reliability.

As to traditional reliability models and algorithm, M. Ashraf and R. Mishra considered all-terminal reliability, i.e. the probability that every pair of nodes can communicate with each other [6]. Another method called inclusion-exclusion principle was showed in [7], where E_1, E_2, …, E_n are independent events and the probability that at least one event in E_1, E_2, …, E_n occur is calculated by Eq. (1):

$$\begin{aligned}&\Pr(E_1 \cup E_2 \cup \cdots E_n)\\&= \Sigma_{1\le i\le n} P(E_i) - \Sigma_{1\le i<j\le n} P(E_i E_j) + \cdots + (-1)^{n-1}\\&\quad - P(E_1 E_2 \cdots E_n)\end{aligned} \tag{1}$$

Disjoint products was used to compute 2-terminal reliability [8-9], namely the probability that there exists at least one operational path between source node and terminal node,

$$\begin{aligned}&\Pr(E_1 \cup E_2 \cup \cdots E_n)\\&= \Pr(E_1) + \Pr(\overline{E_1} E_2 + \cdots + (\overline{E_1}\,\overline{E_2} \cdots E_n) E_2)\end{aligned} \tag{2}$$

where $E_i \cap \overline{E_l} = \phi$. Factoring theorem (see Eq. (3)) was introduced to calculate k-terminal reliability [9], defined as the probability that there is at least one operational path between any pair of given k nodes.

$$R(G) = pR(G*e) + (1-p)R(G-e) \tag{3}$$

where e is an edge of the graph, p is the operational probability of e.

Diameter constraint, introduced by Petingi [10], restricts that lengths of all paths in a network are not greater than a given integer D and is effective for the consideration of reliability model. Petingi proposed a polynomial-time topological reduction algorithm to detect and delete irrelevant edges in diameter constrained source-to-terminal reliability [11]. The computation complexity of network reliability with diameter constraint is NP – hard [12]. Moreover, for k-terminal network reliability with diameter constraint, H. Cancela et al. used Monte Carlo method [13]. Further, computing diameter constrained k-terminal reliability is NP-hard if $k \ge 2$ and $D \ge 3$ [14]. On the other hand, E. Canale et al. proved that diameter constrained k-terminal reliability problem can be solved in polynomial-time when D = 2 [15]. E. Canale also efficiently determined in the most basic and used case

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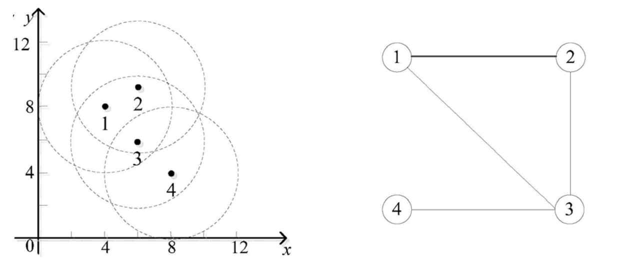
for source-to-terminal reliability model [16]. The model of network reliability for ensuring quality of service and proposed λ-SAT reliability was considered in [17]. R. Li and W. Dang investigated a new reliability parameter named source-to-k-out-of-N-terminal reliability (S(k/N)T reliability) for active network [18]. The S(k/N)T reliability describes the connection from a source node s to at least k random nodes of a terminal node set with N nodes in a network G for a given time period.

In WSN, there exists a topological structures according to the communication range of sensor nodes, which mentioned in [4]. There exists a C-link (u, v) if the distance of u and v satisfies $d(u, v) \leq R$, where R denotes the communication range of sensor nodes. E_c denotes the set of all C-links, and $G(V, E_c)$ is called the communication graph. For example, there are four nodes, and their geometry locations listed in Table 1, where (x, y) is the horizontal and vertical coordinates of node k in a plane, as illustrated in Figure 1(a). Assume that R = 4.0. Considering the distances between a pair of nodes in the WSN, Figure 1(b) is the corresponding communication network.

Table 1. Four Nodes In a Plane

N	1	2	3	4
(x, y)	(4.0, 8.0)	(6.0, 9.0)	(6.0, 6.0)	(8.0, 4.0)



(a) WSN: the real circle denotes the sensing area and the dashed denotes the communication range

(b) the communication graph

Figure 1.

For a given period, it could not be possible to keep all nodes working well in WSN. There are always several node failures in the WSN test, and its performance is actually determined by the percentage of the sensor nodes. On the other hand, it is necessary to analyze the reliability around a group of nodes nearby since too many hops will affect the efficiency of data transfer among these nodes. A problem is how to get better a method to evaluate WSN and ensure that these two aspects. Fortunately, the issues can be solved by refining reliability model. The mathematical model of network reliability with diameter constraint and node-proportion constraint is proposed to meet the performance-evaluation requirements of WSN. In order to simplify the computation of the proposed reliability, a reduction algorithm is designed to detect and remove irrelevant subgraphs. The examples show the drastic reduction in the number of subgraphs and spanning trees that need to be calculated.

