A Further Study of Optimal Matrix Construction for Matrix Embedding Steganography

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Abstract

Matrix embedding is a general approach that can be applied to most steganographic schemes to improve their embedding efficiency. In order to apply matrix embedding to voice-over-IP (VoIP) steganography better, this paper analyses the means to realizing fast matrix embedding. For small payloads, we discuss the feasibility of combining several Hamming codes into parity check matrix (PCM) construction and propose a novel PCM structure. On this basis, a corresponding optimization algorithm is proposed. It can adaptively generate specific PCMs to accommodate to the given cover and provide the best performance while guaranteeing the allowable computational complexity. For large payloads, another PCM structure is presented by combining the PCM of syndrome trellis codes (STCs) and several referential columns. The corresponding optimal construction algorithm is also given. Experimental results show that compared with existing methods, two novel matrix embedding methods achieve higher embedding efficiency and faster embedding speed.

Keywords: Steganography, Matrix embedding, Parity check matrix, Embedding efficiency, Embedding speed

1 Introduction

Steganography is a covert communication technology. Up to now, steganographic covers have been extended from images to almost all kinds of multimedia. Voice over IP (VoIP) is the most popular real-time service in IP networks at present. The study on VoIP steganography is becoming extensive, and various approaches [1-5] have been developed.

No matter what the steganographic cover is, for a given message and cover, the scheme that introduces fewer changes will be more secure in general. Based on this understanding, matrix embedding (ME, also known as syndrome coding) was proposed by Crandall [6]. ME can improve embedding efficiency. It requires the sender and the recipient to agree in advance on a parity check matrix (PCM). Using the PCM, the sender selects the coset leader of error correction codes as the modification vector, and the recipient extracts secret messages by calculating the syndrome of the stego data. ME was made popular by Westfeld who incorporated it into F5 algorithm. After that, Fridrich et al. systematically analyzed ME [7-8] and got its upper bound of embedding efficiency. They also proved that random linear code-based ME can approach this theoretical bound. Recently, ME has been extended to convolution codes, such as syndrome trellis codes (STCs) [9-10]. STCs can embed a given payload with minimal total distortion if the cost of changing each cover element is assigned. This task can be viewed as a generalization of initial ME or writing on wet paper.

To date, existing VoIP steganography methods mainly hide information in the LSBs of speech streams [5]. Since direct LSB replacement degrades speech quality obviously, many methods [11-12] improve their security through using initial ME which can minimize the number of embedding changes. Convolution codes that minimize total distortion are rarely applied to VoIP steganography for the following reasons: On the one hand, different from image cover, the relations of cover elements in speech streams are much more complicated and not intuitive. This leads to extremely rare research achievements in VoIP distortion function. On the other hand, speech streams are generated and transmitted in real time, which gives a short time to perform embedding or extracting process. Whereas minimizing total distortion usually needs larger amount of computation and the process of calculating single-letter distortions costs a certain time. Beyond that, convolution codes are more suitable for long covers. But in VoIP scheme, encoder often divides a cover into small parts and performs embedding operation on each part to maintain the real-time requirement [1]. In this case, the embedding efficiency of convolution codes needs to be further improved.

Our goal is to propose a novel fast method for ME, so that it can be better applied to VoIP steganography. To reduce computational complexity of ME, researchers have developed many improved methods through two ways [13]. The core idea of the first class is to construct special PCM. Typical examples include
Hamming code-based ME [14] and random linear code-based ME [15]. Though they are early methods, their embedding efficiencies are relatively high benefited from their excellent PCM structures. To further improve Hamming code-based ME, Mao proposed a fast method [16] in which the positions of PCM columns are changed to make all columns array in ascending (or descending) order in decimal form, then the coset leader can be found by using a lookup table algorithm. Aiming at the shortcoming of immobility of Hamming codes, Tian et al. presented an adjustable ME method [1] which can adaptively generate a guide matrix to accommodate to various cover lengths and achieve the optimal embedding performance. To increase embedding speed of random linear code-based ME, Wang et al. proposed a new method by translating several random columns to referential columns [17]. For the second class, its core idea is to find a sub-optimal solution as the modification vector instead of the coset leader. Hence, Gao et al. turned to finding a vector in the coset which has relatively small Hamming weight [18]. After that, similar methods such as [19-20] were proposed. Compared with the first kind, these approaches achieve faster embedding speed but at the cost of a fall of embedding efficiency.

We make a further study on the optimization of PCM construction in this paper. Two special matrix forms for small payloads (payloads that are smaller than 0.5) and large payloads (payloads that are larger than 0.5) are presented, respectively. The paper is organized as follows: In Section 2, we review a few elementary concepts of ME and the related works that will be needed for the rest part. Section 3 and Section 4 explain two novel ME methods. Experimental results and their analyses appear in Section 5. Finally, the paper is concluded in Section 6.

2 Related Works

2.1 Matrix Embedding

Without loss of generality, the cover and the secret message are regarded as binary sequences in this paper. Matrix $H$ of dimension $(n-k) \times n$ is the PCM of binary linear $[n,k]$ codes $C$. Based on $H$, the sender can embed $n-k$ secret bits $m^T=(m_1,m_2,\ldots,m_{n-k})$ into an $n$-length cover $c^T=(c_1,c_2,\ldots,c_n)$ . The key problem of ME is to find a modification vector with minimum Hamming weight. Hence, first calculate the difference between $m$ and $Hc$. The result is denoted by $u$, i.e., $u=m \oplus Hc$ . Then, get its coset with respect to $H$.

$$C_u(u)=\{x \in GF(2^n) \mid Hx=u\}$$  \hspace{1cm} (1)

$C_u(u)$ contains $2^k$ vectors. Among them, the one that has the smallest Hamming weight is called coset leader.

