# Joint Coding and Transmission Scheduling for Underwater Acoustic Networks

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## Abstract

Efficient data broadcast is of critical importance for Underwater Acoustic Networks (UANs) since it provides essential services for various tasks. Due to the unique features of UANs including the long propagation delay, low bandwidth and high error probability, efficient data broadcast has been facing great challenges. In this paper, we study the problem of distributing data to a group of underwater sensor nodes in partially connected cooperative network using network coding. In such a scenario, the transmission conflicts occur from simultaneous transmissions of multiple nodes, where the scheduling decision should be made not only on the encoded packets but also on the set of transmitting nodes. We formulate the joint optimization problem over the set of transmitting nodes and the packet combinations with a conflict free graph model, which contains both coding conflict and transmission conflict. We also propose a heuristic solution for this setup by finding the maximum independent set in the conflict free graph. Simulation results show that our coding scheme significantly reduces the number of transmission slots.

Keywords: Broadcast, Underwater acoustic networks, Network coding, Transmission scheduling

# **1** Introduction

#### 1.1 Underwater Acoustic Networks

With the increasing role of ocean in human life, discovering these largely unexplored areas has gained more importance during the last decades. On one side, traditional approaches used for under water monitoring missions have several drawbacks and on the other side, these inhospitable environments are not feasible for human presence as unpredictable underwater activities, high water pressure and vast areas are major reasons for un-manned exploration. Due to these reasons, Underwater Acoustic Networks (UANs) are attracting the interest of many researchers lately [1-2].

Underwater Acoustic Networks (UANs) consist of a variable number of sensor nodes that are deployed to

per- form collaborative monitoring over a given underwater environment which are capable of sending/receiving data using acoustic communication channels [3]. The general scenario of the underwater acoustic networks architecture is shown in Figure 1.



Figure 1. General scenario of the underwater acoustic networks

#### **1.2 Broadcast in UANs**

Similar as in terrestrial wireless sensor networks (TWSNs), reliable and efficient data broadcast has been a desirable feature for UANs [4]. Applications such as sending an important data file from a source node to multiple sink nodes or transmitting a critical command from a control base station to a couple of UAN nodes are common and required by various tasks [5]. Various broadcast schemes have been proposed for wireless sensor networks [6-8]. However, the aforementioned mechanisms tend to be impractical and inefficient for UANs considering the special features of the underwater environment [9].

Different from TWSNs, in UANs, an acoustic channel instead of radio is usually employed for signal transmission in the water. This imposes several challenges for communication within UANs. First, the data propagation delay is long due to the low

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propagation speed of the acoustic signals. Second, UAN channels have very low data rates because of the absorption, multi-path and fading. Third, UANs usually suffer from high error probabilities because of the error prone underwater acoustic channels. In addition to the difficulties brought by acoustic communications, UANs also face other challenges. The harsh underwater environment causes tremendous dynamics within UANs in terms of channel quality and network topology [10]. Besides, UANs have always been energy constrained by nature [11]. The energy constraint means protocols and services in UANs have to be energy efficient.

The limitations with conventional TWSN broadcast schemes motivate researchers to explore new methodologies in UANs. To counter the high error probability of acoustic channels, coding is commonly employed. Random Linear Network Coding (RLNC) has also been applied to UANs [12-13]. However, RLNC can suffer from large decoding delays, since a receiver needs to collect enough linear combinations to perform block-wise decoding [14]. Recently, Instantly Decodable Network Coding (IDNC) attracts a significant number of works [15-17] ac- cording to its fast decoding potential, which is essential for real-time applications [18]. However, the existing scheme do not consider the cooperative recovery. By introducing the cooperative recovery, the traffic of the source node can be offloaded to serve additional nodes and the number of transmissions can be reduced. On the other hand, it can also increase the coverage zone of the network as UAN node can communicate to other node directly. Furthermore, it can reduce the cost associated with the deployment of new infrastructure required for the growing network size and UAN node's traffic demand. Finally, short-range communication provide more reliable delivery of the packets compared to the longrange communication due to small distances between the UAN nodes. Different from the existing RLNC and IDNC broadcast scheme. We study the broadcast problem in underwater acoustic networks with a two phases broadcast scheme, we mainly focus the data recovery phase where sensor nodes perform data recovery according to the cooperative communication.