This paper is organized as follows. Section 2 proposes the concept of reliability with diameter constraint and node-proportion constraint for WSN. To simplify the computation, section 3 gives propositions to identity irrelevant subgraphs and irrelevant trees. A reduction algorithm is proposed in section 4 and demonstrates experimental results. Section 5 highlights the contributions. Last section concludes the paper.

2 Reliability Model

2.1 System Model

A WSN is modeled by an undirected graph G (V, E), where $V = \{v_1, v_2, \cdots, v_n\}$ is the node set, $E = \{e_1, e_2, \cdots, e_m\}$ is the edge set, n is the number of nodes and m is the number of edges. We assume that nodes do not fail. Due to environment factors, channel interference, etc., each edge of the network is not reliable and assigned an independent probability of operation p. The states of the edges are supposed to be either operational state or failed state.

For convenience, three definitions in [21], [17] nd [22] are referred.

Definition 2.1 [21]: The probability that all nodes in a network G communicate to each other is called the network reliability of G, which is denoted as $R(G)$.

Follow the Definition 2.1, the reliability model was refined by inserting a percentage of sensor nodes:

Definition 2.2 [17]**:** In $G(V, E)$, the reliability with λ is the probability that at least $k = \lceil \lambda n \rceil$ ($\lceil * \rceil$ is the ceiling function) nodes communicate in n-node group, denoted as $R(G, \lambda)$.

From Definition 2.2, the larger λ is, the higher of the requirement will be. For example, $\lambda = 0.9$ means that at least 90% of nodes in a network communicate with each other without the information of functioning nodes. On the other hand, too many hops will affect the efficiency of data transfer among these nodes. The reliability model was mentioned as:

Definition 2.3 [22]**:** In $G(V, E)$, the reliability with D is the probability that between each pair of nodes, there exists a path consisting of operational edges whose number is upper bounded by a given integer D, denoted as $R(G, D)$.

Definition 2.3 shows local performance and reliability of some nodes that the constraint of the diameter of G is not greater than D. Further, Definition 2.2 and 2.3 do not consider both diameters and node-proportion in the meantime. To meet the requirement of node-proportion and local performance of WSN, the reliability model with diameter constraint and node-proportion constraint is considered:

Definition 2.4: In $G(V, E)$, the probability that at least $k(=\lceil \lambda n \rceil)$ nodes communicate in n-node group and the diameter of the subgraphs consisting of these k nodes is not greater than a given integer D is the reliability with λ and D, denoted as $R(G, \lambda, D)$.

To better understand Definition 2.4, four definitions are compared by the following example:

Example 2.1: in Figure 2, A network $G(V, E)$ with $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_1v_4, v_3v_4, v_2v_4\}$. Let $A(G)$ be the diameter of G and the operational probability for each edge be $p = 0.9$. And $\lambda = 0.7$ this is, $k = \lceil 0.7 \times 4 \rceil = 3$.

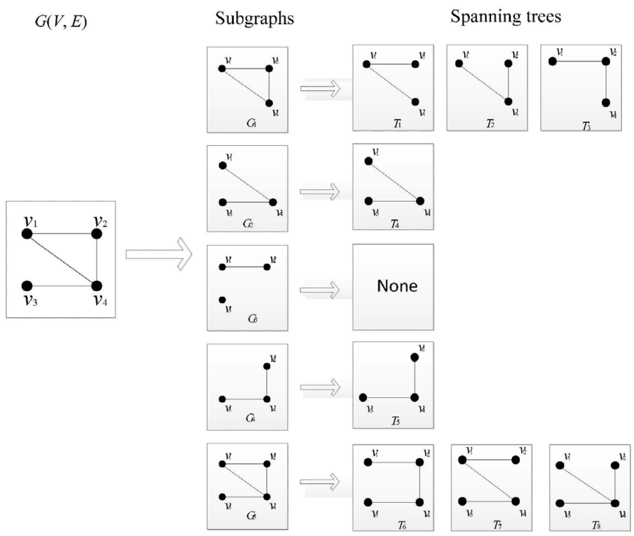


Figure 2. A graph $G(V, E)$ and its subgraphs and spanning trees

As to Definition 2.1, the spanning trees of G are T_6, T_7, T_8. The reliability can be computed by Eq. (1).

$$R(G) = \Pr(E_6 \cup E_7 \cup E_8) = 0.8748$$

And to Definition 2.2, $\binom{4}{3} + \binom{4}{4} = 5$ subgraphs and their corresponding spanning trees can be found. According to Eq. (1),

$$R(G, 0.7) = \Pr(E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7 \cup E_8) = 0.9882$$