$$e_L(u)=\arg \min_{x \in C_u(u)} \omega(x)$$  \hspace{1cm} (2)

$e_L(u)$ represents the optimal modification vector, so the stego $s$ is obtained as follows.

$$s=c \oplus e_L(u)$$  \hspace{1cm} (3)

The recipient extracts secret messages by computing

$$Hs=H(c \oplus e_L(u))=Hc \oplus H \cdot e_L(u)$$

$$=Hc \oplus (m \oplus Hc)=m$$  \hspace{1cm} (4)

2.2 Wang et al.’s Fast Matrix Embedding

The first ME with feasible complexity was proposed by Fridrich et al. [15]. Their PCM structure is

$$H=(I_{n-k}, R)$$  \hspace{1cm} (5)

In which $I_{n-k}$ is an $(n-k) \times (n-k)$ unit matrix, and $R$ is an $(n-k) \times k$ random matrix. The coset leader can be found with $O(n2^k)$ computations.

Based on structure (5), Wang et al. proposed a novel fast method by extending the PCM via some referential columns [17]. Its computational complexity is reduced to $O(n2^k)$ . The PCM they construct is

$$H=(I_{n-k}, R, D)$$  \hspace{1cm} (6)

where $R$ is a random matrix of dimension $(n-k) \times k_i$, $D$ is an $(n-k) \times k_2$ matrix and $k_1+k_2=k$ . The $i$th referential column in $D$ is in the following form:

$$d_i=\left[0,\ldots,0,c_i,1,0,0,\ldots,0\right], 1 \leq i \leq k_2$$  \hspace{1cm} (7)

$t_i$ is usually taken as

$$t_i=\begin{cases} \frac{n-k}{k_2} & \text{if } i<k_2 \\ \frac{(n-k) - (k_2 - 1) \cdot n-k}{k_2} & \text{if } i=k_2 \end{cases}$$  \hspace{1cm} (8)

The following matrix is a specific form of $H$ when $(n,k,k_1,k_2)=(11,5,2,3)$.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$  \hspace{1cm} (9)
According to (6), each modification vector \( x \) can be split into three parts \( x^f = (x_0^f, x_1^f, x_2^f) \), and they satisfy the condition \( x_i \oplus Rx_i \oplus D_{x_2} = u \). Divide both \( x_i \) and \( u \oplus Rx_i \) into \( k_i \) segments (\( |x_i| = |u \oplus Rx_i| = t_j \)). Consequently, the coset leader of \( C_H(u) \) can be found by minimizing the following quantity:

\[
\omega(x_i) + \sum_{i=1}^{k_i} \min\{\omega(u \oplus Rx_i), t_j - \omega(u \oplus Rx_i), +1]\]  \( (10) \)

### 3 Optimal Matrix Construction for Small Payloads

#### 3.1 Structure of the Proposed PCM

The referential columns in (6) can effectively improve embedding efficiency of ME when \( k_i \) is small. But, as \( k_i \) increases, the impact of the referential columns becomes smaller. When \( (n-k)/k_i \geq 1 \) (i.e., payload is smaller than 0.5), the referential columns will not work.

By changing positions of the referential columns, the PCM in (6) can be transformed into the following form.

\[
H = (I_{n-k}, R, D) = (A, R) \]  \( (11) \)

where \( A \) is an \( (n-k) \times (n-k+k_i) \) matrix. For instance, \( A \) in (9) can be rewritten as

\[
A = \begin{pmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_p \end{pmatrix}, \quad B_i = B_2 = B_p = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \]  \( (12) \)

When \( (n-k)/k_i = 2 \), we note that submatrix \( B \) is actually the PCM of [3, 1] Hamming codes (as shown in (12)). It inspires us to use Hamming codes to expand the application scope of Wang et al.’s method. The PCM we construct for small payloads is shown below.

\[
H = (A, R), \quad A = \begin{pmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & B_p \end{pmatrix} \]  \( (13) \)

where \( B_1, B_2, \ldots, B_p \) are all PCMs of Hamming codes, and they could be different.

#### 3.2 PCM Optimization

For a given message \( m \) and cover \( c \), the dimension of \( H \) is definite. The restriction on computational complexity can determine the number of random columns. Therefore, to achieve optimal embedding efficiency, we only need to discuss how to construct \( A \).

Let \( p \) denote the number of submatrices in \( A \). \( r_i, w_i \) denote the height and width of \( B_i \) respectively. There is a certain function relation between \( r \) and \( w_i \).

\[
w_i = f(r_i) = 2^{r_i} - 1, \quad i \in \{1, 2, \ldots, p\} \]  \( (14) \)

On this basis, the structure of \( A \) could be expressed as \( r = \{r_1, r_2, \ldots, r_p\} \), and we can get Theorem 1, 2 below.

**Theorem 1.** For a local matrix \( A \) containing \( p \) submatrices, when its height is fixed as \( n-k \), the number of columns in \( A \) has a certain range:

\[
p(2^{r} - 1) \leq \sum_{i=1}^{p} w_i \leq p + 2^{n-k-1} - 2 \]  \( (15) \)

**Proof.** Matrix height is fixed, thus \( \sum_{i=1}^{p} r_i = n-k \).

According to the average value inequality, we can derive a relationship as follows:

\[
\sum_{i=1}^{p} w_i = f(r_1) + f(r_2) + \cdots + f(r_p) = (2^{r_1} - 1) + (2^{r_2} - 1) + \cdots + (2^{r_p} - 1) = 2^{r_1} + 2^{r_2} + \cdots + 2^{r_p} - p \geq p(\sum_{i=1}^{p} r_i^{n-k-1}) - p = p(2^{r} - 1)
\]

If and only if \( r_1 = \cdots = r_p = (n-k)/p \), the equation holds.

