Gaussian Mixture Model Based Image Denoising with Adaptive Regularization Parameters

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Abstract

Recently, Gaussian mixture model have been studied extensively in image denoising, for the reason that it can better represent image prior. However, the current Gaussian mixture model based image denoising approach commonly employs global regularization parameter, therefore leading to limited denoising performance. To further enhance the performance this method, we exploit a new scheme for spatially adaptive regularization parameter selection, which utilizes scale space technique and residual image statistics to set regularization parameter value according to image details. The experiment results show that our proposed image denoising method can obtain relatively well results both in vision and the value of peak signal to noise ratio.

Keywords: Image denoising, Gaussian Mixture Model, Regularization parameter selection

1 Introduction

Digital image has been widely applied to many scientific and industrial fields, such as computer science, remote sensing, medical science and so on. In reality, digital images are unavoidably degraded by the noise generated from the imaging device or external environment in acquisition and transmission process. In order to enhance the accuracy of image analysis and understanding, image denoising plays a significant role in digital image processing.

Commonly, image degradation process can be modelled as a linear model: \( u_n = u + n \), where \( u \) and \( u_n \) are noisy image and original image, respectively, and \( n \) often denotes the additive Gaussian white noise (GWN). In general, estimating ideal image \( u \) from noisy image \( u_n \) is an ill-posed inverse problem \cite{1}. It is well known that regularization technique is a powerful tool to handle this troublesome problem \cite{2}. In the past decades, various nonlinear regularization models has been introduced \cite{3-4}. Among them, image regularization term has drawn much attention, which is closely related to image prior learning. Employing image prior as driving force for image regularization has been a hot issue.

To date, numerous image priors have been presented, for instance, such as gradient based \cite{5}, non-local based \cite{6-9} and sparse representation based method \cite{10-12}, and so on \cite{13}. Traditional image denoising method as the total variation (TV) \cite{5} uses priors of image gradient distributions to remove noise in a local way. The TV model can obtain well good denoised results on cartoon images. However, it tends to generate staircase effect and over-smoothed results. The non-local prior \cite{6-9} is primarily used for the texture image processing and analysis. Suppose similar patches are contained in a whole image, the non-local self-similarity prior is utilized to by the non-local image restoration methods, and has obtained favorable achievement. In recent years, the sparsity priors \cite{10-12} has been employed to image restoration, with the observation that image patches can be sparsely represented over a dictionary. Among proposed adaptive sparse representation approaches, K-SVD \cite{10} is a classical one. This method assumes that digital image can be expressed by linear combination of atoms in the sparse dictionary. Therefore, the key is to learn the redundant dictionary and calculate its sparse coefficient. However, K-SVD can be able to produce good denoised images with specific structures, because of the independence assumption on the dictionary atoms.

In fact, certain atoms of dictionary are usually strongly correlated. Considering interdependency between atoms in the sparse dictionary, many structured sparse representation method have been introduced recently, including non-local self-similarity based method \cite{14}, group and block sparse based method \cite{15} and mixture model based method \cite{16-19}. Among them, mixture model based method has been proved amazingly competitive in terms of image restoration. In order to learn image patch priors, mixture model based method established the statistical model of image patches by few mixture components.
Image structures are easier to be modeled in a local small windows. Mixture model based image restoration method has comparatively lower computation complexity and more comprehensible mathematical mechanism. Compared with other mixture models, Gaussian Mixture Model (GMM) model [16, 18-19] has been widely applied in image restoration for its good clustering performance and flexibility. Recently, Expected Patch Log Likelihood (EPLL) based prior and its variants [16, 20-22] has been shown to be surprisingly competitive in image denoising. Nevertheless, regularization parameter selection is still an open problem for the EPLL-based image denoising method [23-25].

As well known that regularization parameters selection drastically affects the performance of regularization method. When regularization parameter value is too large, there will be residual noise in the results. In contrast, when the regularization parameter value is too small, the denoised image will probably lose important structure, such as the edges and textures. A variety of parameter estimation methods have been introduced, including Lagrange multiplier method [26], the L-curve based method [27-28], the structure tensor based method [9, 29], the discrepancy principle based method [30], the scale space based method [31-32], and the residual image statistics based method [33]. In this paper, we devote to the regularization parameter selection using residual image statistics based technique for EPLL based image denoising problem. We propose an adaptive regularization parameter selection method through using the local variance of the residual image as spatially varying constraints, which estimate regularization parameter adaptively for EPLL based image denoising method.

The rest of this paper is organized as follows. In section 2, we briefly review the EPLL based image denoising methods. Then, we describe our proposed methods with adaptive regularization parameters in details in section 3. The experimental results are shown in section 4. Finally, section 5 summarizes the whole work.