For Definition 2.3, the spanning trees of G are T_6, T_7, T_8. If $D = 2$, $A(T_6) = A(T_7) = 3 > D$, $A(T_8) = 2 \leq D$. Hence,

$$R(G, 2) = P(E_8) = 0.9^3 = 0.729.$$

First, to Definition 2.4, 5 subgraphs G_1, G_2, G_3, G_4, G_5, and their corresponding spanning trees (see Figure 2) can be obtained according to λ. If $D = 2$, since $A(T_6) = A(T_7) = 3 > D$, T_6, T_7 should be deleted when computing $R(G, \lambda, D)$. Then

$$R(G, 0.7, 2) = \Pr(E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_8) = 0.9882$$

$R(G, 0.7)$ is bigger than $R(G)$ since the former compute the connectivity of 3 random nodes and 4 nodes while $R(G)$ just consider 4 nodes. $R(G, 2)$ is smaller than $R(G)$, because the length of paths in G is limited by 2. Meanwhile, $R(G, 0.7, 2)$ includes both factors.

2.2 The Computation of $R(G, \lambda, D)$ Subprogram 1.

To compute $R(G, \lambda, D)$, it is naturally to consider $\binom{n}{k} + \binom{n}{k+1} + \cdots + \binom{n}{n}$ subgraphs and their all-terminal network reliability.

To calculate the reliability of a graph with diameter constraint D and node-proportion constraint λ, the following algorithm is designed by searching subgraphs and spanning trees and deleting trees whose diameter is greater than D.

Algorithm 2.1. The reliability of $R(G, \lambda, D)$

Input: $G(V, E)$, λ, D, $k = \lceil \lambda n \rceil$

Output: $R(G, \lambda, D)$

Step 1: If $k < n$, go to step 2; else go to step 5;
Step 2: Find all subgraphs and delete disconnected subgraphs;
Step 3: Find the spanning trees of remaining subgraphs;
Step 4: Delete trees whose diameter is greater than D; $k=k+1$, go to step 1;
Step 5: Calculate $R(G, \lambda, D)$ by Eq. (1).

The following example is given by using algorithm 2.1.

Example 2.2. A network $G(V, E)$ in Figure 3, $V = \{v_1, v_2, \ldots, v_8\}$, $E = \{v_1v_2, v_1v_8, v_2v_3, v_2v_7, v_3v_4, v_3v_7, v_4v_5, v_4v_6, v_5v_6, v_6v_7, v_7v_8\}$, $D = 3$, $\lambda = 0.7$, $k = \lceil 0.7 \times 8 \rceil = 6$, the operational probability for each edge is $p = 0.9$.

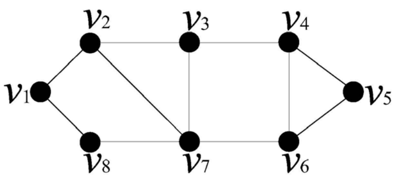


Figure 3. A graph $G(V, E)$ in Example 2.3

Step 2: $\binom{8}{6} + \binom{8}{7} + \binom{8}{8} = 37$ subgraphs are obtained and are shown in Appendix I Table 1.

Step 3: Delete disconnected subgraphs G_{18}, G_{19}, G_{112}, G_{113}, G_{116}, G_{117}, G_{123}.

Step 4: 456 spanning trees of remaining subgraphs are founded by the Matrix-Tree theorem [19].

Step 5: The relevant trees are shown as Figure 4. The events of trees are denoted as E_1, E_2, $\cdots$, E_{12}.

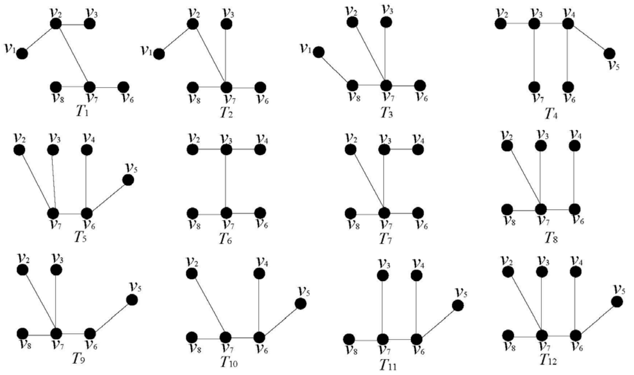


Figure 4. Spanning trees in Example 2.3 for $D \leq 3$

Step 6: $R(G, \lambda, D) = \Pr(E_1 \cup E_2 \cup \cdots \cup E_{12}) = 0.953936595$

From the procedures in Example 2.1 and Example 2.2, Algorithm 2.1 is to traverse all subgraphs and spanning trees. There is a lot of redundancy for constraints D and λ. Thus, it has great potential for algorithm of the computing $R(G, \lambda, D)$.