## **1.3 Methodology and Contributions**

In this paper, we are interested in distributing a block of data packets to a group of under-water sensor nodes. A typical application scenario is that a networkwide software update/patch is required. A source node is initially updated/patched manually by an operator. Afterwards, the source node divides the update/patch data into multiple packets and broadcasts into the network. The objective of this task is that every node can receive all the data packets.

Data packets can be transmitted in two phases. In the first phase, source node broadcasts the original packets sequentially. However, the underwater sensor nodes receive partial content in those transmissions due to erasures in acoustic channels. To recover the missing packets, the sensor nodes communicate with each other using their short-range acoustic channels. Moreover, depending on the location of a sensor node, it can be connected to all other nodes directly (i.e., single- hop transmission) or via intermediate nodes (i.e., multihop transmissions). Therefore, in the second phase, the sensor nodes perform data recovery according to the cooperative communication. In this paper, we consider the conflict free scheduling problem for the data recovery in underwater acoustic networks with network coding, our aim is to minimize the number of transmission slots for sensor nodes to obtain all data contents. The main contributions of this paper are summarized as follows:

- We study the broadcast problem in underwater acoustic networks by introducing the data cooperative recovery phase where sensor nodes perform data recovery according to the cooperative communication.
- The joint optimization scheduling problem for data recovery with network coding is analyzed to obtain a lower bound of the number of transmission slots, which provides an insight into the system performance.
- Heuristic solution is proposed based on a conflict graph model to find an efficient transmission schedule.
- Simulations results demonstrate significant performance benefits in terms of number of transmission slots, compared to the traditional fully connected solution.

The remainder of this paper is organized as follows. Section 2 introduces the related work. In Section 3, we will give the system framework and problem statement. Section 4 obtains the lower bound of network coding gain in terms of number of transmission slots by formulating a relaxed scheduling problem with an integer linear programming. Section 5 introduces a conflict free clique model for the joint coding and scheduling problem in underwater acoustic networks and presents a heuristic solution by finding the maximum independent set in the conflict free graph. Simulation results will be shown in Section 6. Finally, we will conclude the paper in Section 7.

# 2 Related Work

This work combines ideas from broadcast scheduling, network coding, and cooperation communication in underwater acoustic networks. In this section, we discuss the most relevant literature from these areas.

Reliable and efficient broadcast has been extensively studied for UANs. In [19], the paper analyzed three kinds of stop-and-wait protocols for UANs and showed

that the performance of the basic stop-and-wait protocols can be significantly improved by transmitting packets in group and selective acknowledgment. A modified finite-difference time-domain method was proposed in [20] to compute broadband acoustic scattering model of underwater complex object. A novel modeling based on deep learning framework was proposed in [21] to manifest the characteristics of nonlinear system in UAN sensing. In [22], a hop-byhop HARQ based scheme, called SDRT, was proposed for UANs. To counter the high error probability of acoustic channels, coding is commonly employed. Network coding has been applied to UAN like networks featuring time division duplex channels [23]. However, RLNC can suffer from large decoding delays, and incurs heavy decoding computational load, which is not suitable for energy constrained underwater sensors.

Numerous network coding schemes for broadcasting problem have been developed to meet different requirements of applications. The authors in [18] studied IDNC for wireless broadcast aiming at serving the maximum number of devices with any new packet. Moreover, the authors in [15] formulated the problem of minimizing the number of time slots required for broadcasting into a stochastic shortest path framework. Several other works in IDNC considered different importance of packets in coding decisions [24-25]. In particular, IDNC for streaming transmission was adopted in [24] and the proposed IDNC schemes are asymptotically throughput optimal subject to deadline constraints. IDNC is not throughput optimal since each IDNC transmission typically benefits only some of the receivers. However, the aforementioned works developed network coding based broadcast schemes for conventional point to multipoint (PMP) networks, which are fundamentally different from wireless broadcast with cooperative communication considered in this paper.