## 2 EPLL based Image Denoising Method

Image \( u \) containing \( N \) pixels can be separated into \( N \) overlapping image patches with the size of \( \sqrt{D} \times \sqrt{D} \). The vectorized image patches \( u_i = P_i u \) is obtained from image \( u \) at position \( i \) by extracting operator \( P_i \). Suppose that image patches are independent of each other and there exist \( K \) mixture components, the density function of the GMM on \( u_i \) can be defined as:

\[
P_i(u_i) = \sum_{j=1}^{K} \pi_j N(u_i | \mu_j, \Sigma_j)
\]

Where \( \pi_j \) is the mixing coefficient, \( \mu_j \) and \( \Sigma_j \) are the mean and covariance matrix respectively, and \( N(u_i | \mu_j, \Sigma_j) \) expresses the Gaussian distribution [34], which can be written as:

\[
N(u_i | \mu_j, \Sigma_j) = \frac{1}{(2\pi)^{D/2} \sqrt{|\Sigma_j|}} \exp\left(-\frac{1}{2} (u_i - \mu_j)^T \Sigma_j^{-1} (u_i - \mu_j)\right)
\]

Then, the EPLL for image \( u \) is modeled as follows:

\[
EPLL(u) = \log p(u)
\]

\[
p(u) = \prod_{i=1}^{K} \pi_j N(u_i | \mu_j, \Sigma_j)
\]

With (3) and (4), the EPLL based image denoising model can be written as:

\[
\min_{u} \left\{ \lambda \frac{1}{2} ||u - u_0||^2 - \sum_{i=1}^{K} \log p(P_i u) \right\}
\]

Where \( \lambda \) denotes the regularization parameter. In general, given the noise level \( \sigma^2 \), regularization parameter can be computed by \( \lambda = \frac{D}{\sigma^2} \). (5) can be solved by the Half Quadratic Splitting algorithm [16] by introducing a set of auxiliary variables \( z' \) into (5):

\[
\min_{u,z'} \left\{ \frac{\lambda}{2} ||u - u_0||^2 + \sum_{i=1}^{K} \left( \frac{D}{2} \right) ||P_i u - z_i||^2 - \log p(z_i) \right\}
\]

Where \( \beta \) is the penalty parameter which often is set to be large enough to ensure that the solution of (6) is close to that of (5). Then formula (6) can be minimized by alternatively updating \( z' \) and \( u_i \).

## 3 Proposed Method with Adaptive Regularization Parameters

The cartoon pyramid model (CPM) [32] consists of three components as follows:

\[
u_0 = u_c + u_{NC} + u_n
\]

where \( u_c \) denotes the cartoon image, \( u_{NC} \) is the non-cartoon image and \( u_n \) is the GWN. \( u = u_c + u_{NC} \) is the original image and non-cartoon image \( u_{NC} \) is often rich in small scale details. With (7), the residual image \( u_k \) generally contain non-cartoon image \( u_{NC} \) and noise \( u_n \).

\[
u_k = u_0 - u = \tilde{u}_{NC} + \tilde{u}_n
\]

In image denoising, we firstly use the following
constrained model to smooth the noisy image \( u_0 \) and generate residual image \( u_R \) for estimate regularization parameters:

\[
\begin{align*}
\text{min}_{u} \{- \sum_{i=1}^{N} \log p(Pu)\} \\
\text{s.t.} \frac{1}{|\Omega|} \| u - u_0 \|_2^2 = \alpha \sigma^2
\end{align*}
\]

(9)

where \( \alpha \) is a scale factor related to the noise variance. The noise and relevant textures can be separated by formula (16); hence, using formula (16):

\[
\lambda(x,y) = \frac{(u-u_0-C)\sum_{i} \beta R_i^T (R_i u - z_i)}{(u-u_0-C)^2} = \frac{Q(x,y)}{X(x,y)} \quad (17)
\]

\[
Q(x,y) = (u-u_0-C) \sum_{i} \beta R_i^T (R_i u - z_i) \\
C = \int_{\Omega} \lambda(x,y)(u(x,y)-u_0(x,y))dxdy \\
\int_{\Omega} \lambda(x,y)dxdy
\]

(18)

(19)

In summary, the proposed denoising algorithm is implemented as follows:

**Step 1.** Input corrupted image \( u_0 \), model parameters \( \alpha, \beta, \tau \) and iteration stopping tolerance \( \epsilon \), initialize regularization parameter \( \lambda \);

**Step 2.** Compute the residual image \( u_R \) and minimize equation (9) for separating the noise and Textures;

**Step 3.** Calculate the local variance of the residual image \( P_R(x,y) \) by (10);

**Step 4.** Compute the local constraints \( S(x,y) \) by

\[
S(x,y) = \frac{\sigma^4}{P_R(x,y)}
\]

**Step 5.** Choose the most likely Gaussian mixing coefficient \( \beta_{\text{max}} \) for each image patch and calculate auxiliary variables \( z_i \) using formula (16);

**Step 6.** Compute \( u' \) using formula (15) with updated regularization parameters \( \lambda(x,y) \) and constant \( C \) according to (17) and (19);

