

# Aggregate Signature without Pairing from Certificateless Cryptography

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## Abstract

In some real-world applications, many messages must be processed at the same time with low computational costs. In an aggregate signature scheme, anyone can combine  $n$  signatures on  $n$  messages from  $n$  users into a single signature, the resulting signature can convince a verifier that the  $n$  users indeed signed the  $n$  corresponding messages. All of the aggregate signature schemes currently known used bilinear pairings, however, the computational cost of the pairing is much higher than that of the exponentiation in a RSA group and that of the scalar multiplication over the elliptic curve group. In this paper, we propose a certificateless aggregate signature based on RSA and discrete logarithm (DL) problem, and prove the security in the random oracle model. To the best of author's knowledge, the scheme is the first certificateless aggregate signature scheme without pairing.

**Keywords:** Certificateless cryptography, Aggregate signature, RSA, DL problem

## 1 Introduction

It is required that a large amount of data must be processed simultaneously in some real-world applications.

In a high-density traffic scenario, each roadside monitoring equipment needs to verify around 500-2000 messages. In a shopping spree day (For example, on November 11 in China), electronic payment platform needs to process about 200 transactions per minute. In some multicast, the root node needs to collect the data from leaf nodes, when lots of data be transmitted simultaneously, the root node will be swamped.

In traditional public key infrastructure (PKI), there is a trusted certification authority (CA) to issue digital certificate binding the user to his public key. So the certificate management problem arises. To solve the

problem, Shamir [12] introduced identity-based public key cryptography. In this setting, there is a trusted private key generator (PKG) to generate private key for the user through his identity. However, which brings the key escrow problem. To solve the two problems, Al-Riyami et al. [1] put forward the notion of certificateless public key cryptography. In this notion, there is a semi-trusted key generation center (KGC), which generates partial private key for the user with respect to his identity. A user's full private key includes two parts: partial private key issued by KGC and a secret value chosen by himself.

### 1.1 Related Work

Al-Riyami and Paterson [1] presented the first certificateless signature (CLS) scheme, however, they did not give the formal proof of security. Yum and Lee [20] proposed a generic construction of CLS scheme. Huang et al. [11] showed a security drawback of the original scheme and proposed a secure one. Hu et al. [10] pointed out that Yum and Lee's construction is insecure and proposed a new one in the standard model. Xiong et al. [14] presented a security model for certificateless authenticated key agreement protocols and proposed a construction from bilinear pairings. Xiong [17] put forward a scalable certificateless remote authentication protocol, which achieves forward security and anonymity for wireless body area networks. He et al. [9] constructed a certificateless public auditing scheme for cloud-assisted wireless body area networks, which yields better performance over a previously proposed scheme. Xiong and zhang [18] presented a remote authentication protocol, which achieves client anonymity, non-repudiation, key escrow resistance, and revocability in the wireless body area networks. Zhang and Mao [24] constructed a CLS scheme based on RSA without bilinear pairing. He et al. [7] proposed a CLS scheme on the elliptic curve group, which does not use the bilinear pairing. Xiong et al. [15] proposed a certificateless threshold

signature scheme, which is secure against the malicious-but-passive KGC attack in the standard model. Xiong et al. [19] put forward a pairing-free key insulated signature scheme based on certificate, which eliminates the costly pairing operations.

Boneh et al. [2] introduced the concept of aggregate signature. In this setting, given  $n$  signatures on  $n$  messages from  $n$  users, anyone can combine all of these signatures into a single signature. The resulting signature can convince a verifier that the  $n$  messages were signed by the  $n$  corresponding users.

Castro and Dahab [4] proposed the first certificateless aggregate signature (CLAS) scheme. Gong et al. [6] presented two CLAS schemes which are provably secure in a relatively weak model. Zhang and Zhang [23] constructed a CLAS scheme which is provably secure in a stronger model. Zhang et al. [21] proposed a CLAS scheme which requires a certain synchronization, i.e., all signers must share the same synchronized clocks to generate an aggregate signature. However, it is not easy to achieve synchronization in many mobile computing scenarios. Recently, Xiong et al. [16] presented a new CLAS scheme which requires constant pairing computations. Zhang et al. [22] gave the security analysis to Xiong et al.'s scheme [16] by showing four kinds of concrete attacks, and they put forward a secure CLAS scheme. Cheng et al. [5] pointed out that Xiong et al.'s scheme [16] is insecure even against "honest-but-curious" KGC attack, and they proposed an improved scheme.

