Reliability Analysis with Diameter Constraint in Social Networks

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Abstract

Social networks provide representations for each user and show behavioral patterns in different groups. Many researches have been done on the area. One of the descriptions on dependence between users and groups is very important and not able to be ignored, especially the cascade of dependence. In addition, we find that the propagation of dependence in users is consistent with reliability issue. In this paper, we propose a reliability model of a social network and quantify the relationship between users and groups. To explore a user's neighbors, we find that the reliability with diameter constraint is proper to describe the neighbor relationship and the reliability between a user v and a selected group K, which can be expanded by (v, K)-trees and the proposed definition of (v, K)-opposite trees. Moreover, we apply reliability to friend recommendation with physical-proximity module. The reliability-based method reveals that intermediate users are reliable in the propagation of reliability. Using synthetic data sets, we validate the results of reliability analysis.

Keywords: Social networks, Reliability, Diameter constraint, Friend recommendation

1 Introduction

Over the past decade, the rapid growths of social networks and smart devices, such as smartphones and tablets, have influenced the way people interact with each other and built social connections with physical world. On one hand, the growths of social networks increase the popularity of related services and applications, such as Twitter, Myspace, Facebook, Skype, QQ, Wechat, etc.; On the other hand, in terms of computation and communication, it would take high overheads for the infrastructure. To reduce the overheads, Proximity-based Mobile Social Networks (PMSNs) employ some short-range communication techniques, such as WiFi and Bluetooth, to communicate with others and provide mobile users a new way to enjoy services and applications.

Social networks provide an intuitive representation about individual connections and display interesting behavioral patterns [1]. For online social networks, data about social interactions is more readily accessible and measurable than in off-line social networks, which need a strict model to capture their evolutionary properties [2]. The analysis about gathering data and statistics becomes an important tool for developing intelligent systems. Papagelis et al. [3] introduced sampling-based algorithms to efficiently explore a user's social network respecting its structure and to quickly approximate quantities of interest by collecting and analyzing information from a user's explicit or implicit social network. About knowledge sharing, Shen et al. [4] studied the online social networks' structural properties, revealing strikingly distinct properties such as low link symmetry and weak correlation between indegree and outdegree. They found that a small number of top contributors answer most of the questions in the system.

Based on the private information of individuals, many researchers have concentrated on such relationship of individuals as friend matching, influence propagation. Li et al. [5] addressed the problem that increasing backbone network traffic and privacy concerns of the information leakage during the friend matching process limit the expansion of users' friend circles, and proposed Small-World to achieve secure friend matching over physical world and social networks simultaneously. Huang et al. [6] investigated the structure of social networks and developed an algorithm for network correlation-based social friend recommendation. A fundamental research problem in social networks is Influence Maximization, which is to select a group of K nodes from a social network so that the spread of influence is maximized over the network. Zeng et al. [7] studied an alternative influence maximization problem, which aims to find top-K influential nodes given a threshold of influence loss due to the failure of a subset of R ($R<K$) nodes. Motivated by resource and time constraints on viral marketing campaigns, Goyal et al. [8] studied alternative optimization problems and developed a simple greedy algorithm to provide bi-criteria approximation. Recently, social networks have new applications in the fields such as cyber-physical systems [25] and other Internet technology [23-24].

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For example, Chen et al. [25] developed a social group utility maximization framework for cooperative wireless networking that takes into account both social relationships and physical coupling among users; Wang et al. [23] presented TOPK algorithm to rank manufacturers and suppliers at the same time.

Actually, dependence can reflect one of users' interactions in social networks. Social peers have a cascade of dependence for each other. Fortunately, reliability can evaluate the dependence within all users. Network reliability is defined as the probability that surviving nodes and edges span an operating sub network [9-10]. Mathematically, for a graph modeling the network, if there exists a path between a pair of nodes, nodes connect to each other [11]. Moskowitz [12] studied a node-failure network model and proposed factoring algorithm to calculate network reliability. The work of Theologou Carlier [13] used a modified method of factoring algorithm and reductions to calculate the reliability in networks with imperfect vertices. As evaluating network reliability is an NP-hard problem, network reduction methods may lead to important computational gains. Cancela et al. [14] investigated the source to terminal reliability model, proposed diameter constraint D to limit that the length of each path from the source node to the terminal node should not be greater than D, and a polynomial-time algorithm based on it to detect and delete irrelevant edges.

