Wireless Network Localization Algorithm Based on Tikhonov Regularization for Anisotropic Networks

Jiandong Yao[1], Xiaoyong Yan[2], Chengshan Qian[3], Huijun Li[4]

[1] School of network and communication, Jinling institute of Technology, China
[2] School of Modern Posts & Institute of Modern Posts, Nanjing University of Posts and Telecommunications, China
[3] School of Computer & Software, Nanjing University of Information Science, China
[4] School of Instrument Science and Engineering, Southeast University, China
swireless@163.com; xiaoyong_yan@126.com; qianchengshan@nuist.edu.cn; lihuijun@seu.edu.cn

Abstract

In anisotropic network, hop-counts between nodes may not match physical distances well. Hence, it may introduce huge errors to employ multi-hop range-free localization algorithm to estimate nodes location. In this paper, we present a novel multi-hop range-free localization algorithm for anisotropy network. Firstly, we build the relationship between hop-counts and distances among nodes under Tikhonov regularization metric. Since the relationship retains all the hop-counts characteristics to all anchors in all directions, then we can precisely capture the anisotropic relationship between hop-counts and physical distances. Finally, we use the multilateration technique to estimate the locations of all nodes. We evaluate our method based on multiple anisotropy factors, and analyze its performance. We also compare our method with state-of-art methods, and demonstrate the high efficiency of our proposed method. Experiments results show that proposed algorithm improves localization accuracy by more than 90%.

Keywords: Wireless network, Tikhonov regularization, Anisotropic network, Multi-hop localization, Range-free localization

1 Introduction

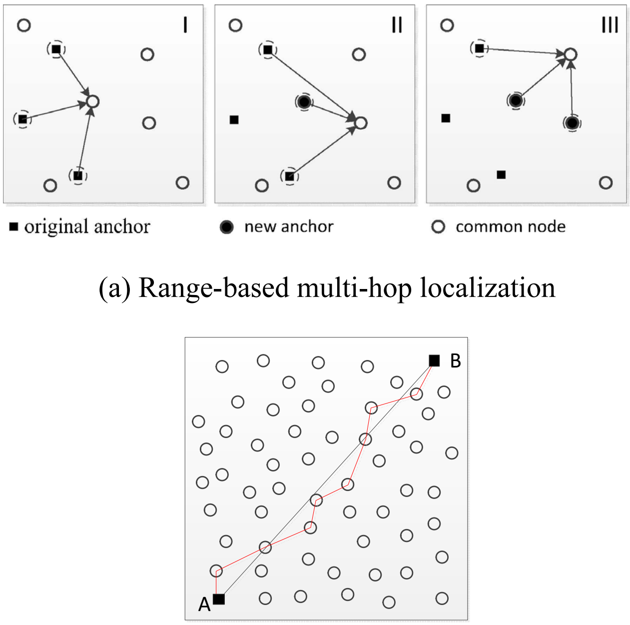
With the miniaturization of micro-electromechanical systems as well as the popularity of wireless communication, more and more users can easily carry portable mobile intelligent terminal equipment. At the same time, people's lives are increasingly dependent on mobile devices. Among the services provided by many mobile devices, location information is generally regarded as the premise of other information services [1-9]. The most convenient way to get the location information is to equip with a satellite positioning system on the mobile terminal, such as global positioning system (GPS) or BeiDou Navigation Satellite System (BDS).

The satellite localization system directly communicates with the receiving terminal (such as GPS or BDS receiver) through the satellite, which forms one-hop localization system. However, the satellite localization system can only be used in outdoor environment without occlusion due to its high cost and high power consumption [10]. In addition, a research about human activity habits found that we human beings spend 80% to 90% of our time in indoors [10-11]. Despite worldwide availability, GPS/BDS signals are largely unavailable indoors. With emerging technologies like ad-hoc, internet of things, etc., one hop localization mode has gradually evolved into the multi-hop localization mode [12-13].

The basic principle of multi-hop localization is that the position of non-anchors can be cooperatively determined by a few anchors equipped with GPS/BDS receiver. According to the method target nodes measured, the multi-hop localization algorithms can be divided into the range-based multi-hop localization method (as shown in Figure 1(a)) and the range-free multi-hop localization method (as shown in Figure 1(b)) [5].