## 1.2 Motivation and Our Contributions

The main goal of aggregate signature is to reduce computation burden and storage burden. In most CLAS schemes, the number of using pairings grows linearly with the number of signers. There are only two CLAS schemes [5, 21] which require constant pairing operations, independent of the number of signers. However, Zhang et al.'s scheme [21] requires all signers to share one-time-use state information to generate an aggregate signature. In fact, it is not applicable in much real life.

In this paper, we constructed a new CLAS scheme and proved the security in the random oracle model, which has the following features:

- The scheme is secure in a strong security model. Namely, the super Type I/II adversaries can obtain the valid signatures for the replaced public key, without additional submission.
- The scheme does not need pairing operation.
- The scheme does not require synchronization for aggregating randomness, which makes it more suitable for practical applications.

## 2 Preliminaries

### 2.1 Elliptic Curve Group

Let  $E/F_p$  denote an elliptic curve  $E$  over a prime finite field  $F_p$ , defined by an equation:

$$y^2 = x^3 + ax + d \pmod{p}, a, d \in F_p$$

And  $4a^3 + 27d^2 \neq 0 \pmod{p}$ .

The points on  $E/F_p$  together with an extra point  $O$  called the point at infinity form a group:

$$\mathfrak{R} = \{(x, y) : x, y \in F_p, E(x, y) = 0\} \cup \{O\}.$$

### 2.2 Complexity Assumption

**Definition 1.** Let  $N = pq$ , where  $p$  and  $q$  are two  $k$ -bit prime numbers. Let  $b$  be a random prime number, greater than  $2^l$  for some fixed parameter  $l$ , such that  $\gcd(b, \varphi(N)) = 1$ . Given  $Y \in Z_N^*$ , RSA problem is to find  $X \in Z_N^*$  such that  $X^b = Y \pmod{N}$ .

**Definition 2.** Let  $\tau = (E, +)$ , where  $E$  is an elliptic curve over a finite field  $F_p$ ,  $P \in E$  is a point having prime order  $b = |E|/2$ . Let  $G = \langle P \rangle \leq \tau$ , given  $xP \in G$ , the discrete logarithm (DL) problem is to compute  $x$ .

### 2.3 System Model

A certificateless aggregate signature scheme consists of the following seven algorithms:

- Setup: This algorithm takes as input a security parameter  $k$  and returns the *params* (system parameters) and *msk* (master secret key).
- Partial-Private-Key-Extract: This algorithm takes as input the *params*, *msk* and a user  $ID_i \in \{0, 1\}^*$ , KGC generates the partial private key  $D_i$  for the user  $ID_i$ .
- Secret-Value-Set: This algorithm takes as input the *params* and a user  $ID_i$ , the user selects a secret value  $t_i$ .
- User-Public-Key-Generate: This algorithm takes as input the *params* and a user  $ID_i$ , the user outputs his public key  $P_i$ .
- Sign: This algorithm takes as input the *params*, signer's full private key  $(t_i, D_i)$  and a message  $m_i$ , then outputs the signature  $\sigma_i$ .
- Aggregate: This algorithm takes as input the *params*, and the signature  $\sigma_i$  on message  $m_i$  under the identity/public key  $ID_i/P_i (i=1, 2, \dots, n)$ , then outputs an aggregate signature  $\sigma$  on a message set

$$M = \{m_1, \dots, m_n\}.$$

- Aggregation verify: This algorithm takes as input the  $params$ , an aggregate signature  $\sigma$  on a messages set  $M = \{m_1, \dots, m_n\}$  under an aggregating set  $A = W \cup \{P_i : ID_i \in W\}$ , where  $W = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities. It outputs 1 if the aggregate signature is valid or 0 otherwise.

**Definition 3.** A certificateless aggregate signature (CLAS) scheme is unforgeable (UNF-CLAS) if the advantage of any polynomially bounded adversary is negligible in the following two games against Type I/II adversaries.