The remainder of this paper is organized as follows. In Section 2, we present problem formulation and propose the system model on reliability of social network. Section 3 analyzes small world effect and introduces diameter constraint in social networks from the perspective of reliability. In Section 4, we conduct experiments to demonstrate the algorithm performance. Finally, we summarize this research.

2 Problem Formulation and System Model

2.1 Problem Formulation

The extent users depend on another user or a selected group can be quantified as probability. The probability evaluation of dependence, i.e. dependability [15], is basically a reliability problem, as reliability can account for the communication between any pair of users in networks. Consequently, reliability is proper to evaluate the trust or dependence relationship between users. Traditional reliability based on (v, K)-trees can evaluate the extent of trust or dependence relationship from v to K but is not able to evaluate the relationship from K to v. Hence, it needs further discussion to quantify the extent K depends on v.

2.2 System Model

2.2.1 Edge Reliability

We consider a social network S where each node represents a user and each user interacts with friends by social connections. In network S, we denote the edge set as E_S and the edge from u to v as edge $e_{u,\,v}$, then

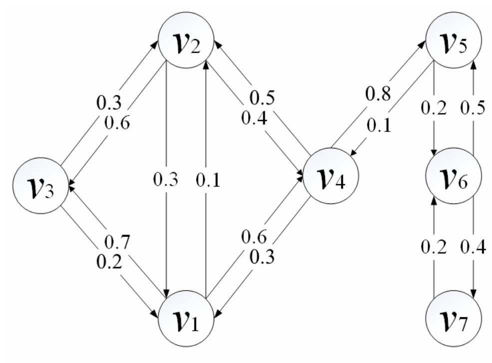
Definition 2.1. The reliability of edge $e_{u,\,v}$, denoted by $p_{u,\,v}$, is defined as the extent that user u trusts or depends on v.

To better description of reliability, we give the following analysis and observations:

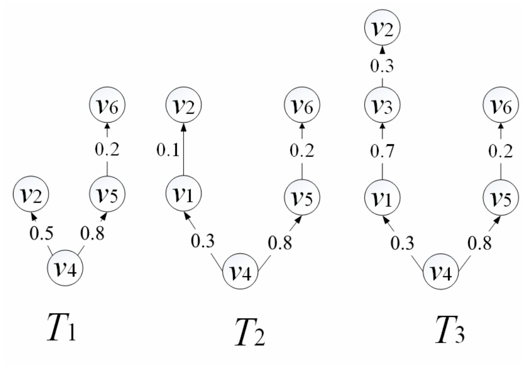
(1) **Randomness:** The reliability between any pair of people is randomly given.

(2) **Dual reliability:** For two nodes v_i and v_j, v_i depends on v_j and conversely v_j depends on v_i. For example, in Figure 1, as $e_{v5,\,v6} = 0.2$ and $e_{v6,\,v7} = 0.4$, user v_5 depends on v_7 at probability 0.2×0.4=0.08 and conversely user v_7 depends on v_5 at probability 0.2×0.5=0.1. Based on two aspects, there is dual influence on reliability for v_i, v_j. In the physical world, if people meet frequently at the same place and at the same time, they are probably socially related [16]; in social network, a potential friend recommended by existing friends would be likely to be a new friend [5].

(3) **Non-reversibility:** The reliability between any two persons cannot be reversed directly and equally. That is, generally $e_{vi,\,vj} \neq e_{vj,\,vi}$. In Figure 1, user v_4 depends on v_5 at probability 0.8 ($e_{v4,\,v5} = 0.8$) while user v_5 depends on v_4 at probability only 0.1 ($e_{v5,\,v4} = 0.1$).



(a)



(b)

Figure 1. An example G with 7 users and its v_4, K-trees

(4) **Multi-hop propagation:** Similar to influence, reliability can also be propagated through social links. It is of high possibility that a friend's friend (1-hop friend) can be a friend and further extend (more than 1-hop friend) in the social network may also result in making more friends, but the probability should be lower than 1-hop friend.

2.2.2 The Reliability Between a User and a Selected Group

In Figure 1(a), we model an example with a directed graph $G\ (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ is the set of users, $K = \{v_2, v_6\}$. We use the method of spanning trees to compute the reliability between a given user and the selected group K.

