

# A Further Study of Optimal Matrix Construction for Matrix Embedding Steganography

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## Abstract

Matrix embedding is a general approach that can be applied to most steganographic schemes to improve their embedding efficiency. In order to apply matrix embedding to voice-over-IP (VoIP) steganography better, this paper analyses the means to realizing fast matrix embedding. For small payloads, we discuss the feasibility of combining several Hamming codes into parity check matrix (PCM) construction and propose a novel PCM structure. On this basis, a corresponding optimization algorithm is proposed. It can adaptively generate specific PCMs to accommodate to the given cover and provide the best performance while guaranteeing the allowable computational complexity. For large payloads, another PCM structure is presented by combining the PCM of syndrome trellis codes (STCs) and several referential columns. The corresponding optimal construction algorithm is also given. Experimental results show that compared with existing methods, two novel matrix embedding methods achieve higher embedding efficiency and faster embedding speed.

**Keywords:** Steganography, Matrix embedding, Parity check matrix, Embedding efficiency, Embedding speed

## 1 Introduction

Steganography is a covert communication technology. Up to now, steganographic covers have been extended from images to almost all kinds of multimedia. Voice over IP (VoIP) is the most popular real-time service in IP networks at present. The study on VoIP steganography is becoming extensive, and various approaches [1-5] have been developed.

No matter what the steganographic cover is, for a given message and cover, the scheme that introduces fewer changes will be more secure in general. Based on this understanding, matrix embedding (ME, also known as syndrome coding) was proposed by Crandall [6]. ME can improve embedding efficiency. It requires the sender and the recipient to agree in advance on a parity check matrix (PCM). Using the PCM, the sender

selects the coset leader of error correction codes as the modification vector, and the recipient extracts secret messages by calculating the syndrome of the stego data. ME was made popular by Westfeld who incorporated it into F5 algorithm. After that, Fridrich *et al.* systematically analyzed ME [7-8] and got its upper bound of embedding efficiency. They also proved that random linear code-based ME can approach this theoretical bound. Recently, ME has been extended to convolution codes, such as syndrome trellis codes (STCs) [9-10]. STCs can embed a given payload with minimal total distortion if the cost of changing each cover element is assigned. This task can be viewed as a generalization of initial ME or writing on wet paper.

To date, existing VoIP steganography methods mainly hide information in the LSBs of speech streams [5]. Since direct LSB replacement degrades speech quality obviously, many methods [11-12] improve their security through using initial ME which can minimize the number of embedding changes. Convolution codes that minimize total distortion are rarely applied to VoIP steganography for the following reasons: On the one hand, different from image cover, the relations of cover elements in speech streams are much more complicated and not intuitive. This leads to extremely rare research achievements in VoIP distortion function. On the other hand, speech streams are generated and transmitted in real time, which gives a short time to perform embedding or extracting process. Whereas minimizing total distortion usually needs larger amount of computation and the process of calculating single-letter distortions costs a certain time. Beyond that, convolution codes are more suitable for long covers. But in VoIP scheme, encoder often divides a cover into small parts and performs embedding operation on each part to maintain the real-time requirement [1]. In this case, the embedding efficiency of convolution codes needs to be further improved.

Our goal is to propose a novel fast method for ME, so that it can be better applied to VoIP steganography. To reduce computational complexity of ME, researchers have developed many improved methods through two ways [13]. The core idea of the first class is to construct special PCM. Typical examples include

Hamming code-based ME [14] and random linear code-based ME [15]. Though they are early methods, their embedding efficiencies are relatively high benefited from their excellent PCM structures. To further improve Hamming code-based ME, Mao proposed a fast method [16] in which the positions of PCM columns are changed to make all columns array in ascending (or descending) order in decimal form, then the coset leader can be found by using a lookup table algorithm. Aiming at the shortcoming of immobility of Hamming codes, Tian *et al.* presented an adjustable ME method [1] which can adaptively generate a guide matrix to accommodate to various cover lengths and achieve the optimal embedding performance. To increase embedding speed of random linear code-based ME, Wang *et al.* proposed a new method by translating several random columns to referential columns [17]. For the second class, its core idea is to find a sub-optimal solution as the modification vector instead of the coset leader. Hence, Gao *et al.* turned to finding a vector in the coset which has relatively small Hamming weight [18]. After that, similar methods such as [19-20] were proposed. Compared with the first kind, these approaches achieve faster embedding speed but at the cost of a fall of embedding efficiency.

We make a further study on the optimization of PCM construction in this paper. Two special matrix forms for small payloads (payloads that are smaller than 0.5) and large payloads (payloads that are larger than 0.5) are presented, respectively. The paper is organized as follows: In Section 2, we review a few elementary concepts of ME and the related works that will be needed for the rest part. Section 3 and Section 4 explain two novel ME methods. Experimental results and their analyses appear in Section 5. Finally, the paper is concluded in Section 6.

## 2 Related Works

### 2.1 Matrix Embedding

Without loss of generality, the cover and the secret message are regarded as binary sequences in this paper. Matrix  $H$  of dimension  $(n-k) \times n$  is the PCM of binary linear  $[n, k]$  codes  $C$ . Based on  $H$ , the sender can embed  $n-k$  secret bits  $\mathbf{m}^T = (m_1, m_2, \dots, m_{n-k})$  into an  $n$ -length cover  $\mathbf{c}^T = (c_1, c_2, \dots, c_n)$ . The key problem of ME is to find a modification vector with minimum Hamming weight. Hence, first calculate the difference between  $\mathbf{m}$  and  $H\mathbf{c}$ . The result is denoted by  $\mathbf{u}$ , i.e.,  $\mathbf{u} = \mathbf{m} \oplus H\mathbf{c}$ . Then, get its coset with respect to  $H$ .

$$C_H(\mathbf{u}) = \{ \mathbf{x} \in GF(2^n) \mid H\mathbf{x} = \mathbf{u} \} \quad (1)$$

$C_H(\mathbf{u})$  contains  $2^k$  vectors. Among them, the one that has the smallest Hamming weight is called coset leader.

$$e_L(\mathbf{u}) = \arg \min_{\mathbf{x} \in C_H(\mathbf{u})} \omega(\mathbf{x}) \quad (2)$$

$e_L(\mathbf{u})$  represents the optimal modification vector, so the stego  $\mathbf{s}$  is obtained as follows.

$$\mathbf{s} = \mathbf{c} \oplus e_L(\mathbf{u}) \quad (3)$$

The recipient extracts secret messages by computing

$$\begin{aligned} H\mathbf{s} &= H(\mathbf{c} \oplus e_L(\mathbf{u})) = H\mathbf{c} \oplus H \cdot e_L(\mathbf{u}) \\ &= H\mathbf{c} \oplus (\mathbf{m} \oplus H\mathbf{c}) = \mathbf{m} \end{aligned} \quad (4)$$

### 2.2 Wang *et al.*'s Fast Matrix Embedding

The first ME with feasible complexity was proposed by Fridrich *et al.* [15]. Their PCM structure is

$$H = (I_{n-k}, R) \quad (5)$$

In which  $I_{n-k}$  is an  $(n-k) \times (n-k)$  unit matrix, and  $R$  is an  $(n-k) \times k$  random matrix. The coset leader can be found with  $O(n2^k)$  computations.