**Game I.** Now we illustrate the first game performed between a challenger  $\ell$  and a Type I adversary  $A_1$  for a CLAS scheme.

**Initialization.**  $\ell$  runs the setup algorithm to generate the master secret key  $msk$  and public system parameters  $params$ .  $\ell$  keeps  $msk$  secret and gives  $params$  to  $A_1$ .

**Query.**  $A_1$  performs a polynomially bounded number of queries.

- Hash functions query:  $A_1$  can ask for the values of the hash functions for any input.
- User public key query:  $A_1$  requests the public key of a user  $ID_i$ ,  $\ell$  returns the corresponding public key  $P_i$ .
- Partial private key query:  $A_1$  requests the partial private key of a user  $ID_i$ ,  $\ell$  responds with the partial private key  $D_i$ .
- User public key replacement:  $A_1$  supplies a new public key value  $P'_i$  with respect to a user  $ID_i$ .  $\ell$  then replaces the current public key with the value  $P'_i$ .
- Secret value query:  $A_1$  requests the secret value of a user  $ID_i$ ,  $\ell$  returns the secret value  $t_i$ . If a user's public key has been replaced,  $A_1$  can not request the corresponding secret value.
- Signature query:  $A_1$  submits the signer's identity/public key  $ID_i/P_i$  and a message  $m_i$  to the challenger.  $\ell$  outputs a valid signature  $\sigma_i$  on the message  $m_i$  under the identity/public key  $ID_i/P_i$ .

**Forge.**  $A_1$  outputs an aggregate signature  $\sigma^*$  on a message set  $M^* = \{m_1, \dots, m_n\}$  under an aggregating set  $A^* = W^* \cup \{P_i : ID_i \in W^*\}$ , where  $W^* = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities. The adversary wins if the result of verify  $(\sigma^*, A^*, M^*)$  is the symbol 1 and the following conditions hold:

1. There exists at least a user  $ID_j \in W^*$  whose partial private key was not queried by  $A_1$ . And the

corresponding tuple  $(ID_j, P_j, m_j)$  has never been queried during the signature queries.

2.  $A_1$  cannot query the secret value for any user if the corresponding public key has already been replaced. The advantage of  $A_1$  is defined as:

$$Adv_{A_1}^{UNF-CLAS} = \Pr[A_1 \text{ wins}].$$

**Game II.** A Type II adversary  $A_2$  plays the second game with a challenger  $\ell$  as follows.

**Initialization.**  $A_2$  runs the setup algorithm to obtain the master secret key  $msk$  and public system parameters  $params$ .  $A_2$  then gives the  $params$  and  $msk$  to  $\ell$ .

**Query.**  $A_2$  adaptively makes a polynomially bounded number of queries as those in Game I. Obviously,  $A_2$  can compute the partial private key of any user by itself with the master secret key.

**Forge.**  $A_2$  outputs an aggregate signature  $\sigma^*$  on a message set  $M^* = \{m_1, \dots, m_n\}$  under an aggregating set  $A^* = W^* \cup \{P_i : ID_i \in W^*\}$ , where  $W^* = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities. The adversary wins if the result of verify  $(\sigma^*, A^*, M^*)$  is the symbol 1 and the following conditions hold:

1. There exists at least a user  $ID_j \in W^*$  whose secret value was not queried and whose user public key was not replaced by  $A_2$ . And the corresponding tuple  $(ID_j, P_j, m_j)$  has never been queried during the signature queries.
2.  $A_2$  cannot query the secret value for any user if the corresponding public key has already been replaced. The advantage of  $A_2$  is defined as:

$$Adv_{A_2}^{UNF-CLAS} = \Pr[A_2 \text{ wins}].$$

### 3 Our Scheme

- Setup: Given the security parameter  $k$ , KGC generates two random  $k$ -bit prime numbers  $p$  and  $q$ , computes  $N = pq$ . For some fixed parameter  $l$  (for example  $l = 200$ ), KGC chooses at random a prime number  $b$  satisfying  $2^l < b < 2^{l+1}$  and  $\gcd(b, \phi(N)) = 1$ . Then it chooses a group  $G$  of prime order  $b$  as defined in Definition 2, a generator  $P$  of  $G$  and computes  $a = b^{-1} \bmod \phi(N)$ . Furthermore, KGC chooses two cryptographic hash functions:

$$H_0 : \{0, 1\}^* \rightarrow Z_N^*, H_1 : \{0, 1\}^* \rightarrow Z_b^*.$$

Finally, KGC outputs the set of public parameters:

$$params = \{N, b, G, P, H_0, H_1\}.$$

The master secret key is  $msk = (p, q, a)$ .

