# Two-dimensional Channel Estimation for Millimeter-wave MIMO Systems with Hybrid Precoding

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## Abstract

Channel estimation for the millimeter wave (mm-wave) MIMO systems with hybrid precoding can be performed by estimating the path directions of the channel and corresponding path gains. Researchers often apply the multiple signal classification (MUSIC) method to estimate angle of arrivals (AoAs) and angle of departures (AoDs), however their schemes are all based on the communication systems equipped with uniform linear arrays (ULAs), which means that only azimuth angle is considered and we cannot get more accurate channel state information (CSI). Different from the previous work, this paper does not ignore the effection of the elevation parameters by proposing a two-dimensional (2-d) channel estimation scheme in the systems with L-shape arrays and uniform planar arrays (UPAs). The simulation results show that the proposed channel estimation scheme can achieve higher accuracy in the two types of arrays, thus obtaining more comprehensive channel information than conventional solutions.

**Keywords:** Millimeter-wave MIMO communications, Two-dimensional channel estimation, Lshape arrays, Uniform planar arrays

## **1** Introduction

Millimeter wave (mm-wave) band has great potential for providing high data rates in wireless local area network and fifth generation (5G) cellular [1-4]. However, the severe path fading of mm-wave limits the transmission distance and reduces the coverage performance, the mm-wave and massive MIMO are combined to obtain enough signal power and improve the communication distance. Due to the traditional channel estimation techniques for MIMO systems fail to characterize the spatial sparsity of the mm-wave channel, the mm-wave channel estimation problem becomes estimating the path directions (angle pairs) and gains rather than estimating the MIMO channel matrix [5].

There are several kinds of channel estimation algorithms from previous research [5, 19]. The first kind of methods belong to beam training method which

search in the angular space by adjusting the steering directions of the beam former because the mm-wave signals are highly directional and consist of a few significant components [5-6]. Moreover, [7] performed an amplitude comparison with respect to the auxiliary beam pair to achieve better angle estimation and in order to avoid being hampered by high training overhead in the actual system, it also developed efficient beam training strategies by utilizing the compressed sensing (CS) theory where the searching beams can be narrower than before to reduce the training overhead. Another method [8] utilized the orthogonal matching pursuit (OMP) algorithm to solve the mm-wave channel estimation problem concerning the hybrid digital/analog precoding structure [9-11]. Similarly, the on-grid CS based methods [12-13] can estimate the channel with reduced training overhead by exploiting the angular channel sparsity. However, the solution assumed that the AoAs and AoDs lie in discrete points in the angle domain, while the actual AoAs and AoDs are continuously distributed in practice. The assumption severely degrades the channel estimation accuracy. To solve this resolution limitation caused by the on-grid angle estimation, [14] propose an iterative reweight (IR)-based superresolution channel estimation scheme to estimate the off-grid AoAs and AoDs.

Different from the works mentioned above, we use the spatial spectrum estimation (SSE) method [15] to solve the mm-wave channel estimation problem. MUSIC method is a classic method for estimating the AoAs and has been employed in [16] for mm-wave communication systems. [17] and [18] extended the method to estimate the AoDs and AoAs jointly. Specifically, [18] considered the fast implementation of the two-dimensional MUSIC. However, the oneand two-dimensional MUSIC algorithms are all performed in the element-space, which are impractical to be implemented with a small number of RF chains. For mm-wave MIMO communications employing hybrid beamforming, the receiving signals are first mixed by the beamformer and then sent to the RF chains. Thus [19] use the two-dimensional beam space MUSIC approach and the least-squares (LS) method to

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estimate the path directions and path gains respectively.

Although all the prior works above have good channel estimation performance, the mm-wave system models are usually based on ULAs. The ULAs can provide only the one-dimensional (1-d) information of CSI. To have a more complete under-standing of the mm-wave channel, our work in this paper aim at two-dimensional (2-d) channel estimation which also values the influence of elevation angles by proposing estimation scheme in the L-shape arrays and UPAs.