The range-based multi-hop localization method is to iterative obtaining the location information between nodes through the signal measurement. Therefore, this method has relatively strict demands on the hardware, so that the equipment cost is high. In addition, in the iterative process of the algorithm, the last round of the estimation error will be accumulated to the next round, resulting in seriously location inaccuracy of the subsequent node [14-16]. In contrast, range-free algorithms locate non-anchors by exploiting the geometry property of the network, and thus achieve lower localization accuracy than range-based ones. Thus, range-free algorithms have drawn much research attention due to their low cost and applicability to large-scale applications [17-20]. However, it is challenging to design an accurate range-free multi-hop

*Corresponding Author: Xiaoyong Yan; E-mail: xiaoyong_yan@126.com
DOI: 10.3966/160792642018051903028



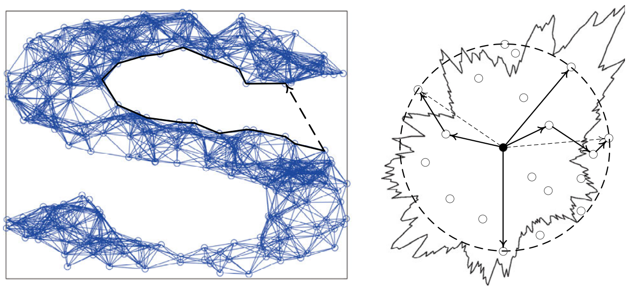
(a) Range-based multi-hop localization

(b) Range-free multi-hop localization

Figure 1. Multi-hop localizations

localization algorithm for anisotropic network, where the deployment and the radio propagation of nodes might be irregular.

Figure 2 shows two typical anisotropic networks [18, 20]. One is a S-shaped network in which nodes are irregular deployment (as shown in Figure 2(a)). The other is a network in which radio irregularity of nodes due to variance in RF sending power or different path losses depending on the propagation direction (as shown in Figure 2(b)).



(a) The irregular deployment of nodes (b) The radio irregular propagation of nodes

Figure 2. two anisotropic networks

In indoor spaces, the distribution area of the nodes is often affected by the problem of non-line of sight (NLOS) propagation. NLOS makes nodes distribution irregular, and makes them fail to obtain accurate distance estimates. As shown in Figure 2(a), the physical distance from A to B is indicated by the dotted line, while the shortest path between A to B is indicated by the solid line. We find that the shortest path is seriously detoured due to the problem of the irregular distribution of nodes or NLOS propagation. In the real world shown in Figure 2(b), radio irregularity is a common phenomenon when the same physical distance among nodes, not the same hop-counts. As shown in Figure 2(b), the distances between A and B, C, D are the same, but their hop-counts are different because of radio irregularity. Classical multi-hop algorithms assume that the network is isotropic and normal distributed. Unfortunately, in practice, network generally may be distribution irregular and radio irregular propagation, which makes the hop-counts deviating the physical distances.

In this paper, we propose a novel multi-hop range-free wireless localization algorithm that called Wireless Network Localization through Tikhonov (WNLT). The proposed algorithm consists of three steps: the measurement step, Tikhonov regularization step, and localization step. First, hop-counts information among nodes of the given network is measured. Second, mapping relationships between physical distances and hop-counts among anchors are modeled using Tikhonov regularization approach under the least-squares metric. Finally, each non-anchor finds its own location in a distributed manner under the help of mapping model.

The rest of this paper is organized as follows: In Section 2, related work about multi-hop range-free localization algorithms in wireless network is described. In Section 3, the motivation of this paper is explained and the new multi-hop range-free localization method is proposed. In Section 4, various simulations are conducted and the results of the proposed methods are compared with those of previous methods. Finally, conclusions are drawn in Section 5.

2 Related Work

Multi-hop range-free algorithms locate nodes without extra the knowledge of internode distance or angle measurements. Therefore, they save the cost of ranging hardware, and they are wider application than range-based ones. Among the much multi-hop range-free localization, the most famous are the Distance Vector-Hop (DV-hop) algorithm [17], the Amorphous algorithm and the Proximity-Distance Map (PDM) [17-19], and their algorithms and running details are as follows:

2.1 DV-hop

DV-hop linearly converts the hop-counts into the physical distances by computing average per-hop distance. The run process of DV-hop can be divided into three steps:

(1) Calculate the least hop-counts between non-anchors and anchors. Without loss of generality, firstly, anchor i floods a message $[x_i, y_i, h_i]$ to the rest of nodes in the network by distance vector exchange

protocol [21-22]. Thereinto, $[x_i, y_i]^{\mathrm{T}}$ represents the coordinate information of the anchor i; h_i denotes a counter to record the least hop-counts to anchor i. The value of counter h_i is initialized to 1 and increases by 1 after each forward. Work as above, every node can obtain its minimum hop-counts to all anchors.

(2) Estimate the average per-hop distance among anchors. Once anchor j obtain the minimum hop-counts from the other anchors, it can report the counter h_i to anchor i. After collecting these values of counter from the rest of anchors in the network, anchor i can calculate the average one-hop distance and broadcasts this to the whole network. For example, the anchor $i's$ average one-hop distance is calculated as:

$$c_i = \frac{\sum_{i \neq j} d_{ij}}{\sum_{i \neq j} h_{ij}} \tag{1}$$

where d_{ij} denotes the physical distance between anchor i and anchor j, and hij denotes the minimum hop-counts between them. The average one-hop distance of every anchor represents the expected distance each hop progresses and can be seen a mapping parameter between hop-counts and distance. In DV-hop, every anchor maintains one its own fixed mapping parameter. In practical application, in order to avoid excessive flooding communication, the parameter of Time To Live (TTL) can be set on the packet to reduce the flooding traffic.