- Partial private key extract: For a user  $ID_i \in \{0,1\}^*$ , KGC computes  $Q_i = H_0(ID_i)$  and sends  $ID_i = Q_i^a$  to the user  $ID_i$  via a secure channel.
- Secret value set: The user  $ID_i$  randomly chooses  $t_i \in Z_b^*$ .
- User public key generate: The user  $ID_i$  computes his public key  $P_i = t_i P$ .
- Sign: For a message  $m_i \in \{0,1\}^*$ , the signer  $ID_i$  performs the following steps:
  1. Randomly selects  $c_i \in Z_b^*$ ,  $A_i \in Z_N^*$ , computes  $T_i = c_i P, B_i = A_i^{b_i} \text{ mod } N, h_i = H_1(m_i, T_i, B_i, ID_i, P_i)$ .
  2. Computes  $r_i = c_i + t_i h_i \text{ mod } b, R_i = A_i D_i^{h_i} \text{ mod } N$ .
  3. Outputs  $\sigma_i = (T_i, B_i, r_i, R_i)$  as the signature.
- **Aggregate:** On receiving message-signature pairs  $(m_i, \sigma_i = (T_i, B_i, r_i, R_i))$  under the identity/public  $ID_i/P_i$  for  $i=1,2,\dots,n$ . Anyone can compute  $r = \sum_{i=1}^n r_i, R = \prod_{i=1}^n R_i$  and outputs an aggregate signature  $\sigma = ((T_1, B_1), \dots, (T_n, B_n), r, R)$  on the message set  $M = \{m_1, \dots, m_n\}$ .
- **Aggregation verify:** To verify the signature  $\sigma = ((T_1, B_1), \dots, (T_n, B_n), r, R)$  on the message set  $M = \{m_1, \dots, m_n\}$  under the aggregating set  $A^* = W^* \cup \{P_i : ID_i \in W^*\}$ , where  $W^* = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities. The verifier performs the following steps:
  1. Computes  $h_i = H_1(m_i, T_i, B_i, ID_i, P_i)$  for  $i=1,2,\dots,n$ .
  2. Checks whether  $rP = \sum_{i=1}^n (T_i + h_i P_i), R^b = \prod_{i=1}^n (B_i Q_i^{h_i})$ . If both of equations hold, accepts the signature. Otherwise, rejects.

## 4 Security

**Theorem 1.** The scheme is unforgeable against the super Type I adversary if the RSA problem is hard in randomly oracle model.

**Proof.** Suppose the challenger  $\ell$  receives a random instance  $(Y, N, b)$  of the RSA problem and has to find an element  $X \in Z_N^*$  such that  $X^b = Y \text{ mod } N$ .  $\ell$  runs  $A_1$  as a subroutine and acts as  $A_1$ 's challenger in the Game I.

**Initialization.**  $\ell$  runs the setup program with the parameter  $k$ , then gives  $A_1$  the system parameters  $params = \{N, b, G, P, H_0, H_1\}$ .

**Queries.** Without loss of generality, we assume that all the queries are distinct and  $A_1$  will make  $H_0(ID)$  query before a user  $ID_i$  is used in any other queries.  $A_1$

sets several lists to store the queries and answers. All the lists are initially empty.