On the other hand, we have \( f'(r) = 2^r \ln 2 > 1 \) and \( f''(r) = 2^{(r+1)} > 0 \) for \( r \geq 1 \). That is to say, \( f(r) \) goes up as \( r \) increases, and the growth range is larger and larger. Therefore,

\[
\sum_{i=1}^{p} w_i = f(r_1) + f(r_2) + \cdots + f(r_p) \leq f(1) + \cdots + f(1) = \cdots \leq f(1) + f(1) + \cdots + f(1) + (r_p - 1) \leq f(1) + f(1) + \cdots + f(1) + (p-1) = p + 2^{n-k-1} - 2
\]

Considering the above analysis, it can be concluded that the column number of \( A \) is related to the diversity of submatrices. The column number achieves the minimum when the sizes of submatrices are all the same, achieves the maximum as \( r = \{1, \ldots, 1, n-k-(p-1)\} \).

**Theorem 2.** For a local matrix \( A \), when its height is fixed as \( n-k \), small number of submatrices (\( p \) is small) is conducive to the increasement of column number.

**Proof.** \( r = \{r_1, \cdots, r_p\} \) when \( A \) contains \( p \) submatrices. If there are \( p' = p+1 \) submatrices, \( r' = \{r_1', \cdots, r_{p'}\} \).

Then we can get consequences follow from Theorem 1:
max $\sum_{i=1}^{n} w_i = p + 2^{n-k-(p-1)} - 2$

$= p + 1 + 2^{n-k-p} - 2 + 2^{-n-k-p} - 1$

$\geq p + 1 + 2^{n-k-p} - 2$

$= \max \{ p, w_i \}$

$\min \sum_{i=1}^{n} w_i = p(2^{n-k/p} - 1)$. Take the derivative of

$\frac{d(\min \sum_{i=1}^{n} w_i)}{dp} = \frac{n-k}{p} - 1 - \frac{n-k}{p} \ln 2 \cdot \frac{n-k}{p^2} = \frac{n-k}{p} - \ln 2 \cdot \frac{n-k}{p} - 1$

Since $(n-k)/p \geq 1$, $(n-k)/p \in N^*$, It’s easy to know $2^n (1-\ln2 \cdot \frac{n-k}{p}) < 1$. So $\min \sum_{i=1}^{n} w_i / dp < 0$, i.e.,

$\min \sum_{i=1}^{n} w_i > \min \sum_{i=1}^{p} w_i$. Hamming codes could embed $r$ bits of messages into $2^r - 1$ bits of cover data with one change at most. The probability of modifying the cover is $(2^r - 1)/2^r$. Thus the embedding efficiency is $r \cdot 2^r/(2^r - 1)$. On this basis, we have the following results.

**Theorem 3.** For a local matrix $A$ containing $p$ submatrices, when its height is fixed as $n-k$, the embedding efficiency of $A$ is related to the diversity of submatrices. The greater the diversity of submatrices is, the higher the embedding efficiency will be. The range of the embedding efficiency is

$$\frac{n-k}{p - p \cdot \frac{1}{2^{n-k/p}}} \leq e \leq \frac{n-k}{p + 1 - \frac{1}{2^{n-k-(p-1)}}}$$

**Proof.** $A$ contains $p$ submatrices. Hence, the average number of embedding changes is

$$E(\omega_d) = \sum_{i=1}^{p} 2^r_i - 1 = p - \sum_{i=1}^{p} \frac{1}{2^r_i}$$

According to the average value inequality, we can learn that

$$\sum_{i=1}^{p} g(r_i) = -2^{-r_1} - 2^{-r_2} \ldots - 2^{-r_p}$$

$$\leq -p \sqrt{2^{-r_1} \cdot 2^{-r_2} \cdot \ldots \cdot 2^{-r_p}}$$

$$= -p \cdot 2^{-\frac{n-k}{p}}$$

where $g(r) = -2^{-r}$. Therefore $E(\omega_d) \leq p - p/2^{n-k/p}$. If and only if $r_i = \ldots = r_p = (n-k)/p$, the equation holds.

On the other hand, we have $g'(r) = 2^{-r} \ln 2 > 0$ and $g''(r) = -2^{-r} (\ln 2)^2 < 0$ for $r \geq 1$. That is to say, $g(r)$ goes up as $r$ increases, but the growth range is smaller and smaller. Therefore,

$$\sum_{i=1}^{p} g(r_i) = g(r_1) + g(r_2) + \ldots + g(r_p)$$

$$\geq g(1) + g(r_1) + \ldots + g(r_p) + (r_1-1) \geq \ldots$$

$$\geq g(1) + g(1) + \ldots + g(r_1) + (r_1-1) + \ldots + (r_{p-1} - 1)$$

$$= g(1) + g(1) + \ldots + g(n-k-(p-1))$$

Consequently,

$$E(\omega_d) \geq p - \left( \frac{p-1}{2} + \frac{1}{2^{n-k-(p-1)}} \right) = \frac{p+1}{2} - \frac{1}{2^{n-k-(p-1)}}$$

Divide $n-k$ (the message length) by $E(\omega_d)$, we will get the expression in (16).

**Theorem 4.** For a local matrix $A$, when its height is fixed as $n-k$, small number of submatrices ($p$ is small) is conducive to the improvement of embedding efficiency.