There are also many works about incorporating coding in cooperative communication network networks. In particular, necessary conditions that characterize the number of transmissions required to recover all missing packets at all devices was provided in [26]. The authors in [27], [28] selected a transmitting device and its XOR encoded packet to service a large number of devices with any new packet. Different from IDNC based broadcast, in a cooperative network, an additional decision on the set of transmitting users is also required. However, the prior works on IDNC based cooperative network [27] only consider fully connected networks, i.e., each node can target all other nodes over one-hop transmission, and thus only one node transmits at each time slot. This fully connected cooperative network is not always practical in underwater acoustic networks due to the limited transmission range of nodes and their widespread over large ocean area. Different from the

above work, we mainly focus on the conflict-free scheduling problem for partial connected cooperative underwater sensor networks with network coding, our aim is to minimize the number of transmission slots for nodes to obtain all data contents.

## **3** System Model and Problem Description

We consider the scenario which consists of n UAN nodes,  $R = \{r_1, r_2, ..., r_n\}$ , which are interested in obtaining the same content from the source node s. Note that the nodes are interested in receiving packets set  $P = \{p_1, p_2, ..., p_m\}$ , where m is the number of data contents. Acoustic is commonly employed as the medium for signal transmission in UANs. A UAN node is equipped with an underwater acoustic modem to send and receive data packets. Acoustic modems are half duplex and can only be in the sending or receiving status at one time. The system model of an underwater acoustic network is shown in Figure 2.



Figure 2. Network model

Data packets are transmitted in two phases. In the first phase, s broadcasts the original packets sequentially. After the first phase, a subset of nodes from R may not receive all packets due to erasures in the acoustic channels. Set  $W(r_i)$  denotes the packets needed at receiver  $r_i$  and set  $H(r_i)$  is referred to as the packet set already had at receiver  $r_i$ . Thus, the following equality holds:  $W(r_i) \cap H(r_i) = \emptyset$  $W(r_i) \bigcup H(r_i) = P, 1 \le i \le n$ . s can set  $W(r_i) = P$  for any  $r_i$ , initially. If  $p_i$  is acknowledged from receiver node  $r_i$ , then  $p_i$  is deleted from  $W(r_i)$ . Therefore, the transmitter always have knowledge on the information of  $W(r_i)$ and  $H(r_i)$ . After broadcasting the original packets sequentially in the first phase, s can maintain a list L=Pto represents for the packets set that do not acknowledged by at least one receiver node. If  $p_i$  is acknowledged by a receiver node, then  $p_i$  is deleted from L. Therefore, at the end of the first phase, it is reasonable to assume that each packet of P is acknowledged by at least one node. Otherwise, the source node keeps broadcasting the packet until the condition is verified. Each receiver will send acknowledgement to transmitter indicating a received or lost packet through the feedback. Note that each

receiver only need to use one bit to acknowledge a received packet, since all the receiver are interested in the same packet set. There are n receivers in total, thus the overall communication overhead from feedback is n bits per time slot. In the second phase, the nodes cooperate with each other to recover their missing packets using cooperative communication. Request will be sent by a node and correspondingly neighbors will coordinate to send the responses.

In this paper, we are interested in distributing data contents to a group of partially connected cooperative underwater nodes. The data packets are broadcasted from a source node to the nodes over acoustic channels. However, the nodes receive partial content in those transmissions due to erasures in acoustic channels. To recover the missing packets, the nodes communicate with each other using their short-range acoustic channels. Moreover, depending on the location of a node, it can be connected to all other nodes directly or via intermediate nodes. In conventional underwater medium access control (MAC) protocol design, we usually assume omnidirectional transmissions and receptions. With this assumption, the transmission range and interference area of these users can be modeled as circles. Without loss of generality, we assume the transmission range and the interference range are the same.

We consider a partially connected cooperative network, where a node is connected to another node directly or via intermediate nodes.  $\forall r_i, r_j \in R$ , we define a  $n \times n$  dimension connection matrix  $C = (c_{ij})_{n \times n}$  as follows.

$$c_{ij} = \begin{cases} 1 & \text{if } r_i \text{ is directly connected to } r_j, \\ 0 & \text{otherwise.} \end{cases}$$
(1)