(3) Location estimation. A non-anchor uses the mapping parameter received from its nearest anchor to estimate its physical distances to anchors. Let h_{ij} denote the hop-counts between node i and node j. Then the corresponding distance d_{ij} can be calculated as:

$$d_{ij} = c_i \times h_{ij} \tag{2}$$

After an unknown node receives the mapping coefficient from more than three adjacent anchors, it uses trilateration to estimate its own location.

2.2 Amorphous

Nagpal [19] proposed another multi-hop range-free localization algorithm similar to DV-hop which use one radio hop coverage method proposed by Kleinrock and Slivers [1] to the average one hop distance. Amorphous's average one hop distance is a fixed parameter of the whole network, and it can be described as

$$d_{hop} = R\left(1 + e^{-n_{local}} - \int_{-1}^{1} e^{-\left(n_{local}/\pi\right)} ar\cos t - t\sqrt{1-t^2}\,dt\right) \tag{3}$$

Thus, the distance d_{ij} between node i and node j can be obtained by $d_{ij} = d_{hop} \times h_{ij}$ .where h_{ij} is the hop-counts between node i and node j. Finally, after an unknown node gets more than three distances from anchors, it estimates its own coordinate.

DV-Hop and Amorphous perform well in isotropic network but encounter severe performance degradation in anisotropic network, because they all rely on a unified mapping parameter to estimate the distances between anchors and non-anchors. Especially the Amorphous's mapping parameter is unique parameters of the whole network, leading to worse performance than DV-hop in anisotropic networks.

2.3 PDM

In recent years, several multi-hop range-free algorithms making use of machine learning have been put forward for the localization problem in anisotropic network. They employ learning algorithm to construct the mapping model between anchors, and a trained mapping model based upon the hop-counts and distances relationship of anchors. The locations of non-anchors are obtained from the physical distances between the anchors and the non-anchors by employing the trained mapping model. Based on this idea, Lim [18] proposed Proximity Distance Map (PDM) localization algorithm that directly builds a mapping model of hop-counts and distances among anchors. In the PDM algorithm, firstly, anchors directly construct a mapping model using truncated singular value (TSVD) [24] under the help of least-squares, and broadcast this model to all common nodes. Secondly, each non-anchor obtains the hop-counts from its anchor list, severally estimates its distances to anchors by multiplying this model. Lastly, a common node estimates its location by multilateration.

Unlike the previous multi-hop range-free algorithm, PDM algorithm don not using unified mapping parameter, but use a trained mapping model. Due to this trained mapping model of PDM retains all the hop distance characteristics to all anchor nodes in all directions. PDM can tolerate multiple anisotropy factors of the network and achieves high localization accuracy.

Inspired by the PDM, Lee etc. [20, 25] have proposed two multi-hop range-free algorithms based on kernel regression which called Localization through Support Vector Regression (LSVR) and Localization through Multi-dimensional Support Vector Regression (LMSVR). LSVR and LMSVR can partly solve the non-line problem between hop-counts and distance, however, the core algorithm of them involves more parameters which usually need to be optimized by cross-validation or the grid search [26]. Therefore, they are not suitable for the applications of wireless networks.

Interestingly, PDM transfer hop-counts to physical distances by formulating the localization into a regression problem. However, PDM never discussed the problem of the different measurement unit between hop-counts and distances. Besides that, PDM have not described the selection of TSVD's truncated parameter. In this paper, by introducing another regularized technique different from TSVD and exploiting the more precise relationship between hop-counts and distances in the network, the proposed algorithm develops a mapping model under Tikhonov's supervision to represent the relationship between the hop-counts and distances of the network. Contrary to TSVD, Tikhonov regularization does not neglect any singular value component, but by adding a filter factor to damp or filter out these high frequency oscillations and thereby ensure more stability and accuracy of the solution. In addition, our proposed approach considers the selection of the filter factor and measurement unit transformation.

3 WNLT Localization Algorithm

3.1 Problem Statement

There are n nodes $\{S_i\}_{i=1}^{n}$ in a two-dimensional plane, where the first $m(m \ll n)$ nodes represent the anchors and the remaining $n-m$ represents the non-anchors. The anchors $\{S_i\}_{i=1}^{m} \in R$ are defined as a kind of node that are aware of their own positions, either through GPS/BDS or manual recording and entering positions during deployment. Non-anchors $\{S_i\}_{i=1}^{n-m} \in U$ are other kinds of nodes which estimate their positions through the help of anchors and localization algorithm. The coordinates of the nodes can be rewritten as follows

$$\mathbf{cor}(S_p) = (x_p, y_p)^T \quad for\ p = 1,\cdots m,\cdots,n \tag{4}$$

For every pair of nodes i and j, the physical distance is defined as

$$\begin{aligned} d(S_i,S_j) &= \|cor(S_i) - cor(S_j)\| \\ &= \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \in \mathbb{R} \end{aligned} \tag{5}$$