Theorem 4 is the conclusion follows from Theorem 3. The proof process is the same as Theorem 2. According to Theorem 2 and Theorem 4, to improve embedding efficiency and embedding speed, minimizing the number of submatrices should be our primary goal in PCM construction. According to Theorem 1 and Theorem 3, we found that given the number of submatrices, enlarging the diversity of submatrices can reduce the number of random columns (i.e. reduce the computational complexity) and is conducive to the improvement of embedding efficiency. For instance, 5 bits need to be embedded into 20 bits of cover data. Let $r_1 = 2$, $r_2 = 3$, we can embed the message using a PCM combined by a [3, 1] code and a [7, 4] code. Let $r_1 = 1$, $r_2 = 4$, we can also embed the message by combining a first-order unit matrix and a [15, 11] code. But the former PCM has 10 random columns; the latter only has 4 random columns. Besides that, the embedding efficiency of the former PCM is 28/9, lower than 80/23 of the latter.

In conclusion, the optimization of local matrix $A$ can be accomplished in two steps: Calculate the optimal number of submatrices in accordance with the message length. And then determine the size of each submatrix. In more specific terms, the first step is to find the minimum $p$ meeting the relation $p(2^{n-k/p} - 1) \leq n \leq p + 2^{n-k-(p-1)} - 2$, namely to solve the following optimization problem:
is the goal of maximizing the diversity of them.  

\begin{align}
\text{minimize } & p \\
\text{subject to } & \left[ \frac{n-k}{p} \right] + (p-1) \left[ \frac{n-k}{p} \right] = n - k \\
& p \in N^*, \; l \in \{1, 2, \ldots, p\} \\
& l(2^{\left\lfloor \frac{n-k}{p} \right\rfloor} - 1) + (p-1)(2^{\left\lfloor \frac{n-k}{p} \right\rfloor} - 1) \leq n \\
& p + 2^{n-k}(p-1) - 2 \geq n \\
\end{align}

Its searching process is

Step 1: Initialize \( p = 1 \);

Step 2: Determine \( l \) according to the equation

\[ l \left[ \frac{n-k}{p} \right] + (p-1) \left[ \frac{n-k}{p} \right] = n - k, \; l \in \{1, 2, \ldots, p\}; \]

Step 3: If \( l(2^{\left\lfloor \frac{n-k}{p} \right\rfloor} - 1) + (p-1)(2^{\left\lfloor \frac{n-k}{p} \right\rfloor} - 1) \leq n \), go to next step. Otherwise \( p = p + 1 \) and return to Step 2;

Step 4: Output \( p \).

After that, optimal submatrices can be determined with the goal of maximizing the diversity of them.

\begin{align}
\text{maximize } & \sum_{i=1}^{p} r_i = 2^i + 2^{i+1} + \cdots + 2^p - p \\
\text{subject to } & \sum_{i=1}^{p} r_i = n - k \\
& \sum_{i=1}^{p} w_i \leq n \\
& r_i \in N^+, \; i \in \{1, 2, \ldots, p\} \\
\end{align}

The optimization process is

Step 1: Initialize \( r_p = n - k - (p - 1), \; r_1 = \cdots = r_{p-1} = 1, \; j = p - 1, \; p_r = 2; \)

Step 2: If \( \sum_{i=1}^{p} (2^{r_i} - 1) \leq n \), go to Step 5. If not, go to next step;

Step 3: \( r_p = r_p - 1 \). If \( r_j \leq \left\lfloor \frac{(n-k-(p-p_c))}{p_r} \right\rfloor \), update \( r_j = r_j + 1 \) and return to Step 2. If not, \( j = j - 1 \) and go to next step;

Step 4: If \( j > p - p_c \), update \( r_j = r_j + 1 \) and return to Step 2. If not, \( p_r = p_r + 1, \; j = p - 1 \) and reinitialize \( r_p = n - k - (p - 1), \; r_1 = r_2 = \cdots = r_{p-1} = 1 \), return to Step 3;

Step 5: \( k_1 = \sum_{i=1}^{p} (2^{r_i} - 1) - (n - k), \; k_i = k - k_z \), Output \( r_1, \; r_2, \; \ldots, \; r_p, \; k_1, \; k_2 \).

3.3 Computational Complexity Analysis

For the proposed PCM, each modification vector can be written as \( x^t = (x^2_t, x^1_t) \), and they satisfy the condition \( Ax_x \oplus Rx_x = u \). Therefore, searching for the coset leader of \( C_{s}(u) \) is a two-step process: First, get coset leaders of \( C_{s}(u \oplus Rx_x) \) under different \( x_x \). Second, choose a vector that minimizing the quantity \( \omega(e_{\;s}(u \oplus Rx_x)) + \omega(x_x) \) as the final modification vector.

In order to reduce time cost in the first step, we adopt the method proposed in [16], change the positions of the columns in \( A \) to make all columns array in ascending (or descending) order in decimal form as follows:

\[
B_i = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}
\]

By this mean, the syndromes \( (u \oplus Rx_x) \), will indicate the coset leaders. Supposing that \( (u \oplus Rx_x)^T = (1, 0, 0) \), since \((1, 0, 0)\) is 4 in decimal form, \((x_x^{opt})^T = (0, 0, 0, 1, 0, 0, 0, 0)\). The computational complexity to find the coset leader of \( C_{s}(u \oplus Rx_x) \) is \( O(p) \).

Theorem 5. According to the embedding features of Hamming code-based ME, for a local matrix \( A \) containing \( p \) submatrices, no matter what \( x_x \) is, the inequality below would always hold.

\[
\omega(e_{\;s}(u \oplus Rx_x)) + \omega(x_x) \leq p
\]

Therefore, the optimal modification vector satisfies the condition \( \omega(x_x) \leq p \). This fact leaves us a clue to find the coset leader with reduced computational complexity in the second step. We only need to process and store vectors in \( C_{s}(u \oplus Rx_x) \) when \( \omega(x_x) \leq p \), and select one having the smallest Hamming weight among them. The number of combinations we need to deal with is

\[
\mu_{h,p} = \begin{cases} \sum_{i=0}^{p-1} C_{h}^i & \text{if } p - 1 \leq k_i \\ \sum_{i=0}^{h} C_{h}^i & \text{if } p - 1 > k_i \end{cases}
\]

Hence, the computational complexity of the novel fast ME method is \( O(p \mu_{h,p}) \).