3 Irrelevant Subgraphs and Irrelevant Spanning Trees

To detect subgraphs which have no contribution to $R(G, \lambda, D)$, the following definitions and propositions are proposed.

Definition 3.1: A tree T satisfying $R(G, \lambda, D) = R(G\text{-}T, \lambda, D)$ is irrelevant.

For diameter constraint, if a tree's diameter is greater than D, it is irrelevant. For example, T_6, T_7 are irrelevant since $A(T_6) = A(T_7) = 3 > D = 2$ in Figure 2.

Definition 3.2: A subgraph G_0 satisfying $R(G, \lambda, D) = R(G\text{-}G_0, \lambda, D)$ is irrelevant.

There are two classes of irrelevant subgraphs:

1. Disconnected subgraphs, such as G_3 in Figure 2;
2. All spanning trees of a subgraph are irrelevant, such as G_{15} in Appendix I Table 3.

Actually, irrelevant subgraphs of class 1 have nothing to do with D while that of class 2 do not.

Proposition 3.1: A subgraph is irrelevant if and only if its diameter is greater than D.

Proof: If the diameter of a subgraph, say G_0, is greater than D, it is irrelevant based on the definition of diameter constrained reliability. Reversely, if G_0 is irrelevant, it must be disconnected or contain at least one path whose length is greater than D. According to the definition of diameter, both cases indicate $A(G_0) > D$. □

Irrelevant subgraphs of class 2 can be totally detected by proposition 3.1. Further, the distance between node v and node w is denoted as $d(v, w)$, then

Proposition 3.2: Subgraphs containing node v and node w is irrelevant if $d(v, w) > D$, where $v, w \in V$.

Proof: Let G_0 be a subgraph containing nodes v and w. It can be inferred from $d(v, w) > D$ that G_0 contains a path whose length is greater than D (i.e. the path between v and w). Thus, the diameter of subgraph G_0 satisfies $A(G_0) \geq d(v, w) > D$. Based on Proposition 3.1, G_0 is irrelevant. □

Example 2.2 is re-computed based on Propositions 3.1 and 3.2.

Example 3.1. As $d(v_1, v_5) = 4 > D$, the subgraphs containing nodes v_1, v_5 are irrelevant. The remaining subgraphs are illustrated in Appendix I Table 2. As $A(G_{15}) = 5$, $A(G_{114}) = 4$, $A(G_{119}) = 4$, $A(G_{123}) = \infty$, $A(G_{124}) = 4$, G_{15}, G_{114}, G_{119}, G_{123}, G_{124} are irrelevant. The remaining subgraphs are given in Appendix I Table 3.

Table 2. The Comparison Between Example 2.2 And Example 3.1

	Example 2.2	Example 3.1
The number of subgraphs	37	10
The number of spanning trees	456	161
Saved subgraphs	0	27
Saved spanning trees	0	295
Efficiency of saved subgraphs	0	72.97%
Efficiency of saved spanning trees	0	64.69%

Table 3. Experimental Results Before And After Reduction

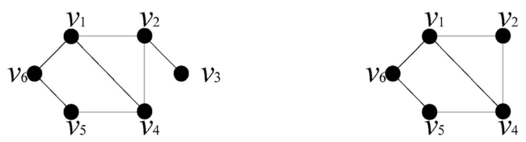
G	A(G)	λ	D	The number of Subgraphs		The number of Spanning trees	
				before	after	before	after
GUN	4	0.7	3	232	31	1276	192
		0.8	3	67	0	430	0
		0.7	2	232	0	1276	0
5*5GRID	8	0.8	6	68406	20964	N/A	N/A
		0.9	6	326	124	7319	1346
		0.8	7	68406	5761	N/A	N/A
EON	4	0.8	3	1160	337	174653	41249
		0.9	3	20	14	1747	986
		0.8	2	1160	29	174653	1786
Arp	6	0.8	4	6196	1192	71362	6251
		0.9	4	211	65	6963	1937
		0.8	3	6196	282	71362	2654

These subgraphs generate 161 spanning trees, the number of which is less than that of Example 2.2 (456 trees). With diameter constraint D, the remaining trees are same as Example 2.2 (see Figure 3). Table 2 shows the comparison of numbers of subgraphs and spanning trees in Example 2.2 and Example 3.1. It can be seen that propositions 3.1 and 3.2 are effective in the detection of irrelevant subgraphs and irrelevant trees when calculating $R(G, \lambda, D)$.

Proposition 3.3. The value of D should satisfy $min\ \{d_1, d_2, \ldots, d_m\} \le D \le d$, where $d_1, d_2, \ldots, d_m$ is the diameter of all the spanning trees, d is the length of the longest path in G.