Definition 2.2. A tree T is called a (v, K)-tree if it is rooted at v (i.e. v has indegree 0), covering all vertices in set K and any pendant vertex u (i.e. u has indegree 1 and outdegree 0) of T belongs to K [21].

Figure 1(b) shows all v_4, K-trees for node v_4 and $K = \{v_2, v_6\}$. According to reliability theory, the reliability between v_4 and nodes in K is the union of all v_4, K-trees.

Proposition 2.1. Let $T_1, T_2, \ldots, T_m$ be all (v, K)-trees, the reliability between node v and nodes in K can be computed by

$$R(v,K) = \Pr(T_1 \cup T_2 \cup \cdots \cup T_m) \quad \textbf{(1)}$$

By the inclusion-exclusion method [12], the probability that at least one event of $C_1, C_2, \ldots, C_n$ happens can be calculated by

$$\begin{aligned}&\Pr(C_1 \cup C_2 \cup \cdots \cup C_n)\\ &= \sum_{i=1}^{n} \Pr(C_i) - \sum_{i<j} \Pr(C_i C_j) + \cdots + (-1)^{n+1} \Pr(C_1 C_2 \cdots C_n)\end{aligned} \quad \textbf{(2)}$$

Each (v, K)-tree represents one kind of dependent relationship between v and K. The reliability is the extent that node v depends on group K, i.e. the event that at least one (v, K)-tree holds, which is consistent with the property of Eq. (2). Hence, the reliability between node v_4 and $K = \{v_2, v_6\}$ is

$$\begin{aligned}R(v_4,K) &= \Pr(T_1 \cup T_2 \cup T_3)\\ &= Pr(T_1) + Pr(T_2) + Pr(T_3) - Pr(T_1 \cap T_2)\\ &\quad - Pr(T_1 \cap T_3) - Pr(T_2 \cap T_3) + \Pr(T_1 \cap T_2 \cap T_3)\\ &= 0.5 \cdot 0.8 \cdot 0.2 + 0.3 \cdot 0.1 \cdot 0.8 \cdot 0.2 + 0.3 \cdot 0.7 \cdot 0.3 \cdot 0.8 \cdot 0.2\\ &\quad - 0.3 \cdot 0.1 \cdot 0.5 \cdot 0.8 \cdot 0.2 - 0.3 \cdot 0.7 \cdot 0.3 \cdot 0.5 \cdot 0.8 \cdot 0.2\\ &\quad - 0.3 \cdot 0.7 \cdot 0.3 \cdot 0.1 \cdot 0.8 \cdot 0.2 + 0.3 \cdot 0.7 \cdot 0.3 \cdot 0.1 \cdot 0.5 \cdot 0.8 \cdot 0.2\\ &= 0.6182662\end{aligned}$$

Proposition 2.1 gives the method of calculating the probability that node v depends on nodes in set K. Actually, the extent that nodes in set K depend on node v should also be evaluated.

Definition 2.3. Suppose T is a (v, K)-tree of G, S is obtained by reversing the direction of each edge in T, then we call S to be a (v, K)-opposite-tree.

Actually, a (v, K)-opposite-tree is not a tree. Node v_4 in subgraph S_1 has no outdegree and nodes in K has no indegree. The reliability from nodes in K to node v_4 can be computed by paths from nodes in K to node v_4.

Proposition 2.2. Suppose $S_1, S_2, \ldots, S_r$ are (v, K)-opposite-trees for given v and K. For each S_i, paths $P_{i_1}, P_{i_2}, \ldots P_{i_{r_i}}$ start with some node in K and end with node v. The reliability from nodes in K to node v is

$$R(K,v) = \Pr\left(\bigcup_{i=1}^{r}\bigcup_{j=1}^{r_i} P_{ij}\right) \quad \textbf{(3)}$$

In Figure 1(b), we denote the corresponding v_4, K-opposite-tree of T_i as S_i.