After a period of time, m training pairs data set are collected from the anchors, i.e., $\{\boldsymbol{H},\boldsymbol{D}\}$, where $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_m]$ and $\boldsymbol{h}_i = [h_{i,1}, \cdots, h_{i,m}]^T$, $\boldsymbol{h}_i$ be least hop-counts between the anchor S_i and the other anchors, $\boldsymbol{D} = [\boldsymbol{d}_1, \boldsymbol{d}_2, \cdots, \boldsymbol{d}_m]$ and $\boldsymbol{d}_i = [d_{i,1}, \cdots, d_{i,m}]^T$, $\boldsymbol{d}_i$ be physical distance between the anchor S_i and the other anchors. Accordingly, the multi-hop range-free wireless localization problem can be formulated as the formula (6):

$$\begin{aligned} &\text{Estimate } \mathbf{cor}(S_k) \\ &\text{Given } \mathbf{cor}(S_i), d(S_i,S_j), and\, h(S_i,S_k) \end{aligned} \tag{6}$$

where $S_i, S_j \in R$, $S_k \in U$, and $h(S_i,S_k)$ is the hop count between the reference node S_i and the unknown node S_k. As a result, the mapping relationship between the hop count and the physical distance can be trained, i.e.,

$$\boldsymbol{D} = \boldsymbol{H}\boldsymbol{T} + \boldsymbol{E} \tag{7}$$

Where $\boldsymbol{D}, \boldsymbol{H}$ are respectively the physical distance matrix and the hop count matrix between the related nodes; $\boldsymbol{T}$ is the mapping relationship between the hop count and the distance; $\boldsymbol{E}$ is the random error.

3.2 Localization Algorithm

Localization methods based machine learning usually consists of three basic steps [27]: (i) the data collection phase, which is also called measure phase, (ii) the model building phase, which is also called the training stage, and (iii) the location estimation phase, which is also called the test stage.

The data collection phase is mainly the collection of hop-counts and distance between anchors. Each anchor and non-anchor exchanges the hop-counts information with one another, and each anchor transmits its position information to the other anchor nodes. In the model building phase, machine learning methods can be used to train and construct mapping model hop-counts and physical distance between anchors. After the model building is completed, location-unknown node can determine their own position locally by the mapping model. The aforesaid three steps are discussed below in greater detail.

The data collection phase. At the beginning of the algorithm, the anchor transmits a broadcast information packet with its own location information to the rest nodes by distance-vector routing protocol within the communication radius. The packet at least contains identification field (ID), the location information and the hop count field (Hop counts, the initial value is 1). The packet format is as follows:

ID	X	Y	Hop_counts

After receiving the packet information, each node records the minimum hop-counts to the connected anchor. Meanwhile incriminating the field value of Hop_counts in the packet by 1. But when the field value of Hop_counts received from the same anchor is not the minimum, the procedure automatically ignores the packet. Though the above method, all the nodes in the network record the smallest hop-counts to their connected anchors. Corresponding physical distance between the anchors can be obtained by the physical distance formula (5) based on their own coordinates.

The model building phase. After obtaining the minimum hop count and the physical distance between the reference nodes, the mapping relationship between the minimum hop count and the actual distance is constructed by using the formula (7). This mapping model of WNLT is really an optimal linear transformation T that gives a mapping from the hop-count H to the physical distance matrix D. It is easy to know that T is a $m \times m$ square matrix. Each column vector t_i of T can be obtained by minimizing the mean square error of the errors:

$$\boldsymbol{e}_i = \sum_{k=1}^{m} \left(d_{ik} - \boldsymbol{h}_k \boldsymbol{t}_i \right)^2 = \left\| \boldsymbol{d}_i - \boldsymbol{H}\boldsymbol{t}_i \right\|^2 \tag{8}$$

Note that the least square solution of the column vector $\boldsymbol{t}_i$ is:

$$\boldsymbol{t}_i = \left(\boldsymbol{H}^{\mathrm{T}} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^{\mathrm{T}} \boldsymbol{d}_i \tag{9}$$

In order to avoid the problem of "the big number eat the little number" caused by the level of number difference in the transformation process of the hop-count and the distance, in the actual operation process the centralized processing will be carried out for the hop-count matrix and the distance matrix. So, the formula (7) is turned into $\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{H}}\boldsymbol{T} + \boldsymbol{E}$, where $\tilde{\boldsymbol{D}}, \tilde{\boldsymbol{H}}$ are the centralized distance matrix and the centralized hop count matrix, respectively. At this point, the matrix expression of the mapping model is:

$$\hat{\boldsymbol{T}} = \left(\tilde{\boldsymbol{H}}^{\mathrm{T}} \tilde{\boldsymbol{H}} \right)^{-1} \tilde{\boldsymbol{H}}^{\mathrm{T}} \tilde{\boldsymbol{D}} \tag{10}$$