4 Optimal Matrix Construction for Large Payloads

4.1 Structure of the Proposed PCM

STCs [9] proposed by Filler et al. is the most famous convolution codes. Its computational complexity is linear with \( h \) and exponential with \( h \) (the height of the submatrix). Embed messages into \( 10^5 \)-length covers and \( 10^6 \)-length covers using STCs, respectively. Figure 1 shows the average embedding efficiency of \( 10^5 \) experiments with different \( h \). Figure 1 indicates that the embedding efficiency of STCs tends to increase along with \( h \), but the improvement is small.
when \( h \) is large. More importantly, we found that, different with small payloads, the embedding efficiency shows little change with various \( h \) when payloads are large. It is stable since \( h \) is small. This phenomenon is more obvious when \( 2^{10^n} \).

\[
A = \begin{pmatrix}
1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0 & 1 & \cdots & 1 & 1 \\
1 & 1 & \cdots & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
1 & 0 & 1 \\
0 & 1 & 1
\end{pmatrix}
\]

(23)

\( A \) consists of two parts: \( A_1 \) has the same form as STCs, and \( A_2 \) is in the form of PCM with referential columns. As the payload is larger than 0.5, we set 2 as the width of submatrices in \( A_1 \). Each part in (23) is shown below.

\[
A_1 = \begin{pmatrix}
1 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0 & 1 & \cdots & 1 & 1 \\
1 & 1 & \cdots & 0 & 1 & 1 & 1 \\
\end{pmatrix},
B_1 = \begin{pmatrix}
1 & 1 \\
0 & 1 \\
1 & 1
\end{pmatrix}
\]

(24)

\[
A_2 = \begin{pmatrix}
B_{21} & 0 & \cdots & 0 \\
0 & B_{22} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & B_{2b}
\end{pmatrix},
B_{21} = B_{22} = \cdots = \begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 1
\end{pmatrix}
\]

(25)

4.2 PCM Optimization

For a given \( m \) and \( c \), the size of \( H \) is definite. And the restriction on computational complexity can determine the size of \( R \). In this case, we only need to discuss how to optimize \( A_1 \) and \( A_2 \).

\( r_1, w_1 \) denote the height and width of \( A_1 \), respectively. Similarly, \( r_2, w_2 \) mean the height and width of \( A_2 \). Then we have

\[
\begin{cases}
    r_1 + r_2 - h = n - k \\
    w_1 + w_2 = n - k_i
\end{cases}
\]

(26)

As the width of submatrices in \( A_1 \) is 2, the size of \( A_1 \) can be expressed as \( r_1 \times 2(r_1 - h + 1) \). And the size of \( A_2 \) is \( (n - k - r_1 + h - 1) \times (n - k_i - 2r_1 + 2h - 2) \) according to (26). Divide local modification vector \( x_0 \) into two segments, \( x_0^T = (x_{0,1}^T, x_{0,2}^T) \) (\( |x_{0,1}| = w_1 \)). Thus, optimal \( A \) can be determined by finding the \( r_i \) meeting the following conditions.

\[
H = (A, R)
\]

(22)
arg min \( \omega(x_{0,1}) + \omega(x_{0,2}) \) \( r_i \in \{0, 3, 4, \cdots, n-k\} \)
subject to
\[
\begin{align*}
2r_i &\le n-k_i \\
2r_i &\le w_2 \\
r_i &\le w_2 \\
r_i &\le n-k_i + h - 1 \\
w_2 &= n-k_i - 2r_i + 2h - 2
\end{align*}
\]

(27)

The expectations of \( \omega(x_{0,j}) \) can be estimated through experiments using STCs, the PCM dimension of which is (1 - 2h) × 2(1 + h) . The red line in Figure 2 indicates \( E(\omega(x_{0,j})) \) when \( h = 3 \).

\[
E(\omega(x_{0,1})) = E(\omega(x_{0,2})) = E(\omega(x_{0,j}))
\]

Figure 2. Variation of Hamming weight of the modification vector with different \( r_i \).

\[
w_2 - r_i = n-k_i - r_i + h - 1 \quad \text{denotes the number of submatrices in } A_1 . \quad \text{According to (8), their heights are}
\]

\[
t_i = \begin{cases} 
\frac{n-k_i + h - 1}{k-k_i - r_i + h - 1} & \text{if } i < k-k_i - r_i + h - 1 \\
\frac{n-k_i + h - 1}{k-k_i - r_i + h - 1} & \text{if } i = k-k_i - r_i + h - 1 \\
\frac{(n-k_i + h - 1) - (k-k_i - r_i + h - 2)}{(n-k_i + h - 1) - (k-k_i - r_i + h - 2)} & \text{if } i > k-k_i - r_i + h - 1
\end{cases}
\]

(28)

Furthermore, we can get the average number of embedding changes using \( A_2 \).

\[
E(\omega(x_{0,1})) = \sum_{j=0}^{t_1} \left( \frac{C_{i_1}^j}{\sum_{k=0}^{t_1} C_{i_1}^k} \cdot j + \sum_{j=0}^{t_1} C_{i_1}^j \cdot (t_j - j - 1) \right) \]

\[
E(\omega(x_{0,2})) = \sum_{j=0}^{t_2} \left( \frac{C_{i_2}^j}{\sum_{k=0}^{t_2} C_{i_2}^k} \cdot j + \sum_{j=0}^{t_2} C_{i_2}^j \cdot (t_j - j - 1) \right)
\]

(29)

For \( n = 60 \), \( n-k = 36 \), \( k_i = 3 \), the average change number using \( A_1 \) is shown by the blue line in Figure 2. Taking \( E(\omega(x_{0,1})) \) and \( E(\omega(x_{0,2})) \) into consideration, the optimal size of \( A_1 \) and \( A_2 \) can be determined as \( 20 \times 36 \) and \( 18 \times 21 \) on this occasion.