In v_4, K-opposite-tree S_1

$$P_{1j} = \begin{cases} v_2 \to v_1 \to v_4, j = 1 \\ v_6 \to v_5 \to v_4, j = 2 \end{cases}$$

In v_4, K-opposite-tree S_2

$$P_{2j} = \begin{cases} v_2 \to v_4 \qquad\quad, j = 1 \\ v_6 \to v_5 \to v_4, j = 2 \end{cases}$$

In v_4, K-opposite-tree S_3

$$P_{3j} = \begin{cases} v_2 \to v_3 \to v_1 \to v_4, j = 1 \\ v_6 \to v_5 \to v_4 \qquad\quad, j = 2 \end{cases}$$

The reliability from nodes in K to node v_4 is

$$\begin{aligned}R(K,v_4) &= \Pr\left(\bigcup_{i=1}^{r}\bigcup_{j=1}^{r_i} P_{ij}\right)\\ &= \Pr(P_{11} \cup P_{12} \cup P_{21} \cup P_{22} \cup P_{31} \cup P_{32})\\ &= 0.5662528\end{aligned}$$

3 Small-world Network and Diameter Constrained Reliability

3.1 Small-world Network and Diameter Constraint

A small-world network is a type of mathematical graph in which most nodes are not neighbors of one another, but the neighbors of any given node are likely to be neighbors of each other and most nodes can be reached from every other node by a small number of hops or steps [17, 22]. The small world property preserves low average distance (or diameter) and high clustering for social networks [2]. Evidence suggests that in most real-world networks, particularly in social networks, nodes tend to create tightly knit groups characterized by a relatively high density of ties; this likelihood tends to be greater than the average probability of a tie randomly established between two nodes [18-19]. Hence, small-world networks tend to contain cliques, and near-cliques, meaning sub-networks, which have connections between almost any two nodes within them.

From the perspective of reliability, diameter constraint, introduced by Cancela and Petingi [19], is consistent with the properties of small-world network because the constraint is to bound lengths of paths not greater than a given value *d*. [20] investigates the source-to-*K*-terminal reliability model and utilizes the definition of domination to computer the proposed reliability with diameter constraint. After the removal of edges that only belong to paths greater than *d*, path lengths between the source node and the terminal node or between the source node and the terminal node set *K* in the residual network do not exceed *d*, which corresponds to the properties of small-world network. Hence, diameter constraint is effective in social network.

3.2 The Reliability Between a User and a Selected Group with Diameter Constraint

In a social network, the reliability may not be propagated to the user far from the given user. By using diameter constraint, we would like to obtain the set of nodes in the neighborhood $N_d(v)$ of a node v at some specific depth $d \in [1, 6]$ (at most d hops away from v). Correspondingly, the reliability of one user and a selected group K is considered in $N_d(v)$.

Definition 3.1. The maximum distance between v and any of the nodes in K is called (v, K)-diameter [21].
In Figure 1, the maximum distance between v_4 and v_2 is 3 and the maximum distance between v_4 and v_6 is 2. Hence, v_4, K-diameter is 3.

Definition 3.2. Given a node v and a selected group K, the reliability between node v and nodes in K with diameter constraint d is the probability that node v depend on any node in set K within a neighborhood $N_d(v)$ of a node v at some specific depth d.

Definition 3.3. A (v, K)-tree is called a (v, K)-d-tree if its (v, K)-diameter is at most d [21].

In Figure 2, if d=2, then tree T_3 is a not (v, K)-d-tree as its (v, K)-diameter is 3.

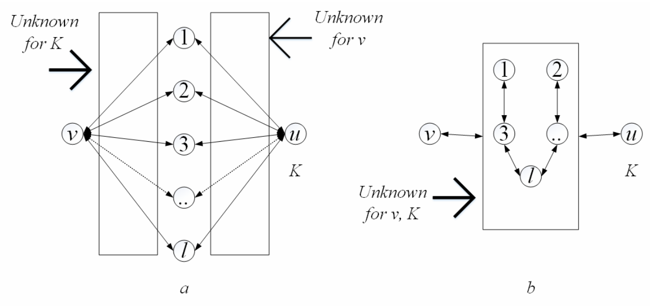


Figure 2. Social reliability between v and u

To evaluate the reliability from K to node v, we also define the maximum distance between any of the nodes in K and v is called (K, v)-diameter.

Definition 3.4. A (v, K)-opposite-tree is called a (v, K)-d-opposite-tree if its (K, v)-diameter is at most d.