Based on structure (5), Wang *et al.* proposed a novel fast method by extending the PCM via some referential columns [17]. Its computational complexity is reduced to  $O(n2^{k_1})$ . The PCM they construct is

$$H = (I_{n-k}, R, D) \quad (6)$$

where  $R$  is a random matrix of dimension  $(n-k) \times k_1$ ,  $D$  is an  $(n-k) \times k_2$  matrix and  $k_1 + k_2 = k$ . The  $i$ th referential column in  $D$  is in the following form:

$$\mathbf{d}_i^T = (\mathbf{0}_{t_i}, \dots, \mathbf{0}_{t_i-1}, \mathbf{1}_{t_i}, \mathbf{0}_{t_i+1}, \dots, \mathbf{0}_{k_2}), 1 \leq i \leq k_2 \quad (7)$$

$t_i$  is usually taken as

$$t_i = \begin{cases} \left\lfloor \frac{n-k}{k_2} \right\rfloor & \text{if } i < k_2 \\ (n-k) - (k_2 - 1) \left\lfloor \frac{n-k}{k_2} \right\rfloor & \text{if } i = k_2 \end{cases} \quad (8)$$

The following matrix is a specific form of  $H$  when  $(n, k, k_1, k_2) = (11, 5, 2, 3)$ .

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \quad (9)$$

According to (6), each modification vector  $\mathbf{x}$  can be split into three parts  $\mathbf{x}^T = (\mathbf{x}_0^T, \mathbf{x}_1^T, \mathbf{x}_2^T)$ , and they satisfy the condition  $\mathbf{x}_0 \oplus \mathbf{R}\mathbf{x}_1 \oplus \mathbf{D}\mathbf{x}_2 = \mathbf{u}$ . Divide both  $\mathbf{x}_0$  and  $\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1$  into  $k_2$  segments ( $|\mathbf{x}_{0,i}| = |(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)_i| = t_i$ ). Consequently, the coset leader of  $C_H(\mathbf{u})$  can be found by minimizing the following quantity:

$$\omega(\mathbf{x}_1) + \sum_{i=1}^{k_2} \min\{\omega((\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)_i), t_i - \omega((\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)_i) + 1\} \quad (10)$$

### 3 Optimal Matrix Construction for Small Payloads

#### 3.1 Structure of the Proposed PCM

The referential columns in (6) can effectively improve embedding efficiency of ME when  $k_2$  is small. But, as  $k_2$  increases, the impact of the referential columns becomes smaller. When  $\lfloor (n-k)/k_2 \rfloor \geq 1$  (i.e., payload is smaller than 0.5), the referential columns will not work.

By changing positions of the referential columns, the PCM in (6) can be transformed into the following form.

$$\mathbf{H} = (\mathbf{I}_{n-k}, \mathbf{R}, \mathbf{D}) = (\mathbf{A}, \mathbf{R}) \quad (11)$$

where  $\mathbf{A}$  is an  $(n-k) \times (n-k+k_2)$  matrix. For instance,  $\mathbf{A}$  in (9) can be rewritten as

$$\mathbf{A} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_3 \end{pmatrix}, \quad \mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (12)$$

When  $\lfloor (n-k)/k_2 \rfloor = 2$ , we note that submatrix  $\mathbf{B}$  is actually the PCM of [3, 1] Hamming codes (as shown in (12)). It inspires us to use Hamming codes to expand the application scope of Wang *et al.*'s method. The PCM we construct for small payloads is shown below.

$$\mathbf{H} = (\mathbf{A}, \mathbf{R}), \quad \mathbf{A} = \begin{pmatrix} \mathbf{B}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_p \end{pmatrix} \quad (13)$$

where  $\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_p$  are all PCMs of Hamming codes, and they could be different.

#### 3.2 PCM Optimization

For a given message  $\mathbf{m}$  and cover  $\mathbf{c}$ , the dimension of  $\mathbf{H}$  is definite. The restriction on computational complexity can determine the number of random columns. Therefore, to achieve optimal embedding

efficiency, we only need to discuss how to construct  $\mathbf{A}$ .

Let  $p$  denote the number of submatrices in  $\mathbf{A}$ .  $r_i$ ,  $w_i$  denote the height and width of  $\mathbf{B}_i$  respectively. There is a certain function relation between  $r$  and  $w$ .

$$w_i = f(r_i) = 2^{r_i} - 1, \quad i \in \{1, 2, \dots, p\} \quad (14)$$

On this basis, the structure of  $\mathbf{A}$  could be expressed as  $\mathbf{r} = \{r_1, r_2, \dots, r_p\}$ , and we can get Theorem 1, 2 below.

**Theorem 1.** For a local matrix  $\mathbf{A}$  containing  $p$  submatrices, when its height is fixed as  $n-k$ , the number of columns in  $\mathbf{A}$  has a certain range:

$$p(2^{\frac{n-k}{p}} - 1) \leq \sum_{i=1}^p w_i \leq p + 2^{n-k-(p-1)} - 2 \quad (15)$$

**Proof.** Matrix height is fixed, thus  $\sum_{i=1}^p r_i = n-k$ .

According to the average value inequality, we can derive a relationship as follows:

$$\begin{aligned} \sum_{i=1}^p w_i &= f(r_1) + f(r_2) + \cdots + f(r_p) \\ &= (2^{r_1} - 1) + (2^{r_2} - 1) + \cdots + (2^{r_p} - 1) \\ &= 2^{r_1} + 2^{r_2} + \cdots + 2^{r_p} - p \\ &\geq p \sqrt[p]{2^{r_1+r_2+\cdots+r_p}} - p \\ &= p(2^{\frac{n-k}{p}} - 1) \end{aligned}$$

If and only if  $r_1 = \cdots = r_p = (n-k)/p$ , the equation holds.

On the other hand, we have  $f'(r) = 2^r \ln 2 > 1$  and  $f''(r) = 2^r (\ln 2)^2 > 0$  for  $r \geq 1$ . That is to say,  $f(r)$  goes up as  $r$  increases, and the growth range is larger and larger. Therefore,

$$\begin{aligned} \sum_{i=1}^p w_i &= f(r_1) + f(r_2) + \cdots + f(r_p) \\ &\leq f(1) + f(r_2) + \cdots + f(r_p + (r_1 - 1)) \leq \cdots \\ &\leq f(1) + f(1) + \cdots + f(r_p + (r_1 - 1) + \cdots + (r_{p-1} - 1)) \\ &= f(1) + f(1) + \cdots + f(n-k - (p-1)) \\ &= p + 2^{n-k-(p-1)} - 2 \end{aligned}$$

Considering the above analysis, it can be concluded that the column number of  $\mathbf{A}$  is related to the diversity of submatrices. The column number achieves the minimum when the sizes of submatrices are all the same, achieves the maximum as  $\mathbf{r} = \{1, \dots, 1, n-k-(p-1)\}$ .

**Theorem 2.** For a local matrix  $\mathbf{A}$ , when its height is fixed as  $n-k$ , small number of submatrices ( $p$  is small) is conducive to the increasement of column number.