4.3 Computational Complexity Analysis

Searching for the coset leader of \( C_H(u) \) with respect to the proposed PCM for large payloads also needs two steps. The computational complexity of finding the coset leader corresponding to \( A_2 \) is linear with \( w_2 \). Therefore, the computational cost of the first step is close to that of STCs, i.e., \( O(w_1 2^k) \). More precisely, considering that extra computation is needed at the juncture of two kinds of submatrices, the whole computational complexity of our method is \( O(2^k \cdot (2^{h-1} \cdot w_1 2^h + w_2 + (2^{h-1} - 1) \cdot v_{h,k} )) \), where

\[
v_{h,k} = \min \{ t'_1, t'_2 \}
\]

subject to \( t'_1 \ge h - 1 \)

(30)

5 Experimental Results

If an ME method has low computational complexity, larger PCM can be utilized and it may lead to higher embedding efficiency. Therefore, some papers only ensure that ME methods have the same computational complexity and embedding rate, ignoring PCM sizes, when compare embedding efficiency. But, in practice, cover data may be divided into small parts and the embedding process is performed on each part [9]. In this case, comparing ME methods should under the condition that PCMs have the same size. So we take PCM sizes into consideration in the following experiments.

5.1 Experiments of ME for Small Payloads

Experiment-1: Take the case of \( n = 60 \) for example. Following the searching process described in Section 3.2, we got the optimal PCMs for small payloads in Table 1. For each payload, 5000 messages and 5000 covers are generated. All of them are random binary sequences. Embed these messages and record embedding efficiencies. Calculate the mean value of 5000 experimental results as the final result.

Comparison of embedding efficiency between Hamming codes [14], Tian et al.’s method [1], Wang et al.’s method [17] and the proposed method is shown in Figure 3.
Table 1. Optimal PCM schemes with different message lengths ($n = 60$)

<table>
<thead>
<tr>
<th>$n - k$</th>
<th>$e$</th>
<th>$r$</th>
<th>$p$</th>
<th>$k_i$</th>
<th>$n - k$</th>
<th>$e$</th>
<th>$r$</th>
<th>$p$</th>
<th>$k_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0000</td>
<td>{1}</td>
<td>1</td>
<td>59</td>
<td>16</td>
<td>4.2172</td>
<td>{1,3,4,4,4}</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>2.6667</td>
<td>{2}</td>
<td>1</td>
<td>57</td>
<td>17</td>
<td>4.0964</td>
<td>{2,3,4,4,4}</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3.4286</td>
<td>{3}</td>
<td>1</td>
<td>53</td>
<td>18</td>
<td>3.9387</td>
<td>{3,3,4,4,4}</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4.2667</td>
<td>{4}</td>
<td>1</td>
<td>35</td>
<td>19</td>
<td>3.7531</td>
<td>{1,3,4,4,4}</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5.1613</td>
<td>{5}</td>
<td>1</td>
<td>29</td>
<td>20</td>
<td>3.8226</td>
<td>{3,3,3,3,4}</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4.6875</td>
<td>{1,5}</td>
<td>2</td>
<td>28</td>
<td>21</td>
<td>3.7433</td>
<td>{2,2,3,3,3}</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>4.4700</td>
<td>{2,5}</td>
<td>2</td>
<td>26</td>
<td>22</td>
<td>3.7581</td>
<td>{3,3,3,3,3}</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>4.5508</td>
<td>{3,5}</td>
<td>2</td>
<td>22</td>
<td>23</td>
<td>3.6095</td>
<td>{1,3,3,3,3}</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>4.7974</td>
<td>{4,5}</td>
<td>2</td>
<td>14</td>
<td>19</td>
<td>3.4595</td>
<td>{2,3,3,3,3}</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4.6153</td>
<td>{1,4,5}</td>
<td>3</td>
<td>13</td>
<td>26</td>
<td>3.3471</td>
<td>{1,3,3,3,3,3}</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>4.4412</td>
<td>{2,4,5}</td>
<td>3</td>
<td>7</td>
<td>27</td>
<td>3.2727</td>
<td>{1,3,3,3,3,3}</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>4.3189</td>
<td>{3,4,5}</td>
<td>4</td>
<td>6</td>
<td>28</td>
<td>3.1638</td>
<td>{1,2,2,2,2,2,2,2,2,2,2,2,2,2,2}</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>4.4412</td>
<td>{3,4,5}</td>
<td>4</td>
<td>4</td>
<td>29</td>
<td>3.0933</td>
<td>{1,2,2,2,2,2,2,2,2,2,2,2,2,2,2}</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>4.1691</td>
<td>{2,3,4,5}</td>
<td>4</td>
<td>0</td>
<td>30</td>
<td>3.0769</td>
<td>{2,2,2,2,2,2,2,2,2,2,2,2,2,2,2}</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 3. Comparison of embedding efficiency when $n = 60$

From Figure 3, we can learn that: (1) the proposed ME method can support different embedding capacities, while Hamming codes can only embed at most 5 bits of secret messages; (2) In Tian et al.‘s method, PCM is made up of several submatrices and columns which are obtained by executing the bit-wise XOR operation between two columns in different submatrices. When the number of extra columns is large, this method may have a good effect. Therefore, rare points in Figure 3 are better than our method, such as $n - k = 10$; (3) However, Tian et al.‘s method takes no account of the combination of the extra columns and potentially can’t make full use of all the cover data. Along with the increase of embedding rate, the number of submatrices has a tendency to increase, and the extra columns become fewer. This is bad for Tian et al.‘s method, but more combinations of extra columns could be dealt with in our method. As a result, the proposed method performs better in this phase; (4) On the whole, the proposed method has the highest embedding efficiency among these four methods.