- $H_0$  queries:  $\ell$  maintains the list  $L_0$  of tuple  $(ID_i, V_i)$ . When  $A_1$  issues a query  $H_0(ID_i)$ ,  $\ell$  responds as follows:  
At the  $s^{th}$   $H_0$  query,  $\ell$  sets  $ID_s = ID^*$  and  $H_0(ID^*) = Y$ . For  $i \neq s$ ,  $\ell$  randomly picks a value  $V_i \in Z_N^*$  and sets  $H_0(ID_i) = V_i^b$ , the query and the answer then are stored in the list  $L_0$ .
  - $H_1$  queries:  $\ell$  maintains the list  $L_1$  of tuple  $(\alpha_i, h_i)$ . When  $A_1$  issues a query  $H_1(\alpha_i)$ ,  $\ell$  randomly picks a value  $h_i \in Z_b^*$ , sets  $H_1(\alpha_i) = h_i$  and adds  $(\alpha_i, h_i)$  to the list  $L_1$ .
  - User public key queries:  $\ell$  maintains the list  $L_U$  of tuple  $(ID_i, t_i)$ . When  $A_1$  issues a user public key query for user  $ID_i$ ,  $\ell$  randomly picks a value  $t_i \in Z_b^*$ , returns  $P_i = t_i P$  and adds  $(ID_i, t_i)$  to the list  $L_U$ .
  - Partial private key queries:  $\ell$  maintains the list  $L_D$  of tuple  $(ID_i, D_i)$ . When  $A_1$  issues a partial private key query for user  $ID_i$ . If  $ID_i = ID^*$ ,  $\ell$  fails and stops. Otherwise,  $\ell$  finds  $(ID_i, V_i)$  in the list  $L_0$ , responds with  $ID_i = V_i$  and adds  $(ID_i, V_i)$  to the list  $L_D$ .
  - User public key replacement requests:  $\ell$  maintains the list  $L_R$  of tuple  $(ID_i, P_i, P'_i)$ . When  $A_1$  issues a user public key replacement request for user  $ID_i$  with a new value  $P'_i$ .  $\ell$  replaces the current public key  $P_i$  with  $P'_i$  and adds  $(ID_i, P_i, P'_i)$  to the list  $L_R$ .
  - Secret value queries:  $\ell$  maintains the list  $L_E$  of tuple  $(ID_i, t_i)$ . When  $A_1$  issues a secret value query for the user  $ID_i$ .  $\ell$  checks the list  $L_U$ , if  $(ID_i, t_i)$  is found in the list  $L_U$ ,  $\ell$  responds with  $t_i$ . Otherwise,  $\ell$  randomly picks a new value  $t_i \in Z_b^*$ , responds with  $t_i$  and adds  $(ID_i, t_i)$  to the list  $L_E$  and  $L_U$ .
  - Signature queries: When  $A_1$  submits a signer's identity/public key  $ID_i/P_i$  and a message  $m_i$  to challenger.  $\ell$  outputs a signature as follow:  
If  $ID_i \neq ID^*$  and  $ID_i \notin L_R$ ,  $\ell$  gives a signature by calling the signing algorithm. Otherwise,  $\ell$  does as follow:
    1. Randomly selects  $R_i \in Z_N^*$  and  $r_i, h_i \in Z_b^*$ .
    2. Computes  $T_i = r_i P - h_i P_i, B_i = R_i^b Q_i^{-h_i}$ .
    3. Adds  $h_i = H_1(m_i, T_i, B_i, ID_i, P_i)$  to the list  $L_1$ . If collision occurs, repeats the steps 1-3.
    4. Outputs  $\sigma_i = (T_i, B_i, r_i, R_i)$  as the signature.
- Forge.**  $A_1$  outputs a forged signature  $\sigma^* = ((T_1, B_1), \dots,$

$(T_n, B_n), r, R)$  on the message set  $M^* = \{m_1, \dots, m_n\}$  under the aggregating set  $A^* = W^* \cup \{P_i : ID_i \in W^*\}$ , where  $W^* = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities, and fulfills the following conditions:

1. There exists at least a user  $ID_j \in W^*$  whose partial private key was not queried by  $A_1$ . And the corresponding tuple  $(ID_j, P_j, m_j)$  has never been queried during the signature queries.