Due to the lack of sufficient information or linear correlation in hop-counts matrix, however, the solution may not be unique or can cause an arbitrarily large perturbation by an arbitrary small perturbation during constructing the mapping model. Theoretically, regularization method is considered a good tool for solving ill-posed problems, and Tikhonov regularization is the most well-known and effectively form of regularization [28-30]. Here, according to the idea of Tikhonov regularization, the mapping model (formula 7) should be satisfies that:

$$\left\| \boldsymbol{HT} - \boldsymbol{D} \right\| \le \Delta \tag{11}$$

where $\Delta = \|v\|$. When the formula (11) takes the equal sign, the mapping relationship $\boldsymbol{T}$ of the formula (7) can be obtained. Thus, the solution of the mapping relationship $\boldsymbol{T}$ can be obtained by the minimization of the formula (12), and the minimization formula is:

$$\min \left\{ \left\| \boldsymbol{HT} - \boldsymbol{D} \right\|^2 + \gamma \left\| \boldsymbol{T} \right\|^2 \right\} \tag{12}$$

where $\gamma \ge 0$ is a regularization parameter that determines the amount of regularization. Note that this is a conditional extremum problem, which can be converted into an unconditional extremum problem to solve by the Lagrange equation. Therefore, we obtain the relationship model of anchors between the hop-counts and the distances.

$$\hat{\boldsymbol{T}} = \left(\boldsymbol{I}\gamma + \tilde{\boldsymbol{H}}^{\mathrm{T}} \tilde{\boldsymbol{H}} \right)^{-1} \tilde{\boldsymbol{H}}^{\mathrm{T}} \tilde{\boldsymbol{D}} \tag{13}$$

The formula (13) is also called the Tikhonov regularization solution, in which $\boldsymbol{I}$ is the identity operator. It is noted that the regularization parameter plays essentially the same role as the bandwidth of a filter when smoothing noisy data. Hence, by choosing the regularization parameter γ too small, the solution will cause an arbitrarily large perturbation because an arbitrary small perturbation of the hop-counts. Otherwise, if the regularization parameter γ is chosen too large, the solution is affected by still too much human disturbance. There are several strategies to determine the regularization parameter, e.g., the generalized deviation criterion, the generalized cross-validation and L-curve method. However, any parameter selection strategy can be lead to an unacceptable run-time complexity for multi-hop localization. Literatures [31] describes the solution of regularization is ill-posed as $\left\| \tilde{\boldsymbol{H}}^{\mathrm{T}} \tilde{\boldsymbol{H}} \right\| < 0.01$. Hence, we choose the regularization parameter $\gamma = 0.01$ to avoid the high computational complexity of regularization parameter in this paper.

The location estimation phase. Each common node S_t starts to estimate the physical distance to anchors after receive the trained mapping model and the hop-counts vectors. Let $\boldsymbol{h}_t$ be the hop-count vector between common node S_t and all anchors and $\hat{\boldsymbol{T}}$ be the trained mapping model, the problem of distance estimation can be written as

$$\hat{\boldsymbol{d}}_t = \left[\hat{d}_{t,1} \cdots \hat{d}_{t,m} \right]^{\mathrm{T}} = \hat{\boldsymbol{T}} \tilde{\boldsymbol{h}}_t + \tilde{\boldsymbol{h}} \tag{14}$$

where $\hat{\boldsymbol{d}}_t$ is the estimated distance vector between S_t and m anchors, $\tilde{\boldsymbol{h}}$ is the centering hop-counts vector of m anchors for avoid the difference between hop-counts and distances.

After the estimates of the distances to the $k\left(k \ge 3\right)$ anchor nodes are calculated, any non-anchor use multilateration to Estimate its own location. For multi-hop localization problem, the square of the distance between the anchors and non-anchor can be expressed as

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = d_1^2 \\ (x - x_2)^2 + (y - y_2)^2 = d_2^2 \\ \vdots \\ (x - x_k)^2 + (y - y_k)^2 = d_k^2 \end{cases} \quad k \ge 3 \tag{15}$$

where (x, y) is the coordinate of the non-anchor. $\left(x_1, y_1\right), \left(x_2, y_2\right), \cdots, \left(x_k, y_k\right)$ are the coordinates of the anchors. If the k -th equation is subtracted from the first equation to the $(k-1)$ -th equation, we can get

$$\begin{cases} 2(x_k - x_1)x + 2(y_k - y_1)y = \\ \quad d_1^2 - d_k^2 + y_k^2 + x_k^2 - y_1^2 - x_1^2 \\ \quad \vdots \\ 2(x_k - x_{k-1})x + 2(y_k - y_{k-1})y = \\ \quad d_{k-1}^2 - d_k^2 + y_k^2 + x_k^2 - y_{k-1}^2 - x_{k-1}^2 \end{cases} \tag{16}$$