**Step 7.** Compute \( z_i^{n+1} \) formula using (16);

**Step 8.** Estimate image \( u^{n+1} \) using (15) with updated \( \lambda(x,y) \) and \( C \);

**Step 9.** Repeat Steps 7-8 until satisfying stopping criterion.

4. Implementation and Experiment Results

In the experiments, we compare our proposed image denoising approach with original EPLL [16], K-SVD method [10], EPLL with gradient fidelity term [24]. The GMM model with 200 mixture components is learned from \( 2 \times 10^6 \) images patches contained in 200 natural images which are sampled from the Berkeley Segmentation Database Benchmark (BSDS300). The noisy images are produced through adding white Gaussian noise with zero mean and standard variance \( \delta = 25 \) into original images. The parameters are set as follows: the image patch size \( \sqrt{D} = 64 \), the weighted
coefficients $\beta = \frac{1}{\delta^2}[14816]$, symmetric smoothing window $\omega_{x,y} = 5$, scale factor $\alpha = 1.5$.

Figure 1 illustrate the denoised results of the original EPLL method, K-SVD method, EPLL with gradient fidelity term and our proposed method on zebra image. Figure 1(a) is a piecewise smooth original zebra image in the Berkeley Database (No. 253027). Figure 1(b) is a noisy image with zero mean and variance $\delta^2 = 25$. Figure 1(c) is the result of the original EPLL based denoising method. We can see that in Figure 1(c) pseudo textures appear in some flat regions. Figure 1(d) shows the denoised image by K-SVD. It can be observed that some textures in Figure 1(d) are smoothed out, for the reason that the K-SVD fails to consider the correlation between the atoms. Figure 1(e) is the result of EPLL with gradient fidelity term. Since the method employs the gradient fidelity term to enhance the performance of EPLL, it can preserve more image detail structures while remove noise. However, image gradient is sensitive to noise. It takes much time to select proper parameters. Figure 1(f) shows the result of our method. Compared with Figure 1(c) to Figure 1(e), we can see that our method can obtain comparatively good tradeoff between preserving details and denoising.

Figure 2 compares the performance of the four image denoising methods on a plane image. Figure 2(a) is a clean building image in the Berkeley Database (No. 126007). Figure 2(b) is a noisy image with zero mean and variance $\delta^2 = 25$, the denoised result of original EPLL method is shown in Figure 2(c). Figure 2(d) displays the result of K-SVD method. We can also observe that K-SVD tends to generate over-smoothed image and EPLL method often produces pseudo texture in image flat regions. Figure 2(e) demonstrates the denoised result of the EPLL method with gradient fidelity term. Compared with the above-mentioned image denoising methods, we find that our proposed method performs relatively well good. In Figure 2(f), our method can better preserve the edges and some textures in images. It is probably because our approach can adaptively select regularization parameters. We also compare the four methods on other kinds of image, such as plane image and human face image. The results are displayed in Figure 3 and Figure 4, respectively. Once again, from the two images, we can see that our method yields satisfying denoised images.
Figure 2. Comparison of the proposed method with other methods on building image

Figure 3. Comparison of the proposed method with other methods on plane image
Additionally, the peak signal-to-noise ratio (PSNR) value is used to quantitatively assess the denoised images, which are shown in Table 1. From Table 1, we can clearly see that the PSNR value of our method are higher than other three methods, that is EPLL and K-SVD, and EPLL with gradient fidelity term. This verifies that adaptive regularization parameters can enable the improved performance of EPLL method.

Table 1. The PSNR (dB) results of different denoising methods

<table>
<thead>
<tr>
<th>Image</th>
<th>Noisy</th>
<th>Original EPLL</th>
<th>K-SVD</th>
<th>EPLL with gradient fidelity term</th>
<th>Our method</th>
</tr>
</thead>
<tbody>
<tr>
<td>zebra</td>
<td>24.10</td>
<td>27.22</td>
<td>26.23</td>
<td>27.56</td>
<td>27.61</td>
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<tr>
<td>building</td>
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<td>30.03</td>
<td>28.59</td>
<td>30.09</td>
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<td>37.06</td>
<td>36.40</td>
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<tr>
<td>human face</td>
<td>25.41</td>
<td>32.49</td>
<td>31.72</td>
<td>32.60</td>
<td>32.75</td>
</tr>
</tbody>
</table>

5 Conclusions

Image prior plays a significant role in various image regularization task. The GMM is a powerful tool for learning image prior and has attracted much attention in image restoration in recent years. In this paper, to improve the performance of image denoising method with GMM, that is EPLL based method, we present a new adaptive regularization parameter estimation method for it through using residual image statistics technique. Because of that we can adaptively assign regularization parameter values to different image structures, the herein proposed image denoising method can adjust the smoothing extent according to the image content. The experiment results illustrate that our method can produce visually satisfying denoised images.

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References


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