Obviously, $R(G, \lambda, D) = R(G, \lambda)$ as $D > d$, $R(G, \lambda, D) = 0$ as $D < min\ \{d_1, d_2, \ldots, d_m\}$

Example 3.2. A network $G(V, E)$ in Figure 5(a), $V = \{v_1, v_2, \ldots, v_6\}$, $E = \{v_1v_2, v_1v_4, v_2v_3, v_2v_4, v_4v_5, v_5v_6, v_6v_1\}$, $D = 2$, $\lambda = 0.8$, $k = \lceil 0.8 \times 6 \rceil = 5$, the operational probability for each edge is $p = 0.9$.



(a) (b)

Figure 5. A graph $G(V, E)$ in Example 3.2

As $d(v_3, v_5) = 3 > D$, $d(v_3, v_6) = 3 > D$, subgraphs containing $\{v_3, v_5\}$ or $\{v_3, v_6\}$ are irrelevant. According to the definition of node-propotion constraint and Proposition 3.2, node v_3 is irrelevant (see Figure 5(b)). Figure 6 shows all the spanning trees of the remaining subgraphs. The diameter of each spanning tree is larger than D, so $R(G, \lambda, D) = 0$.

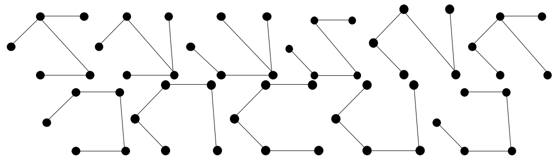


Figure 6. All the spanning trees in Example 3.2

Proposition 3.4. If $min\ \{d_1, d_2, \cdots, d_m\} \le D < A(G)$, there exist irrelevant subgraphs; if $A(G) \le D \le d$, there are no irrelevant subgraphs.

In order to detect more irrelevant subgraphs, the following proposition is proposed.

Proposition 3.5. Suppose the node set $N \subset V$ contains a pairs of nodes whose distance is greater than D, then $b \le |N| \le 2a$, where b is the minimum integer solution of $\binom{b}{2} \ge a$. There exists 4 cases since the different value of $|N|$ will affect the selection of irrelevant subgraphs.

Case 1: the subgraphs containing n-b+2, n-b+3, $\cdots$, n nodes are irrelevant if $|N| = b$;

Case 2: the subgraphs containing n-1, n nodes are irrelevant if $b+1 \le |N| \le a$;

Case 3: the subgraphs containing n nodes are irrelevant if $a+1 \le |N| \le 2a-1$;

Case 4: the subgraphs containing n-a+1, n-a+2, $\cdots$, n nodes are irrelevant if $|N| = 2a$.

Proof: Suppose $N = \{v_1, v_2, \cdots, v_b\} \subset V$ in Case 1, then $d(v_i, v_j) > D$, $d(v_i, v_t) \le D$, where $v_i, v_j \in N$, $v_t \in V$-N. The subgraphs containing n-b+2 must be irrelevant since there must exist two nodes belonging to N. The subgraphs containing n-b+1 nodes may be relevant because it may contain only one node in N. Similarly, other conclusions can be proved. □

According to Propositions 3.1~3.5, a practical algorithm is able to be designed.

4 Reduction Algorithm for $R(G, \lambda, D)$

Subprogram 1.

We use the method of calculating $A(G)$ from Ref. [20], named "Diameter Algorithm".

Subprogram 2.

The subprogram aims to find all node pairs v_i, v_j satisfying $d(v_i, v_j) > D$ and store each pair in set M. Node set N consists of nodes in M. $d(v_i, v_j)$ is calculated by Dijkstra's algorithm [18].

Sub 2. Procedure Find Node Pair

Input: G

Output: N, M, a

```
N = φ, M = φ, a = 0;
for i = 1 to n
 for j = (i+1) to n
  if d(v_i, v_j) > D
   a = a+1, N = N ∪ {v_i, v_j}, M(a) = (v_i, v_j);
  end if
 end for
end for
end
```

Subprogram 3.

Sub 3. Find Subgraph

Input: G, k, M, t

Output: SG

```
SG = φ;
for s = k to t
   Find subgraphs containing s nodes;
   Run sub 2 and delete subgraphs containing node
   pairs of M and store the remaining subgraphs in
   SG(s);
end for
end
```

Subprogram 4.

The subprogram simplifies computation procedures according to proposition 3.5.

Sub 4. Simplification Procedure

Input: a, b, n
Output: SG
1. switch $|N|$
2. **Case** b: $t = n-b+1$
3. run Sub3 to obtain SG;
4. **Case** $[b+1, a]$: $t = n-2$
5. run Sub3 to obtain SG;
6. **Case** $[a+1, 2a-1]$: $t = n-1$
7. run Sub3 to obtain SG;
8. **Case** $2a$: $t = n-a$
9. run Sub3 to obtain SG;
10. **end switch**

Subprogram 5.