Expressing Equation (17) in matrix form

$$\begin{bmatrix} 2(x_1 - x_n) & 2(y_1 - y_n) \\ \vdots & \vdots \\ 2(x_{k-1} - x_k) & 2(y_{k-1} - y_k) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1^2 - x_k^2 + y_1^2 - y_k^2 + d_k^2 - d_1^2 \\ \vdots \\ x_{k-1}^2 - x_k^2 + y_{k-1}^2 - y_k^2 + d_k^2 - d_{k-1}^2 \end{bmatrix} \tag{17}$$

where $A = 2\times\begin{bmatrix} (x_1 - x_n) & (y_1 - y_n) \\ (x_2 - x_n) & (y_2 - y_n) \\ \vdots & \vdots \\ (x_{k-1} - x_k) & (y_{k-1} - y_k) \end{bmatrix}$,

$$\boldsymbol{b} = \begin{bmatrix} x_1^2 - x_k^2 + y_1^2 - y_k^2 + d_k^2 - d_1^2 \\ x_2^2 - x_k^2 + y_2^2 - y_k^2 + d_k^2 - d_2^2 \\ \vdots \\ x_{k-1}^2 - x_k^2 + y_{k-1}^2 - y_k^2 + d_k^2 - d_{k-1}^2 \end{bmatrix}$$

However, in practice the error is inevitable in the measurements which mean the ideal equation (18) cannot be obtained. In fact the equation has the following form: $\boldsymbol{Ax} = \boldsymbol{b} + \boldsymbol{\varepsilon}$ .where $\boldsymbol{\varepsilon}$ denotes error vector. In order to minimize the effect of noise, choose the sum of the squared errors as the measure scale of collective loss, i.e.,

$$\begin{aligned} &\arg\min\left(\boldsymbol{b} - \boldsymbol{Ax}\right)^{\mathrm{T}}\left(\boldsymbol{b} - \boldsymbol{Ax}\right) \\ &\Rightarrow \arg\min\left(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Ax} - 2\boldsymbol{b}^{T}\boldsymbol{Ax} + \boldsymbol{b}^{T}\boldsymbol{b}\right) \end{aligned} \tag{18}$$

So the localization problem transforms to an optimization problem. Set the partial derivative of the formula (19) to be 0, then

$$\begin{aligned} &-2\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} + 2\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Ax} = 0 \\ &\Rightarrow \boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Ax} \end{aligned} \tag{19}$$

If there is an inverse matrix of $A^{\mathrm{T}}A$, that is to say anchors does not lie in a line, the location of the non-anchor can be calculated as the solution of this equation, given by of

$$\hat{\boldsymbol{x}} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} \tag{20}$$

Table 1 compares WNLT with the three state-of-the-art range-free approaches: DV-hop, Amorphous, and PDM. Both DV-hop and Amorphous assume isotropic networks and estimate the node location with their network distances to the three anchors. The difference is that the former floods the network for computing the hop-counts so the communication cost of DV-hop is O(n2), while the latter only broadcast its per-hop distance and the common nodes do not send any requests so the communication cost of Amorphous is O(n). The computation costs of DV-hop and Amorphous are O(n). Both PDM and WNLT are a regression-based approach which with the help of a portion of anchors can handle anisotropic networks. Relying on each node flooding the network to collect the hop-counts to all the anchors, PDM and WNLT.

Table 1. Algorithm comparison

Algorithm	Communication cost	Computation cost	Applicable networks
DV-hop	O(n^2)	O(n)	Isotropic network
Amorphous	O(n)	O(n^2)	Isotropic network
PDM	O(n^2)	O(n^3)	Anisotropic network
WNLT	O(n^2)	O(n^3)	Anisotropic network

4 Performance Analysis

Multi-hop localization algorithm is a popular scheme for large-scale network applications, so it needs a large number of nodes in practical applications. In addition, under the same network environment, the parameter of the localization algorithm is sometimes required to adjust. Due to the above reasons, under limited expenditure and experiment conditions, it may be difficult to continue the verification work. Hence, we evaluate the performance of the proposed algorithm with the simulation software of MATLAB. In order to prevent the influence of single experiment on the experiment results, 100 simulations were conducted in each experiment. During each experiment, the nodes were redeployed in the experiment area, the results of each experiment were summarized, and the mean value of 100 root mean squares (RMS) was used as the evaluation basis, as shown in the following:

$$RMS = \sqrt{\frac{1}{n-m}\sum_{i=1}^{n-m}(X_i - \bar{X})^2} \tag{21}$$

We suppose the nodes are arranged in two different kinds of anisotropic network the topologies of which are very similar to those of the real-world applications. In the simulations, we compare the proposed WNLT algorithm with three related multi-hop localization: (1) the classic DV-hop algorithm proposed in [17]; (2) Amorphous algorithm proposed in [19]; and (3) PDM proposed in [22].