The rest of this paper is organized as follows. Section II presents the typical channel model and introduces the proposed channel model. In section III, we describe the 2-d channel estimation method in detail. Simulation results are presented in section IV. Section V concludes the paper.

Notions: we use the following notations throughout this paper: **A** is a matrix, **a** is a vector,  $diag(\mathbf{A})$  is a vector formed by the diagonal elements of **A** and  $\|\mathbf{A}\|_{F}$  is its Frobenius norm.  $\mathbf{A}^{*}, \mathbf{A}^{T}, \mathbf{A}^{H}, \mathbf{A}^{-1}$  and  $\mathbf{A}^{\dagger}$ are the conjugate, the transpose, the conjugate transpose, inverse and pseudo-inverse respectively. **I** is the identity matrix.  $\mathbf{A} \circ \mathbf{B}$  and  $\mathbf{A} \otimes \mathbf{B}$  are the Khatri-Rao product and the Kronecker product of **A**, **B** respectively.

## 2 System and Channel Model

#### 2.1 System Model

Considering a mm-wave communication system where the hybrid analog/digital beamforming is adopted to reduce the hardware cost [10]. As shown in Figure 1,  $N_t$  and  $N_r$  are the number of transmit and receive antennas respectively. The analog beam former F and W are connected to  $M_t$  and  $M_r$  RF chains and the number of RF chains is usually less than that of the antennas in practice, such that  $M_t < N_t$  and  $M_r < N_r$ . The received signal is processed by the combiner composed of the RF and baseband combiners can be given by:

$$\mathbf{Y} = \mathbf{W}^H \mathbf{H} \mathbf{F} \mathbf{s} + \mathbf{W}^H \mathbf{n} , \qquad (1)$$



Figure 1. The hybrid analog/digital antenna architecture

Where **s** is the vector of transmitted baseband signal, **H** is the channel matrix, and  $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2 I_N)$  is the Gaussian noise corrupting the received signal.

#### 2.2 Channel Model

#### 2.2.1 Channel Model Based on ULAs

The ray-tracing representation is usually used to model the mm-wave channel [19]. In the typical mm-wave massive MIMO system with ULAs, assuming that there are L clusters and each one contributes to a resolvable path. Channel model can be given by Eq. (2).

$$\mathbf{H}(q) = \sum_{l=1}^{L} g_l(q) \mathbf{a}_r(\phi_{r,l}) \mathbf{a}_l^T(\phi_{r,l}), \qquad (2)$$

Where  $L \ll \min(N_T, N_R)$  is the number of propagation paths,  $g_l(q)$  is the channel gain of the  $l^{\text{th}}$  path and q is used to index the time block when the mm-wave channel remain unchanged.  $\phi_{r,l}$  and  $\phi_{l,l}$  are the physical azimuth AoA and AoD of the *l*-th path respectively.  $\mathbf{a}_l(\cdot)$  and  $\mathbf{a}_r(\cdot)$  are the antenna array response at the transmitter and receiver respectively and they can be represented by Eq. (3) if defining  $\alpha = e^{-j2\pi d \sin \phi_l/\lambda}$  and  $\beta = e^{-j2\pi d \sin \phi_l/\lambda}$ .

$$\mathbf{a}_{r}\left(\phi_{r}\right) = \begin{bmatrix} 1, \alpha, \cdots, \alpha^{N_{R}} \end{bmatrix}^{T},$$
  
$$\mathbf{a}_{t}\left(\phi_{t}\right) = \begin{bmatrix} 1, \beta, \cdots, \beta^{N_{T}} \end{bmatrix}^{T},$$
  
(3)

The mm-wave channel matrix  $\mathbf{H}$  in (2) can be also written as

$$\mathbf{H}(q) = \mathbf{A}_{R}(\phi_{r})\mathbf{\Lambda}_{G}(q)\mathbf{A}_{T}(\phi_{r}).$$
(4)