2.  $A_1$  cannot query the secret value for any user if the corresponding public key has already been replaced. **Solve RSA problem.** Note that  $r = \sum_{i=1}^n r_i$ ,  $R = \prod_{i=1}^n R_i$ , the tuple  $(T_i, B_i, r_i, R_i)$  is the signature on the message  $m_i$  under the identity/public key  $ID_i/P_i$  for  $i=1, 2, \dots, n$ . And there exists at least a user  $ID_j \in W^*$  whose partial private key was not queried by  $A_1$ . Which implies that  $(T_j, B_j, r_j, R_j)$  is a forge signature on the message  $m_j$ . Using general forking lemma [3], after replaying  $A_1$  with the same random tape but different  $h_j$  returned by  $H_1$  query of the forged message  $m_j$ ,  $\ell$  gets two aggregate signatures with at least probability  $\varepsilon \cdot (\frac{\varepsilon}{q_{H_1}} - \frac{1}{b})$ :

$$((T_1, B_1), \dots, (T_n, B_n), r, R), ((T_1, B_1), \dots, (T_n, B_n), r', R'),$$

where  $R = \prod_{i=1}^n R_i$ ,  $R' = \prod_{i=1}^n R'_i$ ,  $R_j \neq R'_j$ ,  $R_i = R'_i$  for  $i \neq j$ . If  $ID_j = ID^*$ , then  $R_j = A_j Y^{ah_j}$  and  $R'_j = A_j Y^{ah'_j}$ . It follows that  $(R'R^{-1})^b = Y^{h'_j - h_j} \pmod N$ . Since  $h_j, h'_j \in Z_b^*$ , then  $|h'_j - h_j| < b$ . By the element  $b$  is a prime number, then  $\gcd(b, h'_j - h_j) = 1$ . This means that there exist two integers  $c$  and  $d$  such that  $cb + d(h'_j - h_j) = 1$ . Finally,  $\ell$  solves the RSA problem by computing:

$$X = (R'R^{-1})^d Y^c \pmod N. \text{ In effect, } X^b = (R'R^{-1})^{bd} Y^{bc} = Y^{d(h'_j - h_j)} Y^{bc} = Y^{cb + d(h'_j - h_j)} = Y.$$

**Probability.** Let  $q_{H_i}$  ( $i=0,1$ ) and  $q_D$  be the numbers of  $H_i$  ( $i=0,1$ ) queries and partial private key queries.

The probability that  $\ell$  does not fail during the queries is  $\frac{q_{H_0} - q_D}{q_{H_0}}$ . The probability that  $ID_j = ID^*$  is  $\frac{1}{q_{H_0}}$ . So the combined probability is  $\frac{q_{H_0} - q_D}{q_{H_0}} \cdot \frac{1}{q_{H_0} - q_D} = \frac{1}{q_{H_0}}$ .

Therefore, if the adversary  $A_1$  can win the EUF-CLAS Game I with advantage  $\varepsilon$ , then  $\ell$  can solve the RSA problem with the probability  $\frac{\varepsilon}{q_{H_0}} (\frac{\varepsilon}{q_{H_1}} - \frac{1}{b})$ .

**Theorem 2.** The scheme is unforgeable against the

super Type II adversary if the DL problem is hard in randomly oracle model.

**Proof.** Suppose the challenger  $\ell$  receives a random instance  $(xP, P)$  of the DL problem and has to compute the value of  $x$ .  $\ell$  runs  $A_2$  as a subroutine and acts as  $A_2$ 's challenger in the game II.

**Initialization.**  $A_2$  runs the setup program with the parameter  $k$  to obtain the system parameters  $params = \{N, b, G, P, H_0, H_1\}$  and master secret key  $msk = (p, q, a)$ .  $A_2$  then gives  $\ell$  the  $params$  and  $msk$ .

**Queries.** Without loss of generality, we assume that all the queries are distinct and  $A_2$  will ask for the user public key before a user  $ID_i$  is used in any other queries.  $A_2$  sets several lists to store the queries and answers. All the lists are initially empty.