Table 2. Variation of computational complexity with different message lengths ($n = 60$)

<table>
<thead>
<tr>
<th>$n - k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2$\sum_{i=0}^{28}C_i^{12}$</td>
<td>2$\sum_{i=0}^{28}C_i^{12}$</td>
<td>2$\sum_{i=0}^{22}C_i^{14}$</td>
<td>2$\sum_{i=0}^{22}C_i^{14}$</td>
<td>3$\sum_{i=0}^{13}C_i^{22}$</td>
</tr>
<tr>
<td>$n - k$</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>$C$</td>
<td>3$\sum_{i=0}^{2}C_i^{12}$</td>
<td>3$\sum_{i=0}^{7}C_i^{12}$</td>
<td>4$\sum_{i=0}^{6}C_i^{12}$</td>
<td>4$\sum_{i=0}^{4}C_i^{12}$</td>
<td>4$\sum_{i=0}^{4}C_i^{12}$</td>
<td>5$\sum_{i=0}^{5}C_i^{12}$</td>
<td>5$\sum_{i=0}^{5}C_i^{12}$</td>
<td>5$\sum_{i=0}^{5}C_i^{12}$</td>
<td>5$\sum_{i=0}^{5}C_i^{12}$</td>
<td>6$\cdot 2^i$</td>
</tr>
<tr>
<td>$n - k$</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>$C$</td>
<td>7$\cdot 2^i$</td>
<td>7$\cdot 2^i$</td>
<td>8$\cdot 2^2$</td>
<td>8</td>
<td>9$\cdot 2^1$</td>
<td>9$\cdot 2^1$</td>
<td>10</td>
<td>11$\cdot 2$</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Experiment-2: In order to further test the performance of the proposed ME method, we apply it to StegVoIP [2]. StegVoIP selected 18 LSBs to hide secret messages from each G.723.1 (6.3kbits/s) speech frame. Hence, there are altogether 72 bits in 4 neighbouring frames. We randomly choose 40 or 60 bits from them and take these bits as a unit to perform embedding process. The speech files we used in this experiment are selected from An4 database [21]. They have different lengths. Half of the speech files are
recorded by female speakers and half of them are recorded by male speakers. The secret messages are still binary sequences generated randomly.

Perceptual evaluation of speech quality (PESQ) is proposed by ITU. It's a widely used objective speech quality assessment method. PESQ ranges from -0.5 (the worst) to 4.5 (the best). It can measure the difference between the stego speech and the original speech, so we use it to verify the validity of ME methods. Calculate the mean PESQ of 1000 original female speech files and 1000 original male speech files separately. And compare them with PESQ of stego speeches with different encoding methods. The results are shown in Table 3. From Table 3, we can learn that: PESQ of stego speeches using the proposed method are very close to the original speeches and larger than stego speeches corresponding to other methods, indicating that our method can effectively ensure the speech quality.

Table 3. Comparison of PESQ

<table>
<thead>
<tr>
<th>Embedding rate</th>
<th>Original speech</th>
<th>Tian et al.</th>
<th>Wang et al.</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female (n = 60)</td>
<td>3.769 3.769 3.769</td>
<td>3.769 3.769 3.769</td>
<td>3.346 3.346 3.346</td>
<td>3.315 3.315 3.315</td>
</tr>
</tbody>
</table>

Figure 4 records the processing time of the speech encoder for 100 frame groups (containing 4 neighbouring frames) when the embedding rate is 0.3. As is shown, the proposed curve is more close to the curve without information hiding. The average encoding time delays caused by information hiding with different message lengths can be seen in Table 4 and Table 5, from which we see that our method has the lowest latency. All the experiments were performed on a PC with 3.4 GHz Intel Core i7 CPU and 8GB RAM, and the methods were implemented in C and compiled under Microsoft Visual Studio 2008.

Table 4. Encoding time delay with different message lengths (n = 40)

<table>
<thead>
<tr>
<th>n - k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tian et al. (ms)</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
<td>0.008</td>
<td>0.015</td>
<td>0.027</td>
<td>0.017</td>
<td>0.019</td>
<td>0.024</td>
<td>0.031</td>
</tr>
<tr>
<td>Wang et al. (ms)</td>
<td>1.115</td>
<td>1.116</td>
<td>1.115</td>
<td>1.116</td>
<td>1.117</td>
<td>1.116</td>
<td>1.116</td>
<td>1.117</td>
<td>1.117</td>
<td>1.116</td>
</tr>
<tr>
<td>Proposed method (ms)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.016</td>
<td>0.013</td>
<td>0.005</td>
<td>0.005</td>
<td>0.093</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>n - k</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tian et al. (ms)</td>
<td>0.020</td>
<td>0.021</td>
<td>0.025</td>
<td>0.032</td>
<td>0.027</td>
<td>0.032</td>
<td>0.034</td>
<td>0.026</td>
<td>0.022</td>
<td>0.019</td>
</tr>
<tr>
<td>Wang et al. (ms)</td>
<td>1.116</td>
<td>1.117</td>
<td>1.116</td>
<td>1.116</td>
<td>1.116</td>
<td>1.116</td>
<td>1.116</td>
<td>1.117</td>
<td>1.117</td>
<td>1.117</td>
</tr>
<tr>
<td>Proposed method (ms)</td>
<td>0.019</td>
<td>0.014</td>
<td>0.004</td>
<td>0.036</td>
<td>0.009</td>
<td>0.005</td>
<td>0.022</td>
<td>0.013</td>
<td>0.029</td>
<td>0.007</td>
</tr>
</tbody>
</table>
5.2 Experiments of ME for Large Payloads