Where  $\mathbf{\Lambda}_{G}(q) = diag\{g_{1}(q), \cdots, g_{L}(q)\}, \mathbf{A}_{R} = [\mathbf{a}_{r}(\phi_{1}), \mathbf{a}_{r}(\phi_{2}), \cdots, \mathbf{a}_{r}(\phi_{L})]$  and  $\mathbf{A}_{T} = [\mathbf{a}_{r}(\phi_{1}), \mathbf{a}_{r}(\phi_{2}), \cdots, \mathbf{a}_{r}(\phi_{L})].$ 

#### 2.2.2 Channel Model Based on L-shape Arrays

Considering a x - y plane L-shaped array as shown in Figure 2. The array consists of two subarrays which are an  $N_{Lx}$ -element ULA arranged along the x-axis and an  $N_{Ly}$ -element ULA arranged along the y-axis. The distance between adjacent elements is d, and the origin is set as the reference point, which belongs to subarray x. Defining the elevation angle by  $\theta$ , the channel model can be expressed as Eq. (5).

$$\mathbf{H}_{L}(q) = \begin{bmatrix} \mathbf{A}_{rx}(\phi_{rx}, \theta_{rx}) \\ \mathbf{A}_{ry}(\phi_{ry}, \theta_{ry}) \end{bmatrix} \mathbf{\Lambda}_{G}(q) \begin{bmatrix} \mathbf{A}_{tx}(\phi_{tx}, \theta_{tx}) \\ \mathbf{A}_{ty}(\phi_{ty}, \theta_{ty}) \end{bmatrix}^{T}, \quad (5)$$
$$= \mathbf{A}_{LR}(\phi_{rx}, \theta_{ry}) \mathbf{\Lambda}_{G}(q) \mathbf{A}_{LT}(\phi_{tx}, \theta_{ty})$$



Figure 2. L-shape arrays

The representations of matrixs  $\mathbf{A}_{LR}$  and  $\mathbf{A}_{LT}$  are the same as  $\mathbf{A}_{R}$  and  $\mathbf{A}_{T}$  in the ULAs.

Defining  $\alpha_{rx} = e^{-j2\pi d_x \cos \theta_{rx} \sin \phi_{rx}/\lambda}$ ,  $\alpha_{ry} = e^{-j2\pi d_y \sin \theta_{ry} \sin \phi_{ry}/\lambda}$  $\beta_{rx} = e^{-j2\pi d_x \cos \theta_{tx} \sin \phi_{tx}/\lambda}$  and  $\beta_{ry} = e^{-j2\pi d_y \sin \theta_{ty} \sin \phi_{ty}/\lambda}$ , we have

$$\mathbf{a}_{Lr} = \begin{bmatrix} 1, \alpha_{rx}, \cdots, \alpha_{rx}^{N_{Lx}}; \alpha_{ry}, \cdots, \alpha_{ry}^{N_{Ly}} \end{bmatrix}^{T} \\ \mathbf{a}_{Lt} = \begin{bmatrix} 1, \beta_{rx}, \cdots, \beta_{rx}^{N_{Lx}}; \beta_{ry}, \cdots, \beta_{ry}^{N_{Ly}} \end{bmatrix}^{T}.$$
(6)

#### 2.2.3 Channel Model Based on UPA

Considering a  $N_{Px} \times N_{Py}$  uniform planar arrays as depicted in Figure 3. The channel model is given by

$$\mathbf{H}_{P}(q) = \mathbf{A}_{PR}(\phi_{r}, \theta_{r}) \mathbf{\Lambda}_{G}(q) \mathbf{A}_{PT}(\phi_{t}, \theta_{t})^{T}, \qquad (7)$$