Note that C is a symmetric matrix that depends on the network topology. We assume that each node is able to connect to any other node through single or multi-hop transmission (via the intermediate nodes). In other words, no node is isolated. The coverage zone of transmitting node  $r_i$  (denoted by  $Z_i$ ) is defined as the set of neighboring nodes that are directly connected to it. In other words,  $Z_i = \{r_i \mid c_{ij} = 1, 1 \le j \ne i \le n\}$ . Following the interference model in [29], we define transmission conflict is experienced by a node when it belongs to the coverage zones of multiple transmitting nodes. In other words, when two neighboring nodes  $r_i$  and  $r_k$  of node  $r_i$  transmit simultaneously, their transmissions will collide and node  $r_i$  will not be able to receive any of these transmissions successfully. Let T be the set of possible combinations of nodes that can transmit simultaneously, it can be expressed as follows:

$$T = \{R_s \in P(R) \mid Z_i \cap Z_j = \emptyset, \forall r_i, r_j \in R_s\}$$
(2)

where notation P(R) refers to the set of all the subsets of R,  $R_s$  is a subset of R. A transmission schedule is denoted as  $S = \{R^{s}(t), P^{s}(t) | R^{s}(t) \in T, t \ge 1\}\}$ , where  $R^{s}(t)$  is defined as the set of transmitting nodes at time slot *t*, and  $P^{s}(t)$  is defined as the encoded packets at time slot *t*.

Exploiting cooperative transmission has the potential of improving throughput, and employing network coding will further improve throughput in this setup. It is crucial to determine which node should transmit and which network coded packet should be transmitted. In this paper, the problem is that given the partial connected cooperative underwater acoustic network topology  $C = (c_{ij})_{n \times n}$ , the set of stored packets  $H(r_i)$ , the set of packets W(r<sub>i</sub>),  $1 \le i \le n$ , how to find a transmission schedule  $S = \{R^{s}(t), P^{s}(t)\}$  to satisfy all nodes. We use xor encoding such as IDNC scheme since it is designed for instantaneous and low complexity packet decoding. The objective is to minimize the number of transmission slots, where there may exist more than one transmissions in each transmission slot on condition that there is no transmission conflict. Such an encoding and scheduling decision problem is referred to as Partially Connected

Solving the PCNC problem is very difficult in general, therefore, we give a solution of the relaxed PCNC problem. To relax the PCNC problem, we assume that every transmission slot the same set of transmitting nodes  $R_T$  is selected,  $R_T \in T$ . In other words,  $R^{s}(t) = R_T, t \ge 1$ . According to solve the relaxed PCNC problem, we can obtain a lower bound of the PCNC problem and obtain some insight of the original PCNC problem.

Cooperative Network Coded (PCNC) problem.

# 4 Relaxed Partially Connected Cooperative Network Coded Problem

In the following, we will give an integer linear programming formulation of the relaxed PCNC problem. Let  $d_i$  be the number of transmissions from node  $r_i$  via cooperative link,  $1 \le i \le n$ . Since only nodes without transmission conflict can transmit at the same time slot, the total number of transmission slots with cooperative link is equal to  $max\{d_i | r_i \in R_T, R_T \in T\}$ , therefore the objective of the relaxed PCNC problem is to minimize  $max\{d_i | r_i \in R_T, R_T \in T\}$ . For any subset  $R_u, R_u \subseteq R_T$ , define  $R_u^c = R - R_u$ , which is a complementary set of  $R_u$  and includes elements that are members of R but not members of  $R_u$ . From the following theorem, we will get the constraint conditions of the relaxed PCNC problem.

Theorem 1: In relaxed PCNC problem, for any subset  $R_u$ ,  $R_u \subseteq R_T$ , the following inequality holds,  $\max \{ d_i | r_i \in R_u \} \ge \max_{r_i \in R^c} |W(r_i)|$ .

*Proof*: In relaxed PCNC problem, nodes need to obtain all packets in their need sets. For any subset  $R_u$ ,  $R_u \subseteq R_T$ ,  $R_u^c$  consists of nodes which are not selected in  $R_u$ . With xor encoded packets constructed by IDNC scheme, every receiver can at most decode one needed packet. Consider the receiver  $r_i \in R_u^c$ ,  $W(r_i)$  denotes the needed packets at  $r_i$ . It can be seen that at the ideal case, the number of encoded packets need to recover all needed packet in  $R_u^c$  is  $\max_{ri \in R_u^c} |W(r_i)|$ . In other words, in order to retrieve all needed packets at receivers in  $R_u^c$ , we need at least  $\max_{ri \in R_u^c} |W(r_i)|$  encoded packets, which come from receivers in  $R_u$  via cooperative link without transmission conflict. The number of encoded packets coming from nodes in  $R_u$  without conflict is equal to  $\max\{d_i | r_i \in Ru, Ru \in T\}$ .