In PDM, items containing these small singular values are discarded to in order to maintain the stability of the solution of TSVD [28]. Here we set 3 as the threshold value, items containing the singular values smaller than 3 are discarded. For WNLT, its performance is closely related to the regularization parameter γ. We set $\gamma = 0.01$ in our simulations.

4.1 Problem Statement

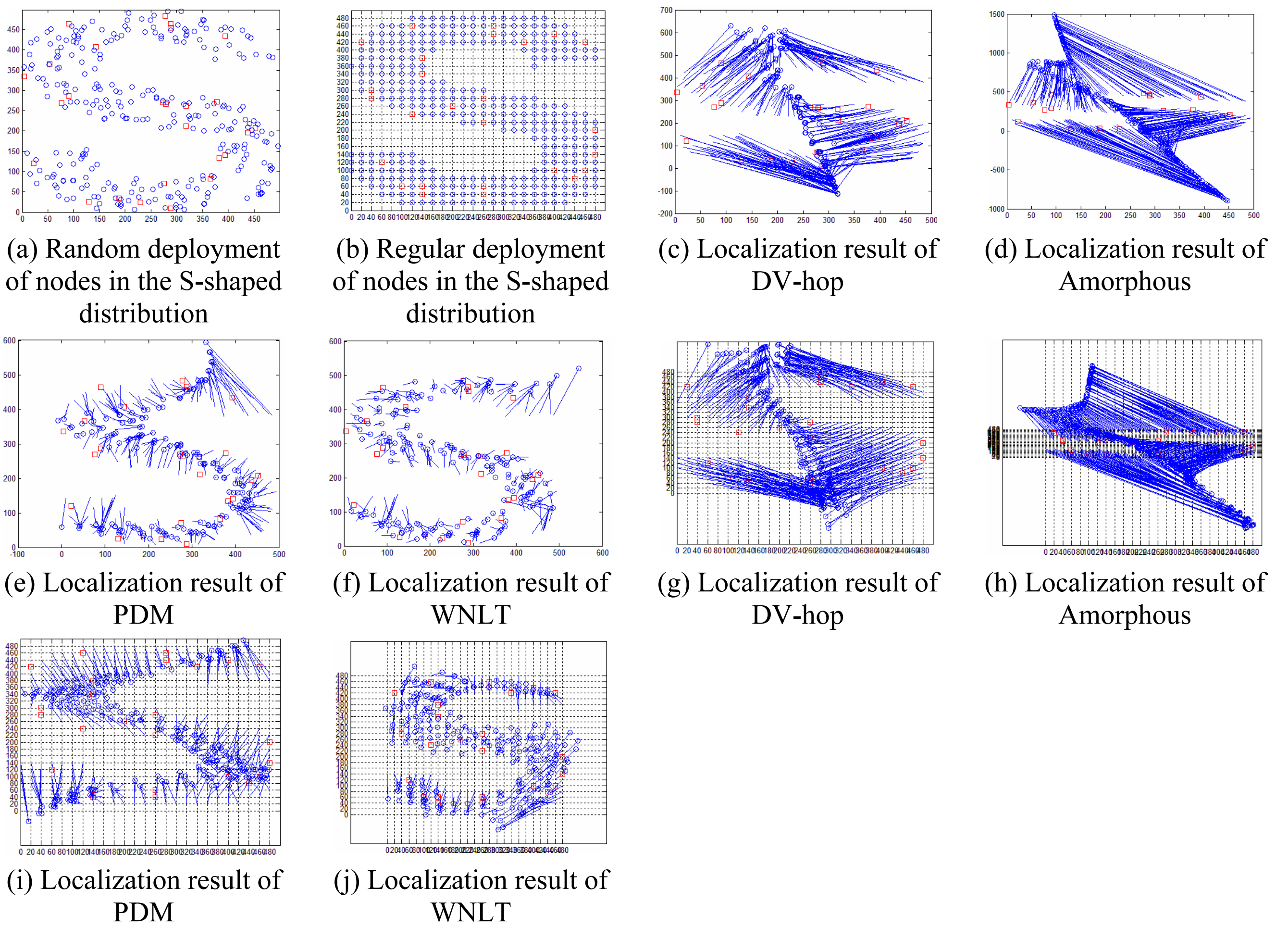
In this set of experiments, barriers were set in the deployment environment to ensure that the propagation path of nodes was not at a straight line. Due to the barriers, the nodes presented S-shape distribution. Nodes are deployed as the following two types.

Random deployment. 300 nodes are randomly and evenly deployed in a 500×500 square area.

Grid deployment. 363 nodes are deployed whose side length of grids is 20 m in a 500×500 square area.

Figure 3(c) to Figure 3(j) show localization results of DV-hop, Amorphous, PDM and WNLT, respectively in the S-shaped area. In Figure 3, the circles refer to common nodes and the squares refer to anchors. The true location of common node and its estimation is connected by a line, and the longer the length of line is, the bigger the localization error.