Figure 3. Uniform planar arrays

The representation of matrixs  $\mathbf{A}_{PR}$  and  $\mathbf{A}_{PT}$  are also the same as  $\mathbf{A}_{LR}$  and  $\mathbf{A}_{LT}$ , we have

$$\mathbf{a}_{Pr} = \begin{bmatrix} 1, \alpha_{rx}, \cdots, \alpha_{rx}^{N_{Lx}} \end{bmatrix}^T \otimes \begin{bmatrix} 1, \alpha_{ry}, \cdots, \alpha_{ry}^{N_{Ly}} \end{bmatrix}^T \\ \mathbf{a}_{Pt} = \begin{bmatrix} 1, \beta_{rx}, \cdots, \beta_{rx}^{N_{Lx}} \end{bmatrix}^T \otimes \begin{bmatrix} 1, \beta_{ry}, \cdots, \beta_{ry}^{N_{Ly}} \end{bmatrix}^T$$
(8)

Due to the basics that the clusters' central angles belong to large-scale fading while the path gains belong to small-scale fading, the channel is determined by the path gains. In particular, the path directions are assumed to be fixed during a frame which consists of N time fading blocks. The buffered N blocks of received signals are utilized to estimate the path directions and the estimated path directions then form the basis of path gains estimation in each block. By using the estimated of path directions, we can estimate the path gains and get the estimation of the entire channel matrix.

## **3 2-D Channel Estimation Scheme**

In this section, the MUSIC algorithm are applied to estimate path directions and the least-squares (LS) method are used to estimate the path gain based on the estimated angle.

### 3.1 1-D Estimation of Path Direction

Before introducing 2-d channel estimation, we first review the channel estimation for mm-wave system with uniform linear arrays in paper [19]. During one fading block, the pilot signal  $p_m(t)$  transmitted at  $m^{th}(m=1,...,M_t)$  RF chain satisfy

$$\int_{M_t T} p_i(t) p_j(t) dt = \begin{cases} \frac{E}{M_t}, i = j\\ 0, i \neq j \end{cases}$$
 (9)

Where *E* is the transmit power allocated equally for each pilot signal. Defining  $P = \frac{E}{M_t}$ , we can obtain the received signal  $\mathbf{y}(q)$  based on the orthogonality of the pilot waveforms.

$$\mathbf{y}(q) = P \mathbf{W}^{H} \mathbf{H}(q) \mathbf{F} + \mathbf{n}(q), \qquad (10)$$

Then, collecting the measurement vectors from the *N* blocks to form the covariance matrix and getting the noise subspace  $\mathbf{U}_n$  by performing the eigenvalue decomposition on the sample covariance matrix. The noise subspace  $\mathbf{B}^H = (\mathbf{F}^H \mathbf{A}_T) \circ (\mathbf{W}^H \mathbf{A}_R)$  satisfies  $\mathbf{B}^H \mathbf{U}_n = 0$ . Finally the 2-d MUSIC method can estimate the *L* path directions by locating the *L* poles in the directional spectrum.

$$P_{ULA} = \frac{1}{\mathbf{B}^{H}(\phi_{t}, \phi_{r})\mathbf{U}_{n}\mathbf{U}_{n}^{H}\mathbf{B}(\phi_{t}, \phi_{r})}.$$
 (11)

## **3.2 2-D Estimation of Path Direction**

Many 2-d DOA estimation methods for the L-shape arrays have been developed which inspires us to obtain the 2-d channel estimation method. Firstly, substituting (4) into (10), we get

$$\mathbf{y}_{L}(q) = P \mathbf{W}^{H} \mathbf{A}_{R}(\phi_{r}, \theta_{r}) \mathbf{\Lambda}_{LG}(q) \mathbf{A}_{T}^{T}(\phi_{t}, \theta_{t}) \mathbf{F} + \mathbf{n}(q), (12)$$