**Proof.**  $\mathbf{r} = \{r_1, \dots, r_p\}$  when  $\mathbf{A}$  contains  $p$  submatrices.

If there are  $p' = p+1$  submatrices,  $\mathbf{r}' = \{r'_1, \dots, r'_p, 1\}$ . Then we can get consequences follow from Theorem 1:

$$\begin{aligned} \max \sum_{i=1}^p w_i &= p + 2^{n-k-(p-1)} - 2 \\ &= p + 1 + 2^{n-k-p} - 2 + 2^{n-k-p} - 1 \\ &\geq p + 1 + 2^{n-k-p} - 2 \\ &= \max \sum_{i=1}^{p'} w_i' \end{aligned}$$

$\min \sum_{i=1}^p w_i = p(2^{(n-k)/p} - 1)$ . Take the derivative of function  $\min \sum_{i=1}^p w_i$  with respect to  $p$ :

$$\begin{aligned} \frac{d(\min \sum_{i=1}^p w_i)}{dp} &= 2^{\frac{n-k}{p}} - 1 - p \cdot 2^{\frac{n-k}{p}} \ln 2 \cdot \frac{n-k}{p^2} \\ &= 2^{\frac{n-k}{p}} (1 - \ln 2 \cdot \frac{n-k}{p}) - 1 \end{aligned}$$

Since  $(n-k)/p \geq 1$ ,  $(n-k)/p \in N^*$ , It's easy to know that  $2^{\frac{n-k}{p}} (1 - \ln 2 \cdot \frac{n-k}{p}) < 1$ . So  $d(\min \sum_{i=1}^p w_i)/dp < 0$ , i.e.,

$$\min \sum_{i=1}^p w_i > \min \sum_{i=1}^{p'} w_i'$$

Hamming codes could embed  $r$  bits of messages into  $2^r - 1$  bits of cover data with one change at most. The probability of modifying the cover is  $(2^r - 1)/2^r$ . Thus the embedding efficiency is  $r \cdot 2^r / (2^r - 1)$ . On this basis, we have the following results.

**Theorem 3.** For a local matrix  $A$  containing  $p$  submatrices, when its height is fixed as  $n - k$ , the embedding efficiency of  $A$  is related to the diversity of submatrices. The greater the diversity of submatrices is, the higher the embedding efficiency will be. The range of the embedding efficiency is

$$\frac{n-k}{p - p \frac{1}{2^{n-k/p}}} \leq e \leq \frac{n-k}{\frac{p+1}{2} - \frac{1}{2^{n-k-(p-1)}}} \tag{16}$$

**Proof.**  $A$  contains  $p$  submatrices. Hence, the average number of embedding changes is

$$E(\omega_A) = \sum_{i=1}^p \frac{2^{r_i} - 1}{2^{r_i}} = p - \sum_{i=1}^p \frac{1}{2^{r_i}}$$

According to the average value inequality, we can learn that

$$\begin{aligned} \sum_{i=1}^p g(r_i) &= -2^{-r_1} - 2^{-r_2} \dots - 2^{-r_p} \\ &\leq -p \sqrt[p]{2^{-r_1 - r_2 - \dots - r_p}} \\ &= -p \cdot 2^{-\frac{n-k}{p}} \end{aligned}$$

where  $g(r) = -2^{-r}$ . Therefore  $E(\omega_A) \leq p - p/2^{n-k/p}$ . If and only if  $r_1 = \dots = r_p = (n-k)/p$ , the equation holds.

On the other hand, we have  $g'(r) = 2^{-r} \ln 2 > 0$  and  $g''(r) = -2^{-r} (\ln 2)^2 < 0$  for  $r \geq 1$ . That is to say,  $g(r)$  goes up as  $r$  increases, but the growth range is smaller and smaller. Therefore,

$$\begin{aligned} \sum_{i=1}^p g(r_i) &= g(r_1) + g(r_2) + \dots + g(r_p) \\ &\geq g(1) + g(r_2) + \dots + g(r_p + (r_1 - 1)) \geq \dots \\ &\geq g(1) + g(1) + \dots + g(r_p + (r_1 - 1) + \dots + (r_{p-1} - 1)) \\ &= g(1) + g(1) + \dots + g(n-k-(p-1)) \end{aligned}$$

Consequently,

$$E(\omega_A) \geq p - \left( \frac{p-1}{2} + \frac{1}{2^{n-k-(p-1)}} \right) = \frac{p+1}{2} - \frac{1}{2^{n-k-(p-1)}}$$

Divide  $n - k$  (the message length) by  $E(\omega_A)$ , we will get the expression in (16).

**Theorem 4.** For a local matrix  $A$ , when its height is fixed as  $n - k$ , small number of submatrices ( $p$  is small) is conducive to the improvement of embedding efficiency.

Theorem 4 is the conclusion follows from Theorem 3. The proof process is the same as Theorem 2.

According to Theorem 2 and Theorem 4, to improve embedding efficiency and embedding speed, minimizing the number of submatrices should be our primary goal in PCM construction. According to Theorem 1 and Theorem 3, we found that given the number of submatrices, enlarging the diversity of submatrices can reduce the number of random columns (i.e. reduce the computational complexity) and is conducive to the improvement of embedding efficiency. For instance, 5 bits need to be embedded into 20 bits of cover data. Let  $r_1 = 2$ ,  $r_2 = 3$ , we can embed the message using a PCM combined by a [3, 1] code and a [7, 4] code. Let  $r_1 = 1$ ,  $r_2 = 4$ , we can also embed the message by combining a first-order unit matrix and a [15, 11] code. But the former PCM has 10 random columns; the latter only has 4 random columns. Besides that, the embedding efficiency of the former PCM is 28/9, lower than 80/23 of the latter.

In conclusion, the optimization of local matrix  $A$  can be accomplished in two steps: Calculate the optimal number of submatrices in accordance with the message length. And then determine the size of each submatrix. In more specific terms, the first step is to find the minimum  $p$  meeting the relation  $p(2^{(n-k)/p} - 1) \leq n \leq p + 2^{n-k-(p-1)} - 2$ , namely to solve the following optimization problem:

$$\left\{ \begin{array}{l} \text{minimize } p \\ \text{subject to } l \left\lceil \frac{n-k}{p} \right\rceil + (p-l) \left\lfloor \frac{n-k}{p} \right\rfloor = n-k \\ p \in N^*, l \in \{1, 2, \dots, p\} \\ l(2^{\lceil \frac{n-k}{p} \rceil} - 1) + (p-l)(2^{\lfloor \frac{n-k}{p} \rfloor} - 1) \leq n \\ p + 2^{n-k-(p-1)} - 2 \geq n \end{array} \right. \quad (17)$$

Its searching process is

Step 1: Initialize  $p = 1$ ;

Step 2: Determine  $l$  according to the equation

$$l \left\lceil \frac{n-k}{p} \right\rceil + (p-l) \left\lfloor \frac{n-k}{p} \right\rfloor = n-k, l \in \{1, 2, \dots, p\};$$

Step 3: If  $l(2^{\lceil (n-k)/p \rceil} - 1) + (p-l)(2^{\lfloor (n-k)/p \rfloor} - 1) \leq n$ , go to next step. Otherwise  $p = p+1$  and return to Step 2;

Step 4: Output  $p$ .

After that, optimal submatrices can be determined with the goal of maximizing the diversity of them.

$$\left\{ \begin{array}{l} \text{maximize } \sum_{i=1}^p w_i = 2^{r_1} + 2^{r_2} + \dots + 2^{r_p} - p \\ \text{subject to } \sum_{i=1}^p r_i = n-k \\ \sum_{i=1}^p w_i \leq n \\ r_i \in N^*, i \in \{1, 2, \dots, p\} \end{array} \right. \quad (18)$$

The optimization process is

Step 1: Initialize  $r_p = n-k-(p-1)$ ,  $r_1 = \dots = r_{p-1} = 1$ ,

$j = p-1$ ,  $p_e = 2$ ;

Step 2: If  $\sum_{i=1}^p (2^{r_i} - 1) \leq n$ , go to Step 5. If not, go to next step;

Step 3:  $r_p = r_p - 1$ . If  $r_j < \lceil (n-k-(p-p_e))/p_e \rceil$ , update  $r_j = r_j + 1$  and return to Step 2. If not,  $j = j-1$

and go to next step;

Step 4: If  $j > p - p_e$ , update  $r_j = r_j + 1$  and return to Step 2. If not,  $p_e = p_e + 1$ ,  $j = p-1$  and reinitialize  $r_p = n-k-(p-1)$ ,  $r_1 = r_2 = \dots = r_{p-1} = 1$ , return to Step 3;

Step 5:  $k_2 = \sum_{i=1}^p (2^{r_i} - 1) - (n-k)$ ,  $k_1 = k - k_2$ , Output  $r_1, r_2, \dots, r_p, k_1, k_2$ .