Hence, the reliability for a node v and a node set K with diameter constraint d can be computed by

$$R(v,K,d) = \Pr\left(\bigcup_i T(d)_i\right) \tag{4}$$

$$R(K,v,d) = \Pr\left(\bigcup_i S(d)_i\right) \tag{5}$$

where $T(d)_i$ is the ith (v, K)-d-tree and $S(d)_i$ is the ith (v, K)-d-opposite-tree.

Based on the above analysis, we propose an algorithm (Algorithm 1) to compute the reliability between node v and a selected group K.

Algorithm 1. Reliability in network G = (V, E)	
Input:	*A connected graph G = (V, E) with the node set V, edge set E, node reliability set P, diameter constraint d, node v and set K.*
Output:	*R(v, K, d) and R(K, v, d)*
Step 1	*If G is not (v, K)-connected, R (v, K, d) = R (K, v, d) = 0, return; else go to step 2.*
Step 2	*If the least distance from node v to any node in K is greater than d, R (v, K, d) = R (K, v, d) = 0, return; else go to step 3.*
Step 3	*Generate all (v, K)-trees.*
Step 4	*For each (v, K)-tree, if its diameter is greater than d, delete it.*
Step 5	*Compute R (v, K, d) and R (K, v, d) with Eq. (4) and Eq. (5).*

We have:

Theorem 3.1. For a given node v and a selected group K, reliability with diameter constraint is less than that without diameter constraint.

Proof. Suppose $G = (V, E)$ is a social network. Given v and set K, we can obtain (v, K) - trees $T_1, T_2, \ldots, T_m$. Hence, the reliability can be computed by

$$R(v,K) = \Pr(T_1 \cup T_2 \cup \cdots \cup T_m)$$

Given constant d, for graph $G = (V, E)$, we can obtain its subgraph $G^* = (V^*, E^*)$ by deleting irrelevant arcs. Let $T(d)_1, T(d)_2, \ldots, T(d)_n$ ($n \le m$) be (v, K)-d-trees, the reliability can be computed by

$$R(v,K,d) = \Pr(T(d)_1 \cup T(d)_2 \cup \cdots \cup T(d)_n)$$

$G^* = (V^*, E^*)$ is a sub graph of $G = (V, E)$, hence

$$\{T(d)_1, T(d)_2, \ldots, T(d)_n\} \subseteq \{T_1, T_2, \ldots, T_m\}$$

Let

$$\{T_1, T_2, \ldots, T_m\} = \{T(d)_1, T(d)_2, \ldots, T(d)_n, T_{n+1}, \ldots, T_m\}$$

where $T_1 = T(d)_1, T_2 = T(d)_2, \cdots, T_n = T(d)_n$

Based on the probability theory, we have

$$R(v,K)=\Pr(T_1\cup T_2\cup\cdots\cup T_m)$$
$$=\Pr(T(d)_1\cup T(d)_2\cup\cdots\cup T(d)_n\cup T_{n+1}\cup\cdots\cup T_m)\ \square$$
$$\geq \Pr(T(d)_1\cup T(d)_2\cup\cdots\cup T(d)_n)=R(v,K,d)$$

3.2 An Application: Friend Recommendation

Friend matching or recommendation is a common problem in social network. Li et al. [5] designs physical proximity module and builds a Katz score-based social strength proximity module to describe the similarity between Social Network users to help match potential friends.

To consider the importance that the co-occurrences at different places are to the social reliability, we use location entropy [21] to calculate the physical proximity. Let l be a location, $V_{l,u}$ be a set of check-ins at location l of user u and V_l be a set of all check-ins at location l of all users. According to [5, 16], the importance function can be denoted as

$$f_{uv}=\sum_{l=1}^{n}c_{uv,l}\exp(-H_l) \quad \textbf{(6)}$$

$$H_l=-\sum_{u,P_{u,l}\neq 0}\frac{|V_{l,u}|}{|V_l|}\log\frac{|V_{l,u}|}{|V_l|} \quad \textbf{(7)}$$

for u and v, where $c_{uv,l}$ denotes the times of the user pair's co-occurrence at location l, $\exp(-H_l)$ means how important the co-occurrences at the place l are to the social strength. As the social-strength is based on reliability, our friend recommendation is based on the combination of physical similarity and social-reliability.