Therefore, the following inequality holds,  $\max \{ d_i | r_i \in R_u, R_u \subseteq R_T \} \ge \max_{r \in R^c} |W(r_i)|.$ 

From Theorem 1, we can obtain an integer linear programming (ILP) formulation of the relaxed PCNC problem as follows. In the ILP formulation, the objective is to minimize the number of transmission slots, and the family of inequalities illustrates the condition for needed packets recovery using xor encoded packets in IDNC scheme. By solving the ILP formulation, we can obtain the lower bound of the PCNC problem.

Minimize  $max \{ d_i | r_i \in R_T, R_T \in T \}$ Subject to  $max \{ d_i | r_i \in R_u \} \ge max_{r_i \in R_u^c} |W(r_i)|,$ 

$$\forall Ru \subseteq RT \tag{3}$$

$$T = \{R_s \in P(R) \mid Z_i \cap Z_j = \emptyset,$$

 $\forall r_i, r_j \in R_s \}$ (4)

$$d_i \ge 0, d_i \in Z; \tag{5}$$

Solving the ILP is NP-hard in general, so we can only solve ILP using optimization tools such as Matlab for small scale networks. For larger scale network, we can use the LP-relaxation method. Using LP-relaxation method, the ILP problem can be transformed to a LP problem by relaxing the integer variables to continuous variables, and the solution of the LP problem can be regarded as a lower bound of the ILP problem, where LP problem can be solved with polynomial time.

## 5 Proposed Solution for PCNC Problem

Different from the traditional cooperative IDNC problem, PCNC problem considers not only how to encode but also how to select the transmitters. There are two conflicts in the PCNC problem, one is

transmission conflict which occurs due to the simultaneous transmissions from multiple nodes to a node in their coverage zones, and the other is coding conflict which occurs due to the instant decodability constraint. In this section, we define a conflict graph G(V, E) to represent both coding and transmission conflicts in one unified framework, and select a set of transmitting nodes and their XOR packet combinations in each time slot based on the conflict graph.

#### 5.1 Conflict Graph Model

Coding and transmission conflicts can be represented in one graph that we refer to as the conflict graph [16]. This graph G is constructed as follows:

*Definition* 1: Given  $R = \{r_1, r_2, ..., r_n\}, P = \{p_1, p_2, ..., p_m\}, W(r_i), H(r_i) \in P, Z_i = \{r_j | c_{ij} = 1, 1 \le j \ne i \le n\}, we construct a conflict graph G(V, E) as:$ 

$$V = \left\{ u_{ijk} \mid \forall r_i \in R, r_j \in Z_i, p_k \in H(r_i) \cap W(r_j) \right\},$$
  

$$E = \left\{ (u_{i_1j_1k_1}, u_{i_2j_2k_2}) \mid r_{i_1} = r_{i_2}, \text{ but } p_{k_1} \notin H(r_{j_2}) \text{ or } \right.$$
  

$$p_{k_2} \notin H(r_{j_1}) \right\}$$
  

$$\cup \left\{ (u_{i_1j_1k_1}, u_{i_2j_2k_2}) \mid r_{i_1} \neq r_{i_2}, \text{ but } r_{j_1} \in Z_{i_2} \text{ or } \right.$$
  

$$r_{j_2} \in Z_{i_1} \right\}$$
  

$$\cup \left\{ (u_{i_1j_1k_1}, u_{i_2j_2k_2}) \mid r_{i_1} \neq r_{i_2}, \text{ but } r_{i_1} \in Z_{j_2} \text{ or } \right.$$
  

$$r_{i_2} \in Z_{i_1} \right\}.$$

Let us first consider the vertex set of conflict graph *G*. For every node  $r_i, r_j \in Z_i$ ,  $p_k \in H(r_i) \cap W(r_j)$ , there exists a vertex  $v_{ijk}$ , which illustrates the packet  $p_k$  that are wanted by the neighbor node  $r_j$  and available at the node  $r_i$ .