### 3.3 Computational Complexity Analysis

For the proposed PCM, each modification vector can be written as  $\mathbf{x}^T = (\mathbf{x}_0^T, \mathbf{x}_1^T)$ , and they satisfy the condition  $\mathbf{A}\mathbf{x}_0 \oplus \mathbf{R}\mathbf{x}_1 = \mathbf{u}$ . Therefore, searching for the coset leader of  $C_H(\mathbf{u})$  is a two-step process: First, get

coset leaders of  $C_A(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)$  under different  $\mathbf{x}_1$ . Second, choose a vector that minimizing the quantity  $\omega(e_L(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)) + \omega(\mathbf{x}_1)$  as the final modification vector.

In order to reduce time cost in the first step, we adopt the method proposed in [16], change the positions of the columns in  $\mathbf{A}$  to make all columns array in ascending (or descending) order in decimal form as follows:

$$\mathbf{B}_1 = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \quad (19)$$

By this mean, the syndromes  $(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)_i$  will indicate the coset leaders. Supposing that  $(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)_1^T = (1, 0, 0)$ , since  $(1, 0, 0)$  is 4 in decimal form,  $(\mathbf{x}_{1,1}^{\text{opt}})^T = (0, 0, 0, 1, 0, 0, 0)$ . The computational complexity to find the coset leader of  $C_A(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)$  is  $O(p)$ .

**Theorem 5.** According to the embedding features of Hamming code-based ME, for a local matrix  $\mathbf{A}$  containing  $p$  submatrices, no matter what  $\mathbf{x}_1$  is, the inequality below would always hold.

$$\omega(e_L(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)) + \omega(\mathbf{x}_1) \leq p \quad (20)$$

Therefore, the optimal modification vector satisfies the condition  $\omega(\mathbf{x}_1) \leq p$ . This fact leaves us a clue to find the coset leader with reduced computational complexity in the second step. We only need to process and store vectors in  $C_A(\mathbf{u} \oplus \mathbf{R}\mathbf{x}_1)$  when  $\omega(\mathbf{x}_1) \leq p$ , and select one having the smallest Hamming weight among them. The number of combinations we need to deal with is

$$\mu_{k_1, p} = \begin{cases} \sum_{i=0}^{p-1} C_{k_1}^i & \text{if } p-1 \leq k_1 \\ \sum_{i=0}^{k_1} C_{k_1}^i & \text{if } p-1 > k_1 \end{cases} \quad (21)$$

Hence, the computational complexity of the novel fast ME method is  $O(p\mu_{k_1, p})$ .

## 4 Optimal Matrix Construction for Large Payloads

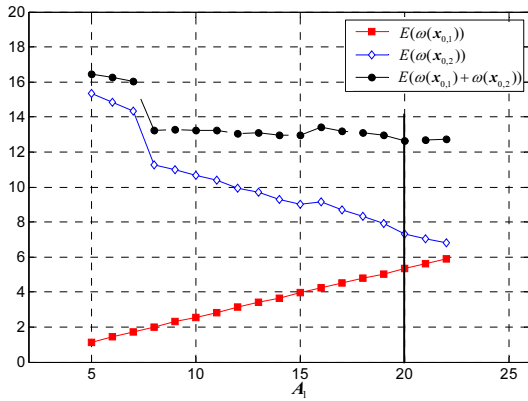
### 4.1 Structure of the Proposed PCM

STCs [9] proposed by Filler *et al.* is the most famous convolution codes. Its computational complexity is linear with  $n$  and exponential with  $h$  (the height of the submatrix). Embed messages into  $10^2$ -length covers and  $10^3$ -length covers using STCs, respectively. Figure 1 shows the average embedding efficiency of  $10^3$  experiments with different  $h$ . Figure 1 indicates that the embedding efficiency of STCs tends to increase along with  $h$ , but the improvement is small



$$\left\{ \begin{array}{l} \arg \min_{r_1} \omega(\mathbf{x}_{0,1}) + \omega(\mathbf{x}_{0,2}) \quad r_1 \in \{0, 3, 4, \dots, n-k\} \\ \text{subject to} \quad 2r_1 \leq n-k_1 \\ \quad \quad \quad r_2 \leq w_2 \\ \quad \quad \quad r_2 = n-k-r_1+h-1 \\ \quad \quad \quad w_2 = n-k_1-2r_1+2h-2 \end{array} \right. \quad (27)$$

The expectations of  $\omega(\mathbf{x}_{0,1})$  can be estimated through experiments using STCs, the PCM dimension of which is  $(r_1-h+1) \times 2(r_1-h+1)$ . The red line in Figure 2 indicates  $E(\omega(\mathbf{x}_{0,1}))$  when  $h=3$ .



**Figure 2.** Variation of Hamming weight of the modification vector with different  $r_1$

$w_2 - r_2 = k - k_1 - r_1 + h - 1$  denotes the number of submatrices in  $A_2$ . According to (8), their heights are

$$t_i = \begin{cases} \left\lfloor \frac{n-k-r_1+h-1}{k-k_1-r_1+h-1} \right\rfloor & \text{if } i < k-k_1-r_1+h-1 \\ (n-k-r_1+h-1) - \\ (k-k_1-r_1+h-2) \left\lfloor \frac{n-k-r_1+h-1}{k-k_1-r_1+h-1} \right\rfloor & \\ \text{if } i = k-k_1-r_1+h-1 \end{cases} \quad (28)$$

Furthermore, we can get the average number of embedding changes using  $A_2$ .

$$E(\omega(\mathbf{x}_{0,2})) = \sum_{i=1}^{k-k_1-r_1+h-1} \left( \sum_{j=0}^{\lfloor (t_i+1)/2 \rfloor} \frac{C_{t_i}^j}{\sum_{k=0}^{t_i} C_{t_i}^k} \cdot j + \sum_{j=\lfloor (t_i+1)/2 \rfloor+1}^{t_i} \frac{C_{t_i}^j}{\sum_{k=0}^{t_i} C_{t_i}^k} \cdot (t_i - j + 1) \right) \quad (29)$$

For  $n=60$ ,  $n-k=36$ ,  $k_1=3$ , the average change number using  $A_2$  is shown by the blue line in Figure 2. Taking  $E(\omega(\mathbf{x}_{0,1}))$  and  $E(\omega(\mathbf{x}_{0,2}))$  into consideration,

the optimal size of  $A_1$  and  $A_2$  can be determined as  $20 \times 36$  and  $18 \times 21$  on this occasion.