Definition 3.5. The Friend Recommendation Index (FRI) between u and v is defined as

$$FRI=\begin{cases}FRI_{uv}=f_{uv}\cdot R(u,v), v\ is\ the\ receiver\\ FRI_{vu}=f_{uv}\cdot R(v,u), u\ is\ the\ receiver\end{cases}$$

For a selected group $K=\{u_1, u_2, \ldots, u_k\}$, the proximity is defined as

$$FRI=\begin{cases}FRI_{vK}=f_{vK}\cdot R(v,K), K\ is\ the\ receiver\\ FRI_{Kv}=f_{vK}\cdot R(K,v), v\ is\ the\ receiver\end{cases}$$

where f_{vk} is the physical proximity between user v and K, and

$$f_{vK}=\frac{1}{k}\sum_{i=1}^{k}f_{vu_i}=\frac{1}{k}\sum_{i=1}^{k}\sum_{l=1}^{n}c_{vu_i,l}\exp(-H_l)$$

Repeat the procedures, user v can obtain the most likely friend based on physical proximity and social reliability

$$U=\max\{FRI_{vu_1},FRI_{vu_2},\ldots,FRI_{vu_k}\}$$

Theorem 3.2. Our method is social connection tree establishment reliable.

Proof. Since weight-aware spatiotemporal matching does not involve intermediate nodes [5], we just need to discuss the reliability of the two-hop and the multi-hop social reliability-aware matching.

(1) $|K|=1$, the model is to estimate the reliability between a pair of users as Figure 3 shows.

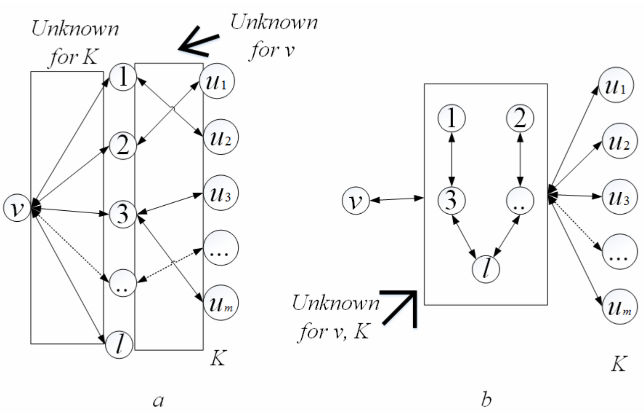


Figure 3. Social reliability between v and K

During the two-hop social reliability-aware matching (Figure 2(a)), $\{1, 2, \ldots, l\}$ is the intermediate nodes set. Users v and u receive and operate on the reliability propagated from intermediate nodes. It is hard for u to know the dependence between v and each intermediate user. Similarly, user v just knows the reliability from $\{1, 2,\ldots, l\}$, but is unable to distinguish the reliability between u and intermediate nodes.

During the multi-hop social reliability-aware matching (Figure 3(b)), both u and v are not aware of the reliability propagation among intermediate nodes in social network because it is impossible for u and v to know explicit social connections in these nodes.

Hence, the dependence between u and v is reliable

(2) Similarly, for $|K|>1$, the model is to estimate the reliability between a user and a selected group as Figure 4 shows.

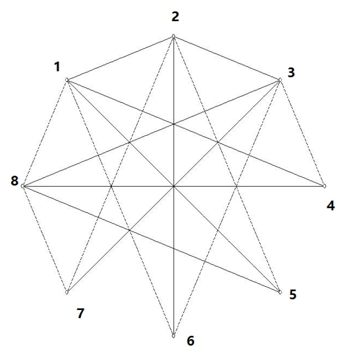


Figure 4. An example with eight nodes

During the two-hop and multi-hop social connections (Figure 3(a)), $\{1, 2, \ldots, l\}$ is the intermediate nodes set. The users v and K receive and operate on the reliability propagated from intermediate nodes. In this case, if K is the receiver, the social reliability that K receives is based on paths from user v

and through intermediate nodes. Users in K know the reliability is propagated from intermediate nodes, but they cannot guess the reliability between v and intermediate nodes. Hence, the intermediate nodes are reliable.