The Eq. (12) can be further formulated as (13), where  $\mathbf{Z}_{TG}(q) = \mathbf{\Lambda}_{LG}(q) \mathbf{A}_{T}^{T}(\phi_{r},\theta_{r}) \mathbf{F} \cdot \mathbf{B}_{LR}(\phi_{r},\theta_{r}) = P \mathbf{W}^{H} \mathbf{A}_{R}(\phi_{r},\theta_{r})$ . In other word, taking  $P \mathbf{W}^{H} \mathbf{A}_{R}(\phi_{r},\theta_{r})$  as a whole with the information of AoAs and taking  $\mathbf{\Lambda}_{G}(q) \mathbf{A}_{T}^{T}(\phi_{r},\theta_{r}) \mathbf{F}$  as whole with the information of path gain.

$$\mathbf{y}_{L}(q) = \mathbf{B}_{LR}(\phi_{r}, \theta_{r}) \mathbf{Z}_{TG}(q) + \mathbf{n}(q), \qquad (13)$$

In order to estimate the paths direction, we collect the measurement vectors from the N blocks to compose the covariance matrix  $R_L$  as follow.

$$\mathbf{R}_{L} = \mathbb{E}\left\{\mathbf{y}_{L}(q)\mathbf{y}_{L}(q)^{H}\right\}$$
$$= \frac{1}{N}\sum_{q=1}^{N}\mathbf{y}_{L}(q)\mathbf{y}_{L}(q)^{H}, \qquad (14)$$

Substituting (13) into (14)

$$\mathbf{R}_{L} = \mathbf{B}_{LR} \mathbf{E} \left\{ \mathbf{Z}_{TG} \mathbf{Z}_{TG}^{T} \right\} \mathbf{B}_{LR}^{H} + \sigma^{2} \mathbf{I}$$
  
=  $\mathbf{B}_{LR} \mathbf{\Lambda}_{TZ} \mathbf{B}_{LR}^{H} + \sigma^{2} \mathbf{I}$ , (15)

Taking the eigenvalue decomposition of the covariance matrix, it can be decomposed into two parts related to signal and noise under the basic assumption that the incident signals and the noise are uncorrelated. The path number is assumed to be known and we get

$$\mathbf{R}_{L} = \sum_{i=1}^{L} \lambda_{i} \mathbf{e}_{i} \mathbf{e}_{i}^{H} + \sum_{j=L+1}^{M_{r}} \lambda_{j} \mathbf{e}_{j} \mathbf{e}_{j}^{H} , \qquad (16)$$
$$= \mathbf{U}_{Ls} \Sigma_{s} \mathbf{U}_{Ls}^{H} + \mathbf{U}_{Ln} \Sigma_{n} \mathbf{U}_{Ln}^{H}$$

Where  $\{\lambda_i\}$  and  $\{\lambda_j\}$  are the eigenvalues sorted in the descending order,  $\{e_i\}$  and  $\{e_j\}$  are the corresponding eigenvectors.  $\mathbf{U}_{Ls}$  and  $\mathbf{U}_{Ln}$  are signal sbspace and noise subspace respectively and  $\Sigma_s = diag\{e_i\}, \Sigma_n = diag\{e_j\}$ 

Since the direction matrix has full-column rank, we have

$$\lambda_i > \sigma^2 \quad for \quad i = 0, 1, \cdots, L$$
  

$$\lambda_i = \sigma^2 \quad for \quad i = L + 1, \cdots, M_r,$$
(17)

Under ideal conditions, the signal subspace and the noise subspace are orthogonal to each other, that is, the steering vectors in the signal subspace are also orthogonal to the noise subspace and we have  $B_{LR}^H U_n = 0$ . Thus the 2-d MUSIC can estimate AoAs by

$$P_{LR} = \frac{1}{\mathbf{B}_{LR}^{H}(\phi_r, \theta_r) \mathbf{U}_{Ln} \mathbf{U}_{Ln}^{H} \mathbf{B}_{LR}(\phi_r, \theta_r)}, \qquad (18)$$

Then we use the same method to estimate the AoDs.