### 4.3 Computational Complexity Analysis

Searching for the coset leader of  $C_H(\mathbf{u})$  with respect to the proposed PCM for large payloads also needs two steps. The computational complexity of finding the coset leader corresponding to  $A_2$  is linear with  $w_2$ . Therefore, the computational cost of the first step is close to that of STCs, i.e.,  $O(w_1 2^h)$ . More precisely, considering that extra computation is needed at the juncture of two kinds of submatrices, the whole computational complexity of our method is  $O(2^{k_1} \cdot (2^{h-1} \cdot w_1 2^h + w_2 + (2^{h-1} - 1) \cdot v_{h,t}))$ , where

$$v_{h,t} = \begin{cases} \min \sum_{i=1}^t t_i & i \in \{1, 2, \dots, k-k_1-r_1+h-1\} \\ \text{subject to} & \sum_{i=1}^t t_i \geq h-1 \end{cases} \quad (30)$$

## 5 Experimental Results

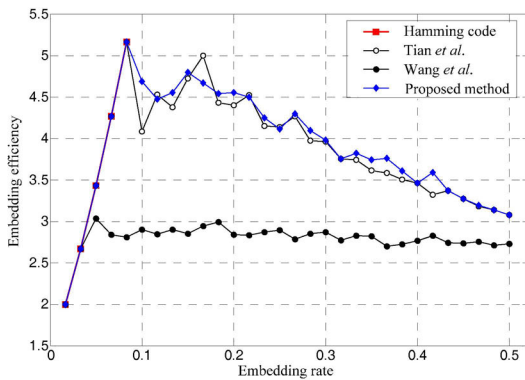
If an ME method has low computational complexity, larger PCM can be utilized and it may lead to higher embedding efficiency. Therefore, some papers only ensure that ME methods have the same computational complexity and embedding rate, ignoring PCM sizes, when compare embedding efficiency. But, in practice, cover data may be divided into small parts and the embedding process is performed on each part [9]. In this case, comparing ME methods should under the condition that PCMs have the same size. So we take PCM sizes into consideration in the following experiments.

### 5.1 Experiments of ME for Small Payloads

Experiment-1: Take the case of  $n=60$  for example. Following the searching process described in Section 3.2, we got the optimal PCMs for small payloads in Table 1. For each payload, 5000 messages and 5000 covers are generated. All of them are random binary sequences. Embed these messages and record embedding efficiencies. Calculate the mean value of 5000 experimental results as the final result. Comparison of embedding efficiency between Hamming codes [14], Tian et al.'s method [1], Wang et al.'s method [17] and the proposed method is shown in Figure 3.

**Table 1.** Optimal PCM schemes with different message lengths ( $n = 60$ )

$n-k$	$e$	$r$	$P$	$k_1$	$n-k$	$e$	$r$	$P$	$k_1$
1	2.0000	{1}	1	59	16	4.2172	{1,3,4,4,4}	5	7
2	2.6667	{2}	1	57	17	4.0964	{2,3,4,4,4}	5	5
3	3.4286	{3}	1	53	18	3.9387	{3,3,4,4,4}	5	1
4	4.2667	{4}	1	35	19	3.7531	{1,3,3,4,4,4}	6	0
5	5.1613	{5}	1	29	20	3.8226	{3,3,3,3,4,4}	6	2
6	4.6875	{1,5}	2	28	21	3.7433	{2,2,3,3,3,4,4}	7	8
7	4.4700	{2,5}	2	26	22	3.7581	{3,3,3,3,3,3,4}	7	3
8	4.5508	{3,5}	2	22	23	3.6095	{1,3,3,3,3,3,3,4}	8	2
9	4.7974	{4,5}	2	14	24	3.4595	{2,3,3,3,3,3,3,4}	8	0
10	4.6153	{1,4,5}	3	13	25	3.5889	{1,3,3,3,3,3,3,3,3}	9	3
11	4.4571	{2,4,5}	3	11	26	3.3471	{2,3,3,3,3,3,3,3,3}	9	1
12	4.4412	{3,4,5}	3	7	27	3.2727	{1,2,3,3,3,3,3,3,3,3}	10	0
13	4.3189	{1,3,4,5}	4	6	28	3.1638	{1,2,2,2,3,3,3,3,3,3,3}	11	1
14	4.1691	{2,3,4,5}	4	4	29	3.0933	{1,1,2,2,2,3,3,3,3,3,3,3}	12	0
15	4.1026	{3,3,4,5}	4	0	30	3.0769	{2,2,2,2,2,3,3,3,3,3,3,3}	12	0



**Figure 3.** Comparison of embedding efficiency when  $n = 60$

From Figure 3, we can learn that: (1) the proposed ME method can support different embedding capacities, while Hamming codes can only embed at most 5 bits of secret messages; (2) In Tian *et al.*'s method, PCM is made up of several submatrices and columns which are obtained by executing the bit-wise XOR operation between two columns in different submatrices. When the number of extra columns is large, this method may

have a good effect. Therefore, rare points in Figure 3 are better than our method, such as  $n-k=10$ ; (3) However, Tian *et al.*'s method takes no account of the combination of the extra columns and potentially can't make full use of all the cover data. Along with the increase of embedding rate, the number of submatrices has a tendency to increase, and the extra columns become fewer. This is bad for Tian *et al.*'s method, but more combinations of extra columns could be dealt with in our method. As a result, the proposed method performs better in this phase; (4) On the whole, the proposed method has the highest embedding efficiency among these four methods.

The computational complexities of Hamming codes and Tian *et al.*'s method are both  $O(n)$ , and Wang *et al.*'s method in this experiment is  $O(n2^6)$ . For  $n = 60$ , actual computational costs of our method are shown in Table 2. It can be concluded that the proposed method has equal or lower computational complexity to that of the previous three methods.

**Table 2.** Variation of computational complexity with different message lengths ( $n = 60$ )

$n-k$	1	2	3	4	5	6	7	8	9	10
$C$	1	1	1	1	1	$2 \sum_{i=0}^1 C_{28}^i$	$2 \sum_{i=0}^1 C_{26}^i$	$2 \sum_{i=0}^1 C_{22}^i$	$2 \sum_{i=0}^1 C_{14}^i$	$3 \sum_{i=0}^2 C_{13}^i$
$n-k$	11	12	13	14	15	16	17	18	19	20
$C$	$3 \sum_{i=0}^2 C_{11}^i$	$3 \sum_{i=0}^2 C_7^i$	$4 \sum_{i=0}^3 C_6^i$	$4 \sum_{i=0}^3 C_4^i$	4	$5 \sum_{i=0}^4 C_7^i$	$5 \sum_{i=0}^4 C_5^i$	5·2	6	$6 \cdot 2^2$
$n-k$	21	22	23	24	25	26	27	28	29	30
$C$	$7 \cdot 2^3$	$7 \cdot 2^3$	$8 \cdot 2^2$	8	$9 \cdot 2^3$	$9 \cdot 2$	10	11·2	12	12

Experiment-2: In order to further test the performance of the proposed ME method, we apply it to StegVoIP [2]. StegVoIP selected 18 LSBs to hide secret messages from each G.723.1 (6.3kb/s) speech frame. Hence, there are altogether 72 bits in 4

neighbouring frames. We randomly choose 40 or 60 bits from them and take these bits as a unit to perform embedding process. The speech files we used in this experiment are selected from An4 database [21]. They have different lengths. Half of the speech files are



recorded by female speakers and half of them are recorded by male speakers. The secret messages are still binary sequences generated randomly.

Perceptual evaluation of speech quality (PESQ) is proposed by ITU. It's a widely used objective speech quality assessment method. PESQ ranges from -0.5 (the worst) to 4.5 (the best). It can measure the difference between the stego speech and the original speech, so we use it to verify the validity of ME methods. Calculate the mean PESQ of 1000 original

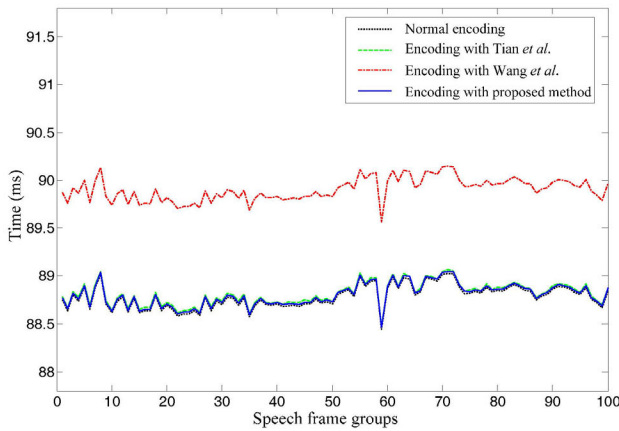
female speech files and 1000 original male speech files separately. And compare them with PESQ of stego speeches with different encoding methods. The results are shown in Table 3. From Table 3, we can learn that: PESQ of stego speeches using the proposed method are very close to the original speeches and larger than stego speeches corresponding to other methods, indicating that our method can effectively ensure the speech quality.