The Spanning tree Algorithm [20] is trying to find all spanning trees.

Main program.

Main. Reduction Procedure $R(G, \lambda, D)$

Input: $G(V, E)$, D, k, n
Output: $R(G, \lambda, D)$
1. Run Sub 1 to obtain $A(G)$;
2. **if** $A(G) > D$
3. Run sub 2 to obtain a, N, M;
4. Run sub 4 to obtain SG;
5. Delete subgraphs whose diameter is greater than D in SG;
6. Run Sub 5 and then obtain the set of all spanning trees T of SG;
7. Delete spanning trees whose diameter is greater than D in T;
8. $R(G, \lambda, D) = \Pr(\bigcup_e T_e)$, $T_e \in T$;
9. **else**
10. Run Algorithm 2.1;
11. **end if**
12. **end**

5 Examples

The Example 3.1 is re-computed and the details of reduction by using the proposed algorithm are shown.

Example 5.1. A network G (V, E) in Figure 3, $V = \{v_1, v_2, \ldots, v_8\}$, $E = \{v_1v_2, v_1v_8, v_2v_3, v_2v_7, v_3v_4, v_3v_7, v_4v_5, v_4v_6, v_5v_6, v_6v_7, v_7v_8\}$, $D = 3$, $\lambda = 0.7$, the operational probability for each edge is $p = 0.9$.

Main. Reduction Procedure $R(G, 0.7, 3)$

Input: G (V, E), $D = 3$, $k = 6$, $n = 8$
Output: $R(G, 0.7, 3)$
1. $A(G) = 4$;
2. $D = 3 < A(G)$;
3. $a = 1$, $N = \{v_1, v_5\}$, $M=\{(v_1, v_5)\}$ and $|N| = 2$;
4. Appendix I Table 2 lists SG;
5. $A(G_{15}) = 5 > D$, $A(G_{114}) = 4 > D$, $A(G_{119}) = 4 > D$, $A(G_{123}) = \infty > D$, $A(G_{124}) = 4 > D$, $SG = SG - \{G_{15}, G_{114}, G_{119}, G_{123}, G_{124},\}$ (see Appendix I Table 3);
6. Sub 5 obtains set T containing 161 spanning trees;
7. Compute the diameter of the remaining spanning trees, Remain the tree whose diameter is not greater than D, update T(see Figure 4);
8. $R(G, \lambda, D) = \Pr(E_1 \bigcup E_2 \bigcup \cdots \bigcup E_{12}) = 0.95393659$;
12. End

The example is illustrated to remove redundancy by the proposed algorithm. Furthermore, networks in Figure 7 [15], [12] are listed to show the advantage of our algorithm.

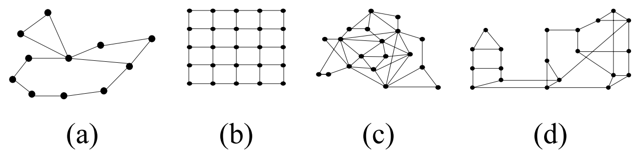


Figure 7. Virtual network and communication network

Table 3 shows the comparison between algorithm 2.1 and the reduction algorithm in section IV. By using the reduction algorithm, numbers of both subgraphs and spanning trees have greatly decreased. For example, in the network (a), it is inevitable to obtain 232 subgraphs and 1276 spanning trees while the number of them in the computation with $\lambda = 0.7$ and $D = 3$ are 31 and 192 respectively. Further, comparisons are performed taking into different values for diameter constraint D and node-proposition constraint λ. For instance, in network (c), for $D = 3$, the number of irrelevant subgraphs are 823 and 6 respectively for $\lambda = 0.8$ and 0.9. Numbers of spanning trees are 41429 and 986. The data indicates that the higher the value of λ is, the less subgraphs and spanning trees will be. In network (d), for $\lambda = 0.8$, the number of subgraphs are 1192 and 282 for $D = 4$ and 3 based on reduction algorithm. The smaller the value of D is, the more the resulting subgraph and spanning tree gains.

6 Conclusion

This paper presents the concept of irrelevant subgraphs and trees that are different from former researches. Experimental results show that the proposed reduction algorithm remove a large number of irrelevant subgraphs and spanning trees. The proposed model and algorithm are useful for evaluating WSN properly. Examples show that the proposed methods are effective.