4 Performance Evaluation

4.1 Social Reliability Evaluation

Let G be a graph depicting connections between users in a social network, where each node in the graph represents a user and each edge stands for the reliability between two nodes. Table 1 shows the reliability evaluation of Figure 1. As the social connections in users of Figure 1 are not complex, there are three cases where the reliability with diameter constraint equals to that with no diameter constraint, because their (v, K)-diameter is less than the value of d. In the other three cases, their reliabilities with diameter constraint is 0 since the shortest path between the given node and one node in K is greater than d. From the perspective of social connections, these cases do not mean that users cannot be friends. The reliability 0 indicates that users, including the given user and K, do not depend on those who are d-hops away from them.

Figure 4 is an example with eight nodes, where the edge reliability for each pair of nodes is listed in Table 2.

Given $|K|$=3, we illustrate several selected groups for each given node in Figure 4. Table 3 shows the results.

Table 1. Reliability evaluation for Figure 1

v	K	d	R((v, K))	R((K, v))	R((v, K), d)	R((K, v), d)
v_1	$\{v_3, v_7\}$	2	0.6584	0.8647	0	0
v_2	$\{v_1, v_5\}$	4	0.6895	0.3689	0.6895	0.3689
v_3	$\{v_2, v_6\}$	3	0.6587	0.9852	0	0
v_4	$\{v_1, v_7\}$	2	0.5974	0.6589	0	0
v_5	$\{v_4, v_6\}$	3	0.2800	0.9000	0.2800	0.9000
v_6	$\{v_1, v_7\}$	3	0.4090	0.2768	0.4090	0.2768

Table 2. Reliability for each pair of nodes in Figure 4

	1	2	3	4	5	6	7	8
1	0	0.6	0	0.5	0.7	0.4	0	0.7
2	0.5	0	0.6	0	0.5	0.8	0.1	0
3	0	0.7	0	0.4	0	0.7	0.8	0.9
4	0.9	0	0.1	0	0	0	0	0.6
5	0.4	0.1	0	0	0	0	0	0.2
6	0.1	0.6	0.9	0	0	0	0	0
7	0	0.7	0.2	0	0	0	0	0
8	0.3	0	0.3	0.5	0.8	0	0	0

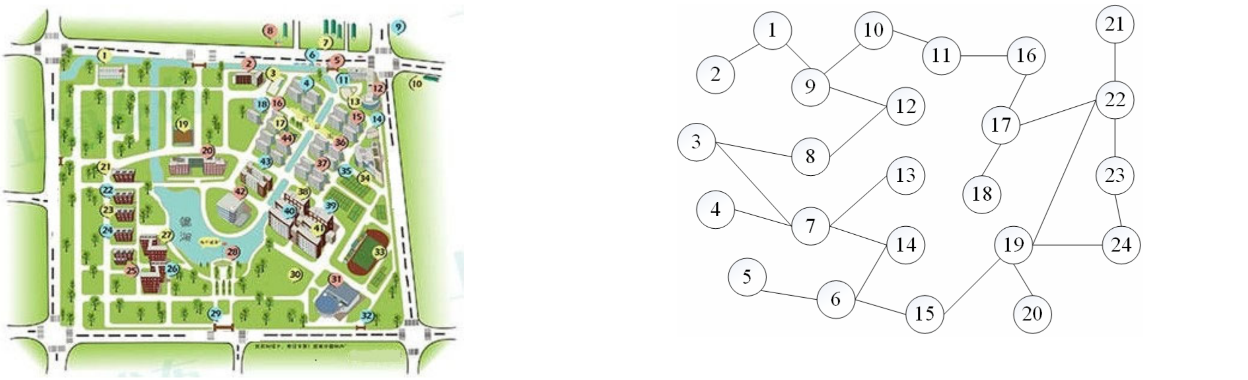
Table 3. Reliability evaluation for Figure 4

v	K	d	R((v, K))	R((K, v))	R((v, K), d)	R((K, v), d)
1	{2, 6, 8}	4	0.9877	0.9588	0.9624	0.9466
1	{2, 5, 7}	3	0.8655	0.8749	0.8655	0.7854
2	{4, 5, 6}	5	0.8744	0.7566	0.8744	0.7022
2	{3, 7, 8}	4	0.9465	0.6574	0.9288	0.6085
3	{5, 6, 8}	3	0.8966	0.7544	0.6877	0.7544
3	{2, 4, 8}	3	0.8674	0.6988	0.7411	0.6988

The social connections in Figure 4 are more complex than Figure 1. The results in Table 3 verifies the practical significance of Theorem 3.1. For example, for given user No. 1, K = {2, 6, 8} and d=4, tree {1, 2, 3, 4, 6, 7, 8} is not taken into consideration, which results that user No. 1 loses the reliability between No. 7 and No. 8. As reliability is to quantify the extent of trust or dependence for users, lacking of the reliability between No. 7 and No. 8 definitely reduces the reliability between No. 1 and {2, 6, 8}.