The S-shape network is a classic anisotropic network. In this group of simulation, we can find when the network is anisotropic, hop-counts between nodes may not mismatch physical distance well. For DV-hop and Amorphous, it may introduce huge errors to use a fixed mapping coefficient for matching hop-counts to distances. Especially for Amorphous, it uses a global network unique fixed coefficient, witch resulting in greater error. In contrast, PDM and WNLT can effectively match this mapping relationship between nodes. PDM and WNLT directly construct the optimal linear transformation between hop-counts and physical distances, so that more accurate hop-counts to distances transformation can be obtained between nodes, and better location estimation can be obtained. The proposed WNLT method has considered that the level of number difference between hop-counts and physical distances. We weaken the measurement level difference between hop-counts and physical distances before building the mapping model by the centralization method. Furthermore, in order to avoid the complicated process of parameter selection and ensuring the localization precision, we also adopt the classic optimal regular parameter. Figure 3(c) to Figure 3(f) show the localization results of random deployment, and the final RMS of DV-hop, Amorphous, PDM and WNLT are 179.6504, 553.039, 57.231, 57.231 and 38.1439 respectively. Figure 3(g) to Figure 3(j) show the localization results of regular deployment, and the final RMS of DV-hop, Amorphous, PDM and WNLT are 162.491, 647.9535, 57.7465 and 38.6313 respectively.



(a) Random deployment of nodes in the S-shaped distribution

(b) Regular deployment of nodes in the S-shaped distribution

(c) Localization result of DV-hop

(d) Localization result of Amorphous

(e) Localization result of PDM

(f) Localization result of WNLT

(g) Localization result of DV-hop

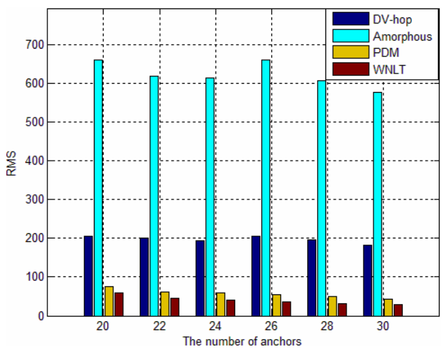
(h) Localization result of Amorphous

(i) Localization result of PDM

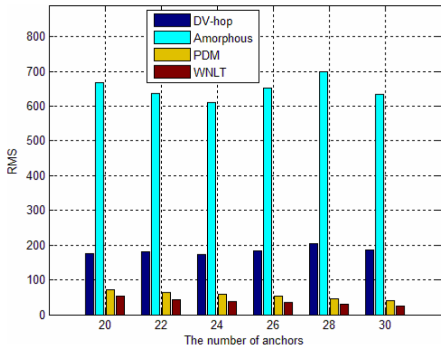
(j) Localization result of WNLT

Figure 3. Localization results of the random and regular deployments in the S-shaped distribution of nodes

Figure 4(a) and Figure 4(b) respectively show the effects of the number of anchors with different multi-hop range-free localization under random deployment and grid deployment. Literature [23] analyze that the number of anchors setting should ensure that there are 6 neighbor nodes around each node in the network. Therefore, we set the number of anchors gradually increases from 20 to 30 with the step size of 2. In accordance with Figure 4, it is easy to see that under both the random deployment and regular deployment, the Amorphous method has the bigger RMS value, which indicates that it is very sensitive to the anisotropic network. The localization performance of DV-hop method is superior to that of Amorphous method, because the Amorphous method only has one fixed coefficient in anisotropic network. However, both PDM and WNLT can be relieved performance degradation due to the mismatch problem between hop-counts and distances in anisotropic network, so they demonstrate much better performance than the previous methods. More specifically, as showed in the Figure 4, the proposed WNLT outperforms the PDM method. This is the inevitable result because the property between hop-counts and distances are captured for the proposed WNLT. Under random deployment, compared to the DV-hop, Amorphous and PDM methods, the localization precision of the proposed WNLT is improved by 79.4%, 93.5% and 28.9% respectively. Under regular deployment, the average localization precision of the WNLT method is enhanced by 79.7%, 94.2% and 32.5% respectively.



(a) Random deployment



(b) Regular deployment

Figure 4. The bar graph of the change of RMS of four algorithms with the different number of anchors

4.2 Radio Irregularity Problem of Nodes

Radio irregularity is a common phenomenon, and is one of the main reasons for the anisotropic network. Radio irregularity results in uneven node connectivity, which further causes that the node will not be connected with node of equal distance. Hence, we employ the parameter DOI (Degree Of Irregularity) to evaluate the adaptability and stability of the proposed algorithm in this section.

Assumed that communication range distributed in the $\left[r-\delta, r+\delta\right]$, δ is maximum range of the communication range, DOI [32] is defined as δ/r, which is exploited to measure degree of irregular communication range. Figure 5 and Figure 6 respectively describe examples of the radio propagation model when DOI=0 and DOI=0.01.