Firstly getting the conjugate transpose  $y_L^H(q)$ .

$$\mathbf{y}_{L}^{H}(q) = \mathbf{B}_{LT}(\boldsymbol{\phi}_{t}, \boldsymbol{\theta}_{t}) \mathbf{Z}_{RG}(q) + \mathbf{n}(q), \qquad (19)$$

Where  $\mathbf{B}_{LT} = P\mathbf{F}^H \mathbf{A}_T$  and  $\mathbf{Z}_{RG}(q) = \mathbf{\Lambda}^H_{LG}(q) \mathbf{A}_R^T \mathbf{W}$ , and the directional spectrum of AoDs is shown as

$$P_{LT} = \frac{1}{\mathbf{B}_{LT}^{H}(\phi_{t},\theta_{t})\mathbf{U}_{Ln}\mathbf{U}_{Ln}^{H}\mathbf{B}_{LT}(\phi_{t},\theta_{t})}.$$
 (20)

#### **3.3 Estimation of Path Gain**

The MUSIC method has been applied to obtain the path directions of the transmitter. In this section, the estimated angles can be used to obtain the path gain. We firstly calculate  $\tilde{\mathbf{B}}_{LR}$  in Eq. (13) and then use LS method to estimate  $\tilde{\mathbf{Z}}_{TG}$  according to Eq. (21). Then the gain  $\tilde{\Lambda}_{LG}(q)$  can be computed as Eq. (22). In particular, the gain here depends on  $\tilde{B}_{LR}$ , because the angle change belongs to large-scale fading and the gain change belongs to small-scale fading. Therefore, the gain is estimated according to the obtained path direction during each time block. In the end,  $\mathbf{H}_L$  can be estimated as shown in Eq. (23) and  $\mathbf{H}_P$  also can be obtained in the same way.

$$\tilde{\mathbf{Z}}_{T}(q) = \left(\tilde{\mathbf{B}}_{LR}^{H}\tilde{\mathbf{B}}_{LR}\right)^{-1}\tilde{\mathbf{B}}_{LR}^{H}\mathbf{y}_{L}(q), q = 1, 2, \cdots, N, \quad (21)$$

$$\tilde{\boldsymbol{\Lambda}}_{LG}(q) = \tilde{\mathbf{Z}}_{T}(q) \left( \tilde{\mathbf{A}}_{T}^{T} \mathbf{F} \right)^{\dagger}, \qquad (22)$$

$$\tilde{\mathbf{H}}_{L} = \tilde{\mathbf{A}}_{R}(\phi_{r}, \theta_{r}) \tilde{\boldsymbol{\Lambda}}_{LG} \tilde{\mathbf{A}}_{T}(\phi_{t}, \theta_{t}).$$
(23)

## **4** Simulation Results

In this section, numerical simulation results are provided to assess the performance of the proposed 2-d channel estimation scheme. Considering the mm-wave MIMO systems with hybrid precoding where both the L-shape arrays and the UPAs are equipped at the transmitter and receiver.

Using the L-shape arrays with  $N_{Lx} = N_{Ly} = 32$  and  $N_{LT} = N_{LR} = 32 + 32 - 1=63$  and the UPAs with  $N_{Px} = N_{Py} = 8$  and  $N_{PT} = N_{PR} = 64$ . The path gain are assumed Gaussian, i.e.  $g_l \sim C\mathcal{N}(0, \sigma_g^2)$  and the signal-to-noise (SNR) is defined as by  $SNR = \frac{E\sigma_g^2}{N_l \sigma_n^2}$ . In order to compare the estimation performance of the proposed algorithm under two different array conditions, the remaining conditions are set to be the same. There are  $M_l = 4$  RF chains at the transmitter,  $M_r = 4$  RF chains receiver and  $d = \lambda/2$  for the two kinds of array

models. All results are averaged over 200 frames with N = 60. A uniform sampling grid is applied over angular space  $[0^{\circ}, 60^{\circ}]$  in both the azimuth and elevation directions and the grid resolutions are 0.05°. Assuming the power is uniformly distributed across all path and setting the number of paths L=1 and 3. For the following simulation results, we first show the channel estimation performance of signal path where the direction is  $(10.05^{\circ}, 15.20^{\circ})$  and  $(12.15^{\circ}, 17.10^{\circ})$ and the result prove the feasibility of the scheme. Then we present the performance of multiple paths L=3and the directional parameters  $(\phi, \theta)$  are given as Table 1.