Once the vertices are generated in G, two vertices  $v_{i_1j_1k_1}$  and  $v_{i_2j_2k_2}$  are connected by an edge due to either a coding conflict or a transmission conflict. Any two vertices  $v_{i_1j_1k_1}$  and  $v_{i_2j_2k_2}$  will be set adjacent if one of the following condition satisfies:

- *r<sub>i1</sub> = r<sub>i2</sub>*, but *p<sub>k1</sub> ∉ H(r<sub>j2</sub>)* or *p<sub>k2</sub> ∉ H(r<sub>j1</sub>)*. In this case, two different nodes *r<sub>j1</sub>* and *r<sub>j2</sub>* require two packets *p<sub>k1</sub>* and *p<sub>k2</sub>*, but at least one of these two nodes does not contain the other needed packet. Therefore, these nodes will not be able to decode a requested packet from an encoded packet *p<sub>j1</sub> ⊕ p<sub>j2</sub>*. This condition is a coding conflict which is the complimentary condition to those used to construct the IDNC graph in [15] since we need to represent coding conflict instead of coding opportunities. *r<sub>i1</sub> ≠ r<sub>i2</sub>*, but *r<sub>j1</sub> ∈ Z<sub>i2</sub>* or *r<sub>j2</sub> ∈ Z<sub>i1</sub>*. In this case, two
- $r_{i_1} \neq r_{i_2}$ , but  $r_{j_1} \in Z_{i_2}$  or  $r_{j_2} \in Z_{i_1}$ . In this case, two vertices representing the transmissions to two different nodes  $r_{j_1}$  and  $r_{j_2}$ , but at least one of these two nodes  $r_{j_1}$  and  $r_{j_2}$  is in the coverage zones of both transmitting nodes  $r_{i_1}$  and  $r_{i_2}$ . This condition prohibits transmission from node  $r_{i_1}$  to node  $r_{j_1}$  in

the case of transmission from node  $r_{i2}$  to node  $r_{j2}$ , and vice versa.

•  $r_{i_1} \neq r_{i_2}$ , but  $r_{i_1}=r_{j_2}$  or  $r_{i_2}=r_{j_1}$ . In this case, two vertices represent the transmissions from two different nodes  $r_{i_1}$  and  $r_{i_2}$ , but at least one of these two nodes  $r_{i_1}$  and  $r_{i_2}$  is targeted by the other node. This prohibits transmission from a node in the case of that node is already targeted by another node, and vice versa. In other words, a node cannot be a transmitting node and a targeted node simultaneously.

Consider a simple example in Figure 3(a). All nodes wants to receive  $\{p_1, p_2, p_3\}$ , according to transmissions at the first stage,  $H(r_1)=\{p_1, p_3\}$ ,  $H(r_2)=\{p_1, p_2\}$ ,  $H(r_3)=\{p_1, p_2, p_3\}$ ,  $H(r_4)=\{p_1, p_3\}$ ,  $H(r_5)=\{p_2, p_3\}$ . Due to the graph definition, the conflict graph is constructed as Figure 3(b) shows. In Figure 3(b),  $v_{123}$  represents that  $r_2$  needs packet  $p_3$ , and its neighbor  $r_1$  has packet  $p_3$ ,  $r_2 \in Z_1$ . Therefore,  $r_2$  can get packet  $p_3$  from neighbor  $r_1$ .

 $v_{123}$  represents that  $r_1$  can send packet to  $r_2$  while  $v_{212}$  represents that  $r_2$  can send packet to  $r_1$ , there exists an edge  $(v_{123}, v_{212})$  since  $r_1$  are not able to be both transmitter and receiver according to the third condition of edge definition. Since  $r_2 \in Z_3$ , there exists an edge  $(v_{123}, v_{351})$  according to the second condition of edge definition. Consider vertices  $v_{323}$  and  $v_{342}$ , since  $p_3 \in H(r_4)$  and  $p_2 \in H(r_2)$ ,  $(v_{323}, v_{342}) \notin G$  according to the first condition of edge definition.

With this graph representation, we can deduce the following theorem which shows that all feasible coding and transmission conflict free decisions can be defined by the set of independent sets in conflict graph G. Independent set in a graph is defined as a set of vertices n a graph, no two of which are adjacent [30].



Figure 3. Conflict graph example

Theorem 2: Finding the optimal transmission schedule and corresponding encoded packets is equivalent to finding an independent set in the conflict graph G.

*Proof*: To prove this theorem, we need to show that there is a one to one mapping between the set of feasible nodes and the independent set in the conflict graph G.