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Appendix I

Table 1. Subgraphs of G in Figure 3

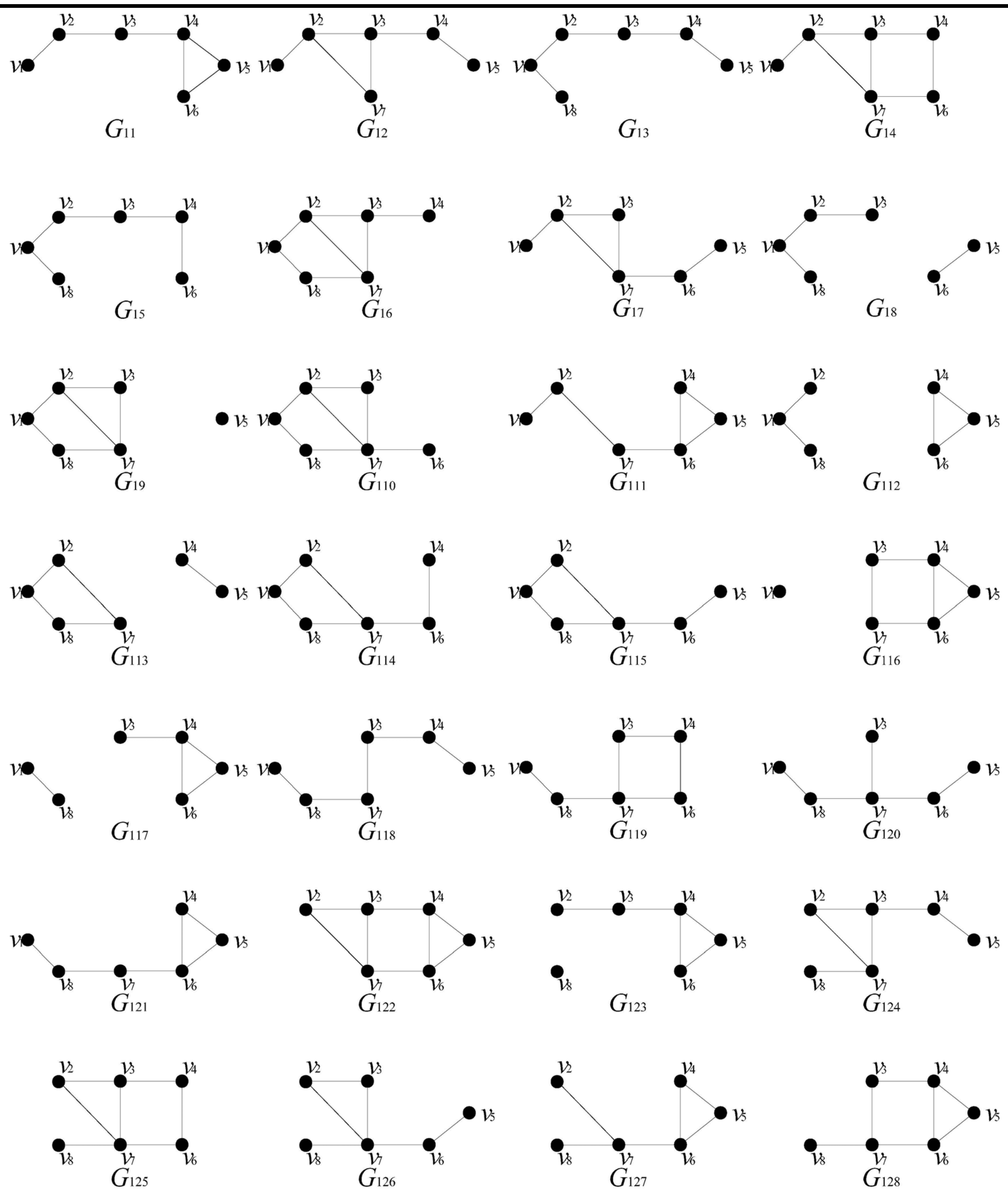
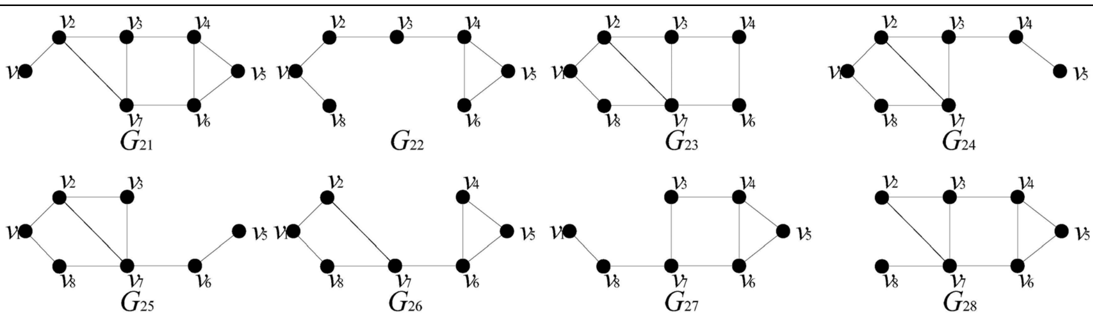
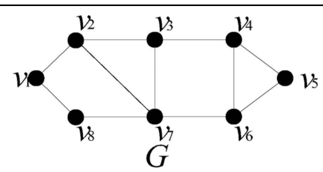
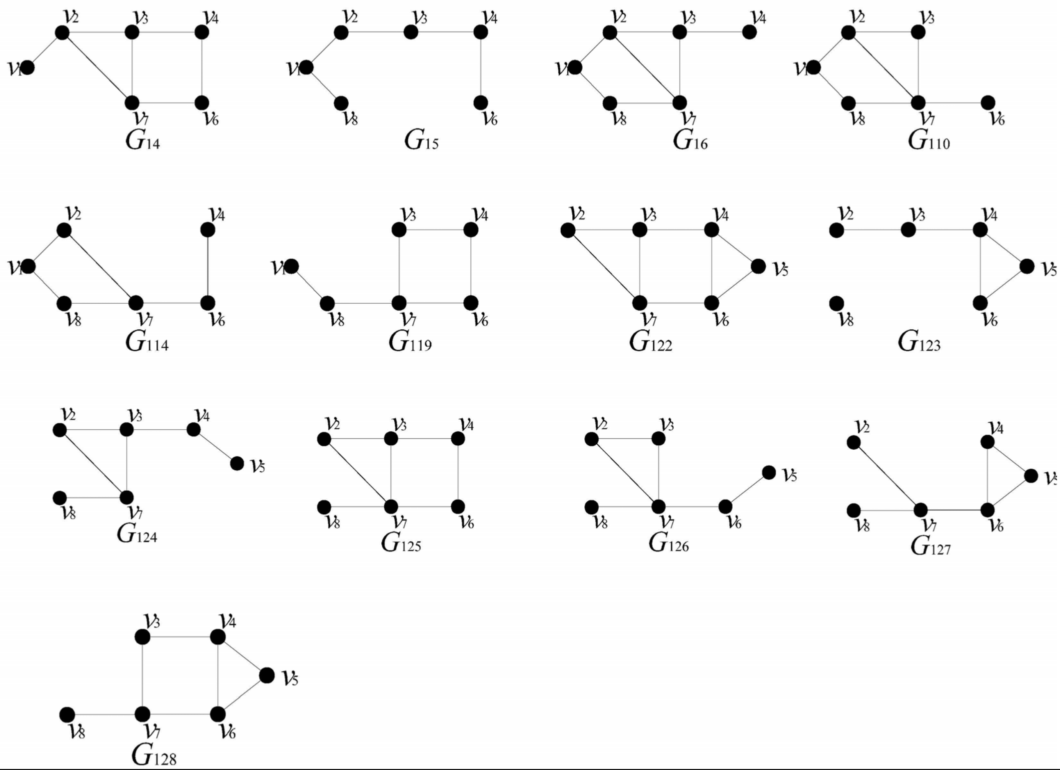
$\binom{8}{6}$	
$\binom{8}{7}$	
$\binom{8}{8}$	