- User public key queries:  $\ell$  maintains the list  $L_U$  of tuple  $(ID_i, t_i)$ . When  $A_2$  issues a user public key query for the user  $ID_i$ ,  $\ell$  responds as follows: At the  $s^{th}$  query,  $\ell$  sets  $ID_s = ID^*$ ,  $P_s = P^* = xP$ . For  $i \neq s$ ,  $\ell$  randomly picks a value  $t_i \in Z_b^*$ , returns  $P_i = t_i P$  and adds  $(ID_i, t_i)$  to the list  $L_U$ .
- $H_0$  queries:  $\ell$  maintains the list  $L_0$  of tuple  $(ID_i, Q_i)$ . When  $A_2$  issues a query  $H_0(ID_i)$ ,  $\ell$  randomly picks a value  $Q_i \in Z_N^*$ , sets  $H_0(ID_i) = Q_i$  and adds  $(ID_i, Q_i)$  to the list  $L_0$ .
- $H_1$  queries: Same as that in the proof of Theorem 1.
- Partial private key queries: Since  $A_2$  knows master secret key  $msk = (p, q, a)$ , he can compute the partial private key for any user by himself. Hence  $A_2$  does not need issue partial private key query.
- User public key replacement requests: Same as that in the proof of Theorem 1.
- Secret value queries:  $\ell$  maintains the list  $L_E$  of tuple  $(ID_i, t_i)$ . When  $A_2$  issues a secret value query for the user  $ID_i$ . If  $ID_i = ID^*$ ,  $\ell$  fails and stops. Otherwise,  $\ell$  finds  $(ID_i, t_i)$  in the list  $L_U$ , responds with  $t_i$  and adds  $(ID_i, t_i)$  to the list  $L_E$ .
- Signature queries: Same as that in the proof of Theorem 1.

**Forge.**  $A_2$  outputs a forged signature  $\sigma^* = ((T_1, B_1), \dots, (T_n, B_n), r, R)$  on the message set  $M^* = \{m_1, \dots, m_n\}$  under the aggregating set  $A^* = W^* \cup \{P_i : ID_i \in W^*\}$ , where  $W^* = \{ID_1, ID_2, \dots, ID_n\}$  is a set of  $n$  identities, and fulfills the following conditions:

1. There exists at least a user  $ID_j \in W^*$  such that his secret value was not queried and his user public key was not replaced by  $A_2$ . And the corresponding tuple

$(ID_j, P_j, m_j)$  has never been queried during the signature queries.

2.  $A_2$  cannot query the secret value for any user if the corresponding public key has already been replaced.

**Solve DL problem.** Note that  $r = \sum_{i=1}^n r_i$ ,  $R = \prod_{i=1}^n R_i$ , the tuple  $(T_i, B_i, r_i, R_i)$  is the signature on the message  $m_i$  under the identity/public key  $ID_i / P_i$  for  $i = 1, 2, \dots, n$ . And there exists at least a user  $ID_j \in W^*$  such that his secret value was not queried and his user public key was not replaced by  $A_2$ . Which implies that  $(T_j, B_j, r_j, R_j)$  is a forge signature on the message  $m_j$ . Using general forking lemma [3], after replaying  $A_2$  with the same random tape but different  $h_j$  returned by  $H_1$  query of the forged message  $m_j$ ,  $\ell$  gets two aggregate signatures with at least probability  $\varepsilon \cdot (\frac{\varepsilon}{q_{H_1}} - \frac{1}{b})$ :

$$((T_1, B_1), \dots, (T_n, B_n), r, R), ((T_1, B_1), \dots, (T_n, B_n), r', R'),$$

where  $r = \sum_{i=1}^n r_i$ ,  $r' = \sum_{i=1}^n r'_i$ ,  $r_j \neq r'_j$  and  $r_i = r'_i$  for  $i \neq j$ . If  $ID_j = ID^*$ , then  $r_j = c_j + xh_j$  and  $r'_j = c_j + xh'_j$ ,  $\ell$  can solve DL problem by computing  $x = (h_j - h'_j)^{-1} \cdot (r - r') \bmod b$ .

**Probability.** Let  $q_{H_i} (i = 0, 1)$ ,  $q_U$ ,  $q_R$  and  $q_E$  be the numbers of  $H_i (i = 0, 1)$  queries, user public key replacement requests, user public key queries and secret value queries.

Without loss of generality, we may assume that  $L_E \cap L_R = \Phi$ .