4.2 Friend Recommendation Evaluation

Figure 5 is a small social network in a campus, where (a) is an ichnography of a schoolyard, (b) shows the dependence of 24 students. Table 4 shows the corresponding reliability of each edge.



(a) (b)

Figure 5. A small social network in a campus

Table 4. Reliability for each pair of nodes in Figure 5

Edge		Reliability		Edge		Reliability	
Node *u*	Node *v*	p_{uv}	p_{vu}	Node *u*	Node *v*	p_{uv}	p_{vu}
1	2	0.9	0.7	9	12	0.3	0.6
1	9	0.3	0.3	10	11	0.5	0.8
3	7	0.4	0.7	11	16	0.7	0.8
3	8	0.8	0.7	15	19	0.4	0.5
4	7	0.7	0.6	16	17	0.3	0.4
5	6	0.6	0.5	17	18	0.8	0.3
6	14	0.3	0.8	17	22	0.6	0.2
6	15	0.2	0.4	19	20	0.7	0.7
7	13	0.4	0.2	19	22	0.3	0.8
7	14	0.7	0.7	19	24	0.4	0.4
8	12	0.5	0.8	21	22	0.5	0.7
9	10	0.1	0.3				

We aim to investigate the study relationship of these students, so we just consider the frequency they go to the library 20 and classroom buildings 21-27. The students share their locations via Wechat. Based on investigation of two months, we obtain the dataset of these students. Taking student No. 11 as an example, we list the FRI to show the likelihood of friend matching in Table 5. Within students {1, 12, 17}, No. 17 is most likely to be friend of No. 11. In addition, we can find that higher social reliability generally corresponds to higher physical proximity. This observation shows the coherence of social connections and physical behaviors. The exception is that the group {2, 19} shares high physical proximity but low social reliability with No. 11. It can be seen that in social networks, the hops between No. 11 and group {2, 19} is not less than 4. Multi-hops reliability propagation reduces the trust or dependence between No. 11 and group {2, 19}, so No. 11 is less likely to join the group {2, 19} in spite of their high physical proximity.

Table 5. Friend recommendation for No. 11

	No. 11	Physical proximity	Social reliability	FRI
Users	1	3.421	0.072	0.2463
	12	2.432	0.072	0.1751
	17	4.223	0.21	0.8868
Groups	{10, 16}	5.211	0.94	4.8983
	{9, 17}	4.409	0.3996	1.7618
	{2, 19}	5.807	0.1124	0.6527

Table 6 gives the results of reliability evaluation for Figure 5. We find that in general, *R* (*v, K*) is less than *R* (*K, v*). In a social network, the construction of a group is based on high trust or dependence of each other. The growth of a group is also related to its dependence on a new comer. For example, the extent the group {6, 20, 22} depends on No. 19 shows that they think No. 19 can join their group.

Table 6. Reliability evaluation for Figure 5

v	K	d	Number of (v, K)-d-tree	R(v, K, d)	R(K, v, d)
8	{1, 3, 10}	4	1	0.6223	0.8245
10	{1, 8, 17}	3	1	0.2306	0.2939
19	{6, 20, 22}	2	2	0.7287	0.8124
22	{16, 18, 19}	2	2	0.6891	0.7166

5 Conclusions

This paper mainly discusses the problem of reliability and diameter constraint in social network. According to the available information of the network, we analyze the reliability between a given node *v* and a selected group *K* and propose an algorithm to calculate it. The diameter constraint efficiently investigates the neighborhood of a given node *v* at a specified depth *d*. We prove that reliability with diameter constraint is less than that without diameter constraint. Further, we apply the reliability method to friend recommendation. This work is helpful in understanding the relationship between a given node *v* and a selected group *K* in social network.

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