Let  $r_{i1}$  and  $r_{i2}$  be any two different selected transmitting nodes and  $r_{j1}$  and  $r_{j2}$  be their corresponding targeted nodes. Since the above two transmissions are conflict free,  $r_{i_1} \neq r_{j_2}$ ,  $r_{i_2} \neq r_{j_1}$ , and  $r_{j_1} \notin Z_{i_2}$ ,  $r_{j_2} \notin Z_{i_1}$ . By definition of G, edge  $(v_{i_1j_1k_1}, v_{i_2j_2k_2}) \notin G$ , which illustrates that  $v_{i_1j_1k_1}, v_{i_2j_2k_2}$ belongs to the same independent set IS. Let  $r_{i1}=r_{i2}$ , the sender can broadcasts  $p_{k_1} \oplus p_{k_2}$  to receivers  $r_{j1}$  and  $r_{j2}$ , and receivers are able to decode successfully. Therefore,  $p_{k_1} \in H(r_{i_2})$  and  $p_{k_2} \in H(r_{i_1})$ . By definition of G,  $v_{i_1 j_1 k_1}$ ,  $v_{i_2 j_2 k_2}$  are not adjacent, which represents that  $v_{i_1 j_1 k_1}$ ,  $v_{i_2 j_2 k_2}$  also belongs to the same independent set IS. In a similar way, let IS be an independent set in conflict graph. Since all the nodes are pairwise nonadjacent, for any  $v_{i_1 j_1 k_1}$ ,  $v_{i_2 j_2 k_2} \in IS$ , if  $r_{i1} \neq r_{i2}$  then  $r_{j_1} \notin Z_{i_2}$  and  $r_{j_2} \notin Z_{i_1}$ , or  $r_{i1} \neq r_{i2}$  and  $r_{i_2} \neq r_{j_1}$  .If  $r_{i_1}=r_{i_2}$  then  $p_{k_1} \in H(r_{j_2})$  and  $p_{k_2} \in H(r_{i_1})$  .Hence a set of transmitting nodes  $\{r_i | v_{iik} \in IS\}$  and a set of targeted nodes  $\{r_j | v_{ijk} \in IS\}$  is a feasible conflict free nodes set, and the encoded packets  $r_i$  need to send is  $\bigoplus_{v_{ik} \in IS} p_k$ . Therefore, there exists a one to one mapping between the set of feasible nodes and the set of independent set in the conflict graph. Thus, finding a set of transmitting nodes and a set of targeted nodes is equivalent to finding an independent set in the conflict graph, which conclude our proof.

Each of the selected transmitting nodes forms a coded packet by XORing the source packets identified by the vertices in independent set IS representing transmission from that node. For example in Figure 3(b),  $\{v_{323}, v_{342}, v_{351}\}$  is an independent set which represents that  $r_3$  is able to broadcast encoded packet  $p_1 \oplus p_2 \oplus p_3$  to nodes  $r_2, r_4, r_5$ .  $\{v_{212}, v_{342}, v_{351}\}$  is also an independent set which represents that  $r_3$  can broadcast encoded packet  $p_1 \oplus p_2$  to  $r_1$  at the same time. There is no transmission conflict since  $r_1$  is out of the coverage zone of  $r_3$ . Note that our conflict graph model only based on the  $H(r_i)$  and  $W(r_i)$ , then which source nodes did the packets in  $H(r_i)$  receive from does not influence the recover process in the second phase.

#### 5.2 Heuristic Solution

In this section, we design a heuristic algorithm that aims to reduce the number of transmission slots in PCNC problem using the developed conflict graph. Given the analysis in the above subsection, we can define our proposed heuristic solution as a maximum independent set search based on the conflict graph. The heuristic solution is shown in Figure 4.