The probability that  $\ell$  does not fail during the queries is  $\frac{q_U - q_E}{q_U}$ . The probability that  $ID_j = ID^*$  is  $\frac{1}{q_U - q_E - q_R}$ . So the combined probability is  $\frac{q_U - q_E}{q_U} \cdot \frac{1}{q_U - q_E - q_R} \geq \frac{1}{q_U}$ .

Therefore, if the adversary  $A_2$  can win the EUF-CLAS Game II with advantage  $\varepsilon$ , then  $\ell$  can solve the DL problem with the probability  $\frac{\varepsilon}{q_U} (\frac{\varepsilon}{q_{H_1}} - \frac{1}{b})$ .

**Table 2.** Comparison of several CLAS schemes

Scheme	Sign	Verify	Execution time/(n=100)
Castro and Dahab's scheme [4]	$2n E_G$	$(2n+1)P+n E_G$	$59.22n+20.04/5942.04$
Cheng et al.'s scheme [5]	$4n E_G$	$3P+2n E_G$	$38.28n+60.12/3888.12$
Gong et al.'s scheme 1 [6]	$2n E_G$	$(2n+1)P$	$52.84n+20.04/5304.04$
Gong et al.'s scheme 2 [6]	$3n E_G$	$(n+2)P+n E_G$	$45.56n+40.08/4596.08$
Zhang et al.'s scheme [22]	$nP+2n E_G$	$2nP+(3n+1) E_G$	$92.02n+6.38/9208.38$
Zhang and Zhang's scheme [23]	$3n E_G$	$(n+3)P$	$39.18n+60.12/3978.12$
Our scheme	$n E_S + 2n E_N$	$(n+1)(E_S + E_N)$	$20.35n+7.52/2042.52$

## 5 Efficiency

In this section, we compare the performance of our scheme with several CLAS schemes in Table 2, we define some notations as follows.

$P$ : a pairing operation.

$E_G$ : a pairing-based scalar multiplication operation.

$E_S$ : a scalar multiplication operation.

$E_N$ : a modular exponent operation in  $Z_N$ .

By using Windows XP operation system and PIV 3-GHZ processor with 512-MB memory. He et al. [8] obtained the running time for cryptographic operations. To achieve 1024-bit RSA level security, Tate pairing was used, which is defined on a supersingular curve  $E/F_p: y^2 = x^3 + x$  with embedding degree 2, where  $q$  is a 160-bit Solinas prime  $q = 2^{159} + 2^{17} + 1$  and  $p$  is a 512-bit prime satisfying  $p + 1 = 12qr$ . To achieve the same security level, the parameter secp160r1 [13] was used too, where  $p = 2^{160} - 2^{31} - 1$ . The running times are listed in Table 1.

**Table 1.** Cryptographic operation time (in milliseconds)

$P$	$E_G$	$E_S$	$E_N$
20.04	6.38	2.21	5.31

We use a simple method to evaluate the computational cost. For example, Zhang and Zhang's scheme [23] requires  $3n$  pairing-based scalar multiplication operations and  $n + 3$  pairing operations. So the resulting computation time is  $6.38 \times 3n + 20.04 \times (n + 3) = 39.18n + 60.12$ . In order to facilitate the comparison, we let  $n = 100$ , then the computation time is  $39.18 \times 100 + 60.12 = 3978.12$ . Based on the above parameter and ways, the detailed comparison results of several different CLAS schemes are illustrated in Table 2.

## 6 Conclusion

All of the known aggregate signature schemes used bilinear pairings. Some good results have been achieved in speeding up the computation of pairing in recent years, however, the computational cost of the pairing is much higher than of the exponentiation in a RSA group and that of the scalar multiplication over the elliptic curve group.

So it is still interesting to design aggregate signature scheme without pairing. In this paper, a new certificateless aggregate signature scheme based on RSA and discrete logarithm problem was proposed, which is unforgeable against type I/II adversaries in the random oracle model. To the best of author's knowledge, the scheme is the first CLAS scheme without pairing and which is more efficient than previous ones in computation. Due to the good properties of the scheme, it should be useful for practical applications.

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