Experiment-3: Embed messages into random cover data using Fridrich et al.’s method [15] and the proposed method for large payloads. The computational complexity of Fridrich et al.’s method is $O(n2^n)$. When embedding rate is not large enough, the ME time will be too long. So we set $n = 60$ and the minimum embedding rate is set to be 0.75. To ensure that the proposed method can realize fast embedding, the computational complexity of our method is limited to be lower than $O(2O)$. Notice that, for a given cover length and computational complexity, there are many parameter combinations of $h$ and $k$ resulting in different PCM. We select the parameters which can yield the least distortion among them. For each payload, we embed 5000 blocks of random messages, and calculate the average embedding efficiency. Experimental results are shown in Figure 5 and Table 6, from which we can draw a conclusion that two ME methods achieve almost equal embedding efficiency, while the embedding speed of our method outperforms Fridrich et al.’s method.

Table 5. Encoding time delay with different message lengths ( $n = 60$ )

<table>
<thead>
<tr>
<th>$n - k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tian et al. (ms)</td>
<td>0.001</td>
<td>0.002</td>
<td>0.004</td>
<td>0.008</td>
<td>0.015</td>
<td>0.041</td>
<td>0.046</td>
<td>0.035</td>
<td>0.032</td>
<td>0.034</td>
</tr>
<tr>
<td>Proposed method (ms)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.008</td>
<td>0.041</td>
<td>0.032</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Table 6. Comparison of embedding speed between Fridrich et al.’s method and the proposed method ($n = 60$)

<table>
<thead>
<tr>
<th>Embedding rate</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fridrich’s method (Kbits/s)</td>
<td>0.05</td>
<td>0.53</td>
<td>6.06</td>
<td>64.13</td>
<td>804.52</td>
</tr>
<tr>
<td>Proposed method (Kbits/s)</td>
<td>2.83</td>
<td>2.88</td>
<td>6.06</td>
<td>64.13</td>
<td>804.52</td>
</tr>
</tbody>
</table>

Fridrich et al.’s method is an exhaustive method. It has the capability of searching global optimal solution within the defined space. Different from this, the cost leader we found using STCs or Wang et al.’s method is a local optimal solution. The larger the number of random columns in PCM is, the more combinations will be considered in searching for the coset leader, and thus a higher embedding efficiency we will get. Therefore, local matrix $R$ tends to be maximized within the range of allowable computational complexity. The computational complexity should be lower than $O(2^n)$ in this experiment, so the number of random columns is 10 at most. When the embedding rate is larger than 0.85, the random columns needed by PCM is less than 10. As a result, the PCMs we constructed using the proposed method are the same as Fridrich et al.’s method at this moment.

Experiment-4: Just like Experiment-2, we apply ME to StegVoIP and make a comparison between the proposed method and existing fast ME methods in [10] and [17]. The computational complexity of Fridrich et al.’s method is too high to be applied to VoIP steganography. So we didn’t use it in this experiment. For $n = 60, C \leq O(2^n)$, we select the best parameters (shown in Table 7) of the proposed method and get their embedding efficiencies (shown in Figure 6(a)). According to the conclusions of Experiment-3, the proposed ME method are the same as Fridrich et al.’s method when $n - k \geq 54$. Therefore, it is not discussed in this experiment. Similarly, Table 8 and Figure 6(b) only show the results when $n = 100, C \leq O(2^n), n - k \leq 92$. Table 7 and Table 8 illustrate that the novel
method turns into Wang et al.’s method when payload is close to 1. Figure 7 records the processing time of speech encoder with different ME methods in several cases. And Table 9, Table 10 show the precise encoding time delay caused by information hiding.

According to these results, we can see that a promising embedding efficiency is obtained by the proposed method while maintaining low computational complexity.

Table 7. Optimal values of PCM parameters with different message lengths (n = 60)

<table>
<thead>
<tr>
<th>n - k</th>
<th>e</th>
<th>r₁</th>
<th>r₂</th>
<th>h</th>
<th>k₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>3.7086</td>
<td>24</td>
<td>9</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>32</td>
<td>3.6296</td>
<td>23</td>
<td>11</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>3.5522</td>
<td>22</td>
<td>13</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>34</td>
<td>3.4681</td>
<td>21</td>
<td>15</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>35</td>
<td>3.4007</td>
<td>20</td>
<td>17</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>36</td>
<td>3.3457</td>
<td>18</td>
<td>20</td>
<td>3</td>
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Table 8. Optimal values of PCM parameters with different message lengths \( (n = 100) \)

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Table 9. Encoding time delay with different message lengths \( (n = 60) \)

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<th>Proposed method (ms)</th>
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Table 10. Encoding time delay with different message lengths \( (n = 100) \)

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6 Conclusion

In this paper, a further study on the PCM construction was proposed. We analyzed the approaches to realizing fast ME. On this basis, two specific matrix structures for small payloads (payloads that are smaller than 0.5) and large payloads (payloads that are larger than 0.5) were presented. Experimental results showed that our fast ME methods can realize better embedding efficiency and faster embedding speed than state-of-the-art works. It’s worth mentioning that though we fixed the cover length to make comparison in our experiments, the two novel methods both can be applied to covers of arbitrary length.

References

A Further Study of Optimal Matrix Construction for Matrix Embedding Steganography


Biographies

Zhanzhan Gao received the B.S. degree in electronic science and M.S. degree in information security from Zhengzhou information science and technology institute in 2011 and 2014, respectively. He is now pursuing the Ph.D. degree in information security. His research interests include information hiding and multimedia processing.

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