Table 1. Simulation parameters

L-Shape	Path 1	Path 2	Path 3
AoA	(52.15°,12.25°)	(27.55°,32.85°)	(12.95°,47.75°)
AoD	(11.05°,10.20°)	(29.45°, 26.10°)	(45.80°, 51.65°)

The directional spectrums of AoAs and AoDs of the system equipped with the L-shape arrays are shown in Figure 4 and Figure 5. The x-axis and y-axis represent the azimuth angles and the elevation angles. As we can see, although SNR is very low, the proposed algorithm can show obvious pole in the directional spectrum. That means, all the three path direction can be estimated accurately and as the SNR increases, the precision of the estimation can be greater.



**Figure 4.** The directional spectrum of AoAs when SNR=5dB

Figure 6 shows the mean-square error (MSE) of the estimated AoAs and AoDs under different number of paths. The MSE of the estimated path angles are defined as Eq. (24). The simulation results also show that the proposed channel estimation scheme can find the path direction with high accuracy and as the number of channel paths decreases, the precise increases. And the influence of the number of paths on the estimation accuracy is greater than the influence of the array shape.

$$MSE = E\left\{ (\phi_{t} - \hat{\phi}_{t})^{2} + (\theta_{t} - \hat{\theta}_{t})^{2} + (\phi_{r} - \hat{\phi}_{r})^{2} + (\theta_{r} - \hat{\theta}_{r})^{2} \right\}, (24)$$



**Figure 5.** The directional spectrum of AoDs when SNR=5dB



**Figure 6.** The angle estimation performance under different number of paths as a function of SNR

The Figure 7 demonstrates the proposed method can accurately implement the 2-d channel estimation in the systems equipped with L-shape arrays and UPAs. In addition, we can also see that the channel estimation performance based on L shape arrays is superior than UPAs under the same conditions. Due to there are few method aimed at 2-d channel estimation, the beam training-based channel estimation scheme [6] and the OMP-based channel estimation scheme [8] are adopted for performance comparison when ULAs is used. Since the estimation errors of both azimuth and elevation angles contribute to the normalized mean squared error (NMSE), under the same conditions, the NMSE performance of UPAs should be higher than that of ULAs. But as we can see, the performance of the proposed estimation scheme under the UPAs has slightly better performance than prior ones under ULAs, which proves the value of our work more powerfully. The NMSE is defined as follow.

$$NMSE = E\left\{\left\|\hat{\mathbf{H}} - \mathbf{H}\right\|_{F}^{2} / \left\|\mathbf{H}\right\|_{F}^{2}\right\}.$$
 (25)



**Figure 7.** NMSE performance of different channel estimation schemes when L=3

## 5 Conclusion

We proposed a 2-d channel estimation scheme for mm-wave MIMO with hybrid precoding. The influence of the elevation angle and horizontal angle parameter both are considered comprehensively, and the extended classical MUSIC method is also used to estimate the CSI in the L-shape arrays and UPAs. Simulation results show that the proposed method can effectively estimate the channel state information in both arrays and even better than other traditional techniques. However the scheme needs high scanning resolution which means increasing the computational overhead. In the angular domain of channel estimation, there are still many topics to be researched, such as high resolution of angle estimation algorithm and low complexity algorithm in the receiving end.

## Acknowledges

This work was supported by NSFC: 61401407 and Engineering Project of Communication University of China: 3132018XNG1851

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# **Biographies**



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