Table 2. Subgraphs without nodes v_1 and v_5

13



2

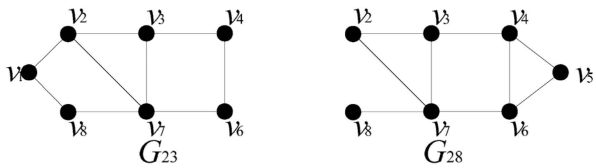
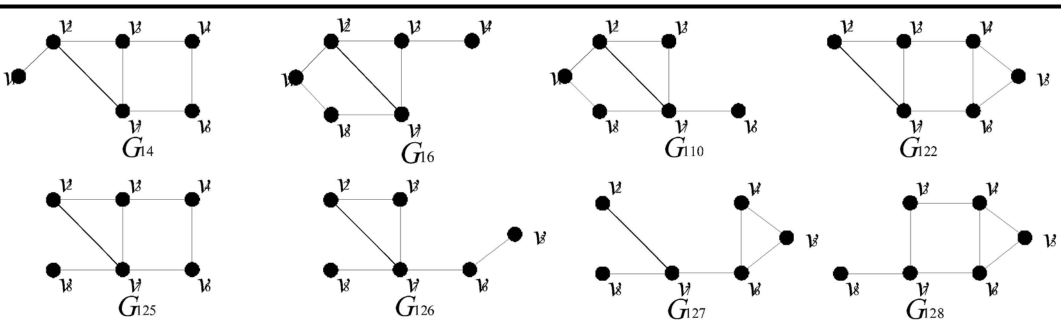


Table 3. Subgraphs whose diameter is not greater than *D*

8



2