**Table 3.** Comparison of PESQ

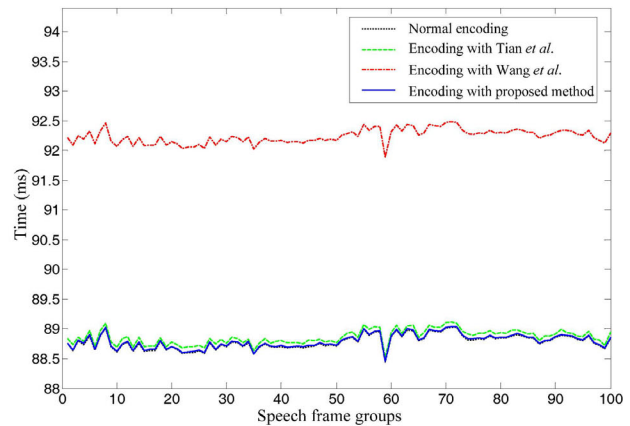
	Female ( $n = 40$ )			Male ( $n = 40$ )			Female ( $n = 60$ )			Male ( $n = 60$ )		
Embedding rate	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
Original speech	3.769	3.769	3.769	3.776	3.776	3.776	3.769	3.769	3.769	3.776	3.776	3.776
Tian et al.	3.738	3.567	3.426	3.745	3.574	3.432	3.687	3.531	3.346	3.694	3.538	3.352
Wang et al.	3.684	3.534	3.399	3.691	3.540	3.404	3.617	3.425	3.267	3.623	3.432	3.275
Proposed method	3.738	3.575	3.426	3.745	3.581	3.433	3.715	3.532	3.346	3.722	3.540	3.352

Figure 4 records the processing time of the speech encoder for 100 frame groups (containing 4 neighbouring frames) when the embedding rate is 0.3. As is shown, the proposed curve is more close to the curve without information hiding. The average encoding time delays caused by information hiding

with different message lengths can be seen in Table 4 and Table 5, from which we see that our method has the lowest latency. All the experiments were performed on a PC with 3.4 GHz Intel Core i7 CPU and 8GB RAM, and the methods were implemented in C and compiled under Microsoft Visual Studio 2008.



(a)  $n = 40$



(b)  $n = 60$

**Figure 4.** Comparing of processing time between different methods

**Table 4.** Encoding time delay with different message lengths ( $n = 40$ )

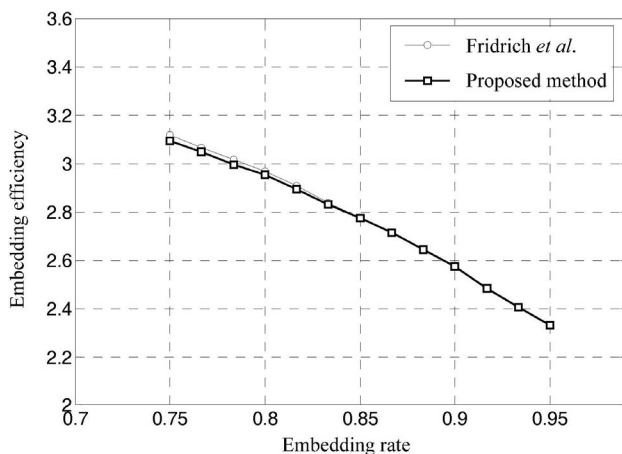
$n - k$	1	2	3	4	5	6	7	8	9	10
Tian et al. (ms)	0.001	0.002	0.004	0.008	0.015	0.027	0.017	0.019	0.024	0.031
Wang et al. (ms)	1.115	1.116	1.115	1.115	1.116	1.117	1.116	1.116	1.117	1.116
Proposed method (ms)	0.001	0.001	0.001	0.001	0.001	0.016	0.013	0.005	0.005	0.093
$n - k$	11	12	13	14	15	16	17	18	19	20
Tian et al. (ms)	0.020	0.021	0.025	0.032	0.027	0.032	0.034	0.026	0.022	0.019
Wang et al. (ms)	1.116	1.117	1.116	1.116	1.116	1.115	1.116	1.116	1.117	1.117
Proposed method (ms)	0.019	0.014	0.004	0.036	0.009	0.005	0.022	0.013	0.029	0.007

**Table 5.** Encoding time delay with different message lengths ( $n = 60$ )

$n-k$	1	2	3	4	5	6	7	8	9	10
Tian et al. (ms)	0.001	0.002	0.004	0.008	0.015	0.041	0.046	0.035	0.032	0.034
Wang et al. (ms)	3.454	3.456	3.455	3.456	3.456	3.456	3.456	3.456	3.456	3.456
Proposed method (ms)	0.001	0.001	0.001	0.001	0.001	0.048	0.041	0.032	0.027	0.214
$n-k$	11	12	13	14	15	16	17	18	19	20
Tian et al. (ms)	0.048	0.052	0.059	0.067	0.056	0.064	0.053	0.055	0.058	0.043
Wang et al. (ms)	3.455	3.456	3.456	3.456	3.458	3.456	3.455	3.455	3.456	3.456
Proposed method (ms)	0.129	0.052	0.137	0.052	0.004	0.405	0.128	0.009	0.005	0.021
$n-k$	21	22	23	24	25	26	27	28	29	30
Tian et al. (ms)	0.055	0.052	0.049	0.043	0.062	0.057	0.043	0.045	0.031	0.028
Wang et al. (ms)	3.456	3.456	3.456	3.456	3.457	3.456	3.456	3.456	3.456	3.456
Proposed method (ms)	0.049	0.050	0.029	0.007	0.061	0.016	0.009	0.020	0.011	0.011

## 5.2 Experiments of ME for Large Payloads

Experiment-3: Embed messages into random cover data using Fridrich *et al.*'s method [15] and the proposed method for large payloads. The computational complexity of Fridrich *et al.*'s method is  $O(n2^k)$ . When embedding rate is not large enough, the ME time will be too long. So we set  $n=60$  and the minimum embedding rate is set to be 0.75. To ensure that the proposed method can realize fast embedding, the computational complexity of our method is limited to be lower than  $O(2^{10})$ . Notice that, for a given cover length and computational complexity, there are many parameter combinations of  $h$  and  $k_1$  resulting in different PCM. We select the parameters which can yield the least distortion among them. For each payload, we embed 5000 blocks of random messages, and calculate the average embedding efficiency. Experimental results are shown in Figure 5 and Table 6, from which we can draw a conclusion that two ME methods achieve almost equal embedding efficiency, while the embedding speed of our method outperforms Fridrich *et al.*'s method.