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1. t \leftarrow 1;
2. while (\bigcup_{r_i \in R} W(r_i) \neq \emptyset)
       Construct conflict graph G
3.
       Find a maximum independent set IS in G
4.
5. R^{s}(t) \leftarrow \{r_{i} | v_{ijk} \in IS\}
6. p(r_i) \leftarrow \bigoplus_{v_{ijk} \in ISPk}, \forall r_i \in R^s(t)
7. P^{s}(t) \leftarrow \{p(r_i), r_i \in R^{s}(t)\}
8. Update the information in H-set and W-set;
9. t \leftarrow t + 1;
10.end while
Output:
Nodes selected to send packet \{R^s(t), t \ge 1\};
Packets the selected node needs to send
\{P^{s}(t), t \geq 1\};
```

Figure 4. The encoding algorithm for PCNC problem

At Step 4, we find a maximum independent set to select the nodes which should transmit. The encoded packet is decided at Step 5 according to xoring all needed packets corresponding to the same sender in the independent set. However, finding a maximum independent set is NP-hard [31] so we just use a simple greedy algorithm. This algorithm starts with an empty set of vertices and keeps adding vertices with the minimum degree into the independent set until no larger independent set can be found, where the degree of a vertex in a graph is the number of edges incident to the vertex. Therefore, the time complexity of the greedy algorithm for finding a maximum independent is  $O(|V|^2)$ .

The cooperative recovery of our proposed scheme is a multi-hop transmission schedule, our transmission schedule focus on how to transmit at each time slot, then the lost packets of a receiver node can be recovered from other nodes far away hop by hop.

## 6 Simulations

In this section, we evaluate the performance of our PCNC scheme via simulations. The simulation scenario consists of a source node and n underwater sensor nodes. The sender needs to send m packets which we denote as packet set P to n receivers, according to the prior transmission at the first stage, every node has already stored some packets. The needed packets are randomly selected from the m packets with probability  $\rho$ , and the stored packets set  $H(r_i) = P - W(r_i), 1 \le i \le n$ . We set the transmission range as 200m, which is commonly used in typical practical UAN settings. The cooperative underwater sensor network topology is illustrated by a connection matrix C, if a pair of nodes are directly connected,  $c_{ii} = 1$ . We compute the connectivity degree in the network as  $\theta = num/n^2$ , where *num* is the number of non-zero

entries in C. In the case of a fully connected network,  $\theta = 1$ .

We implement our proposed scheme, and compare its performance with fully connected cooperative solution [32], which considers a fully connected network and uses IDNC to minimize the decoding delay in each time slot. We investigate the networkwide broadcast completion time, which is equal to the delay between when the source node broadcasts the packets and when all the nodes in the network have completely recovered the data block.

In order to study the impacts of m and n on the network coding gain, we use the number of transmission slots as performance metric. If the broadcast task is completed with small number of transmission slots, then the system throughput is larger, otherwise the system throughput is small. We also compare our heuristic solution with the lower bound using integer linear programming. The ILP is solved with the LP-relaxation method, which can be solved with polynomial time. Figure 5 illustrates the performance of PCNC scheme,  $\rho$  is uniformly distributed in [0.5, 1]. In Figure 6,  $\rho$  is uniformly distributed in [0.2, 1].

Figure 5(a) and Figure 6(a) show the impact of number of packets m on the network coding gain which is measured by the number of transmission slots for n = 5,  $\theta = 0.2$ . This setting reflects the poorly connected topology. Figure 5(b) and Figure 6(b) show the impact of the number of receivers *n* on the network coding gain for m = 20,  $\theta = 0.8$ , which illustrate the dense connected topology. As expected, with the increasing of *m* and *n*, the number of transmission slots increase. The reason is that there are more packets need to be sent to receiver nodes. Figure 5 and Figure 6 show that our solution reduces the number of transmission slots significantly as compared to fully connected cooperative solution when  $\theta$  is small. The reason is that, with poorly connected network topology, our proposed conflict free graph can fully utilize the information of network topology and information receivers already had to give an optimization solution.

## 7 Conclusion

In this paper we consider the scheduling problem for partially connected cooperative underwater sensor networks with network coding. We formulate the joint optimization problem over the set of transmitting nodes and the packet combinations using a conflict free graph model. Based on the maximum independent set of the proposed conflict free graph model, a heuristic solution for this setup is proposed.



**Figure 5.** The performance of PCNC,  $\rho$  in [0:5, 1]



**Figure 6.** The performance of PCNC,  $\rho$  in [0:2, 1]

In our paper, the propagation delays is measured in time slots, and a larger propagation delay results in transmission with multiple time slots, and thus generalizing time slots measurement could be a possible future research direction. With network coding, the packet duration need to the same. If packet duration is different, we can also conduct network coding by padding zero in the end of the short packet. Therefore, generalizing the scenario with different packet duration could be a possible future research direction.

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