**Figure 5.** Comparison of embedding efficiency when  $n = 60$ **Table 6.** Comparison of embedding speed between Fridrich *et al.*'s method and the proposed method ( $n = 60$ )

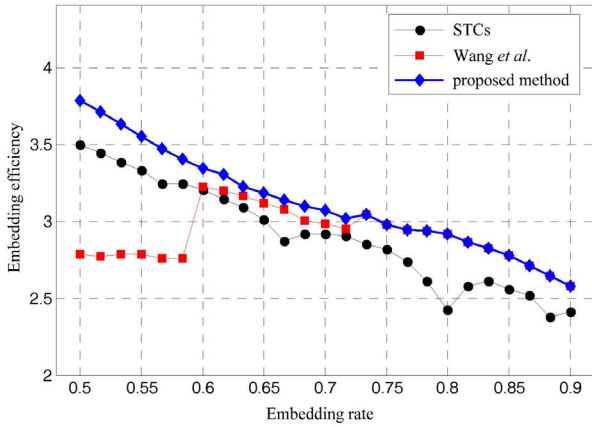
Embedding rate	0.75	0.80	0.85	0.90	0.95
Fridrich's method (Kbits/s)	0.05	0.53	6.06	64.13	804.52
Proposed method (Kbits/s)	2.83	2.88	6.06	64.13	804.52

Fridrich *et al.*'s method is an exhaustive method. It has the capability of searching global optimal solution within the defined space. Different from this, the coset leader we found using STCs or Wang *et al.*'s method is a local optimal solution. The larger the number of random columns in PCM is, the more combinations will be considered in searching for the coset leader, and thus a higher embedding efficiency we will get. Therefore, local matrix  $\mathbf{R}$  tends to be maximized within the range of allowable computational complexity. The computational complexity should be lower than  $O(2^{10})$  in this experiment, so the number of random columns is 10 at most. When the embedding rate is larger than 0.85, the random columns needed by PCM is less than 10. As a result, the PCMs we constructed using the proposed method are the same as Fridrich *et al.*'s method at this moment.

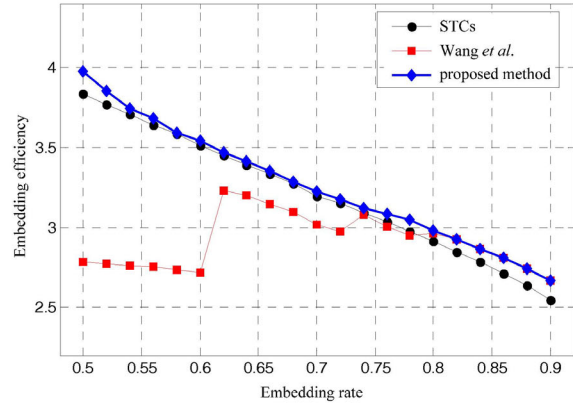
Experiment-4: Just like Experiment-2, we apply ME to StegVoIP and make a comparison between the proposed method and existing fast ME methods in [10] and [17]. The computational complexity of Fridrich *et al.*'s method is too high to be applied to VoIP steganography. So we didn't use it in this experiment. For  $n = 60$ ,  $C \leq O(2^6)$ , we select the best parameters (shown in Table 7) of the proposed method and get their embedding efficiencies (shown in Figure 6(a)). According to the conclusions of Experiment-3, the proposed ME method are the same as Fridrich *et al.*'s method when  $n-k \geq 54$ . Therefore, it is not discussed in this experiment. Similarly, Table 8 and Figure 6(b) only show the results when  $n=100$ ,  $C \leq O(2^8)$ ,  $n-k \leq 92$ . Table 7 and Table 8 illustrate that the novel

method turns into Wang *et al.*'s method when payload is close to 1. Figure 7 records the processing time of speech encoder with different ME methods in several cases. And Table 9, Table 10 show the precise encoding time delay caused by information hiding.

According to these results, we can see that a promising embedding efficiency is obtained by the proposed method while maintaining low computational complexity.

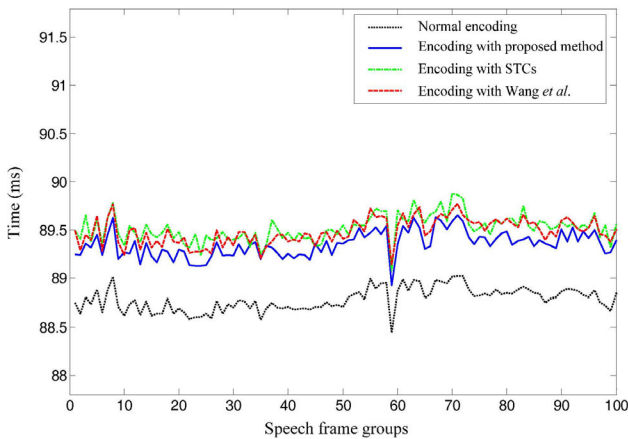


(a)  $n = 60$

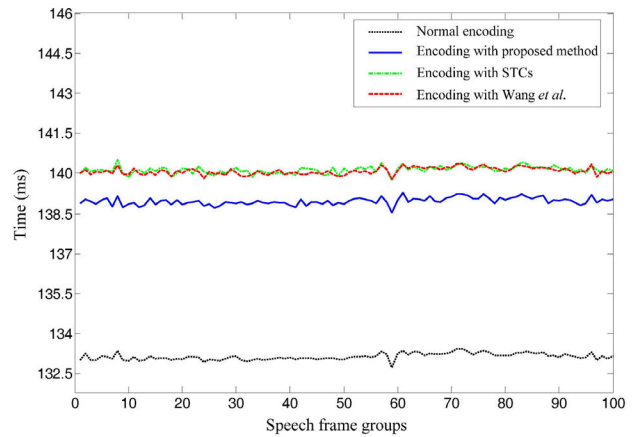


(b)  $n = 100$

Figure 6. Comparison of embedding efficiency between different methods



(a)  $n = 60, n - k = 40$



(b)  $n = 100, n - k = 66$

Figure 7. Comparison of processing time between different methods

Table 7. Optimal values of PCM parameters with different message lengths ( $n = 60$ )

$n - k$	$e$	$r_1$	$r_2$	$h$	$k_1$	$n - k$	$e$	$r_1$	$r_2$	$h$	$k_1$
31	3.7086	24	9	3	2	43	3.0146	8	37	3	2
32	3.6296	23	11	3	2	44	3.0418	0	44	0	6
33	3.5522	22	13	3	2	45	3.9763	0	45	0	6
34	3.4681	21	15	3	2	46	2.9407	0	46	0	6
35	3.4007	20	17	3	2	47	2.9385	0	47	0	6
36	3.3457	18	20	3	2	48	2.9136	0	48	0	6
37	3.3048	17	22	3	2	49	2.8602	0	49	0	6
38	3.2225	15	25	3	2	50	2.8257	0	50	0	6
39	3.1811	14	27	3	2	51	2.7749	0	51	0	6
40	3.1338	12	30	3	2	52	2.7129	0	52	0	6
41	3.0990	10	33	3	2	53	2.6447	0	53	0	6
42	3.0688	9	35	3	2	54	2.5793	0	54	0	6

**Table 8.** Optimal values of PCM parameters with different message lengths ( $n = 100$ )

$n - k$	$e$	$r_1$	$r_2$	$h$	$k_1$	$n - k$	$e$	$r_1$	$r_2$	$h$	$k_1$
52	3.8530	41	13	3	4	72	3.1743	14	60	3	5
54	3.7443	39	17	3	4	74	3.1201	12	64	3	5
56	3.6842	36	22	3	4	76	3.0844	6	72	3	5
58	3.5905	33	27	3	4	78	3.0457	3	77	3	6
60	3.5404	30	32	3	4	80	2.9762	3	79	3	6
62	3.4675	28	36	3	4	82	2.9213	0	82	0	8
64	3.4122	25	41	3	4	84	2.8615	0	84	0	8
66	3.3509	23	45	3	4	86	2.8108	0	86	0	8
68	3.2863	21	49	3	4	88	2.7397	0	88	0	8
70	3.2237	18	54	3	4	90	2.6676	0	90	0	8

**Table 9.** Encoding time delay with different message lengths ( $n = 60$ )

$n - k$	31	32	33	34	35	36	37	38
STCs (ms)	0.481	0.494	0.517	0.535	0.549	0.568	0.574	0.597
Wang et al.'s method (ms)	0.501	0.512	0.525	0.546	0.556	0.574	0.575	0.606
Proposed method (ms)	0.563	0.551	0.542	0.533	0.531	0.520	0.511	0.507
$n - k$	39	40	41	42	43	44	45	46
STCs (ms)	0.601	0.625	0.637	0.652	0.670	0.687	0.695	0.715
Wang et al.'s method (ms)	0.610	0.626	0.634	0.640	0.664	0.688	0.693	0.706
Proposed method (ms)	0.498	0.475	0.451	0.429	0.403	0.682	0.697	0.706
$n - k$	47	48	49	50	51	52	53	54
STCs (ms)	0.720	0.745	0.763	0.787	0.792	0.814	0.827	0.830
Wang et al.'s method (ms)	0.711	0.722	0.744	0.758	0.772	0.780	0.803	0.821
Proposed method (ms)	0.711	0.723	0.744	0.758	0.771	0.780	0.803	0.821

**Table 10.** Encoding time delay with different message lengths ( $n = 100$ )

$n - k$	52	54	56	58	60	62	64	66	68	70
STCs (ms)	5.402	5.614	5.826	6.023	6.218	6.440	6.648	6.845	7.055	7.257
Wang et al.'s method (ms)	5.555	5.721	5.912	6.100	6.313	6.504	6.675	6.862	7.096	7.303
Proposed method (ms)	6.181	6.113	6.059	6.006	5.927	5.821	5.748	5.689	5.608	5.530
$n - k$	72	74	76	78	80	82	84	86	88	90
STCs (ms)	7.450	7.676	7.875	8.082	8.284	8.485	8.704	8.921	9.134	9.316
Wang et al.'s method (ms)	7.485	7.662	7.857	8.030	8.225	8.402	8.613	8.796	9.023	9.247
Proposed method (ms)	7.463	7.136	4.670	5.799	5.941	8.401	8.613	8.796	9.023	9.247

## 6 Conclusion

In this paper, a further study on the PCM construction was proposed. We analyzed the approaches to realizing fast ME. On this basis, two specific matrix structures for small payloads (payloads that are smaller than 0.5) and large payloads (payloads that are larger than 0.5) were presented. Experimental results showed that our fast ME methods can realize better embedding efficiency and faster embedding speed than state-of-the-art works. It's worth mentioning that though we fixed the cover length to make comparison in our experiments, the two novel methods both can be applied to covers of arbitrary length.

## References

- [1] H. Tian, J. Qin, Y.-F. Huang, Optimal Matrix Embedding for Voice-over-IP Steganography, *Signal Processing*, Vol. 117, No. C, pp. 33-43, December, 2015.
- [2] J. Liu, K. Zhou, H. Tian, Frame-bitrate-change Based Steganography for Voice-over-IP, *Journal of Central South University*, Vol. 21, No. 12, pp. 4544-4552, December, 2014.
- [3] Y.-F. Huang, S. Tang, Y. Zhang, Detection of Covert Voice-over Internet Protocol Communications Using Sliding Window-based steganalysis, *IET Communications*, Vol. 5, No. 7, pp. 929-936, May, 2011.
- [4] S.-F. Yan, G.-M. Tang, Y.-F. Sun, Z.-Z. Gao, L. Shen, A Triple-layer Steganography Scheme for Low Bit-rate Speech Streams, *Multimedia Tools & Applications*, Vol. 74, No. 24, pp. 11763-11782, December, 2015.
- [5] Z.-L. Wei, B.-K. Zhao, B. Liu, J. Su, L. Xu, E. Xu, A Novel Steganography Approach for Voice over IP, *Journal of*

- Ambient Intelligence & Humanized Computing*, Vol. 5, No. 4, pp. 601-610, August, 2014.
- [6] R. Crandall, Some Notes on Steganography, <http://www.dia.unisa.it/~ads/corso-security/www/CORSO-0203/steganografia/LINKS%20LOCALI/matrix-encoding.pdf>.
- [7] J. Fridrich, P. Lisoněk, D. Soukal, On Steganographic Embedding Efficiency, *8th International Workshop on Information Hiding*, Alexandria, VA, 2006, pp. 282-296.
- [8] J. Fridrich, T. Filler, Practical Methods for Minimizing Embedding Impact in Steganography, *Security, Steganography and Watermarking of Multimedia Contents IX*, Bellingham, WA, 2007, pp. 201-215.
- [9] T. Filler, J. Judas, J. Fridrich, Minimizing Embedding Impact in Steganography Using Trellis-coded Quantization, *Media Forensics and Security II*, Bellingham, WA, 2010, pp. 175-178.
- [10] T. Filler, J. Judas, J. Fridrich, Minimizing Additive Distortion in Steganography Using Syndrome-trellis Codes, *IEEE Transactions on Information Forensics and Security*, Vol. 6, No. 3, pp. 920-935, September, 2011.
- [11] J. Qin, H. Tian, Y.-F. Huang, J. Liu, Y. Chen, T. Wang, Y. Cai, X. A. Wang, An Efficient VoIP Steganography Based on Random Binary Matrix, *10th International Conference on P2P, Parallel, Grid, Cloud and Internet Computing*, Krakow, Poland, 2015, pp. 462-465.
- [12] Z.-J. Wu, H.-J. Gao, D.-Z. Li. An Approach of Steganography in G.729 Bitstream Based on Matrix Coding and Interleaving, *Chinese Journal of Electronics*, Vol. 24, No. 1, pp. 157-165, January, 2015.
- [13] X.-L. Li, S.-R. Cai, W.-M. Zhang, B. Yang, A Further Study of Large Payloads Matrix Embedding, *Information Sciences*, Vol. 324, No. C, pp. 257-269, December, 2015.
- [14] A. Westfeld, F5—A Steganographic Algorithm: High Capacity Despite better Steganalysis, *4th International Workshop on Information Hiding*, Pittsburgh, PA, 2001, pp. 289-302.
- [15] J. Fridrich, D. Soukal, Matrix Embedding for Large Payloads, *IEEE Transactions on Information Forensics and Security*, Vol. 1, No. 3, pp. 390-395, September, 2006.
- [16] Q. Mao, A Fast Algorithm for Matrix Embedding Steganography, *Digital Signal Processing*, Vol. 25, No. 1, pp. 248-254, February, 2014.
- [17] C. Wang, W.-M. Zhang, J.-F. Liu, N. Yu, Fast Matrix Embedding by Matrix Extending, *IEEE Transactions on Information Forensics and Security*, Vol. 7, No. 1, pp. 346-350, February, 2012.
- [18] Y.-K. Gao, X.-L. Li, T.-Y. Zeng, B. Yang, Improving Embedding Efficiency via Matrix Embedding: A Case Study, *16th IEEE International Conference on Image Processing*, Cairo, Egypt, 2009, pp. 109-112.
- [19] J.-J. Wang, H.-S. Chen, A Suboptimal Embedding Algorithm with Low Complexity for Binary Data Hiding, *IEEE International Conference on Acoustics, Speech and Signal Processing*, Kyoto, Japan, 2012, pp. 1789-1792.
- [20] J.-J. Wang, C.-Y. Lin, H.-S. Chen, T.-Y. Yang, A Suboptimal Embedding Algorithm for Binary Matrix Embedding, *International Symposium on Computer, Consumer and*

*Control*, Taichung, Taiwan, 2012, pp. 165-168.

- [21] An4 Database, <http://www.speech.cs.cmu.edu/databases/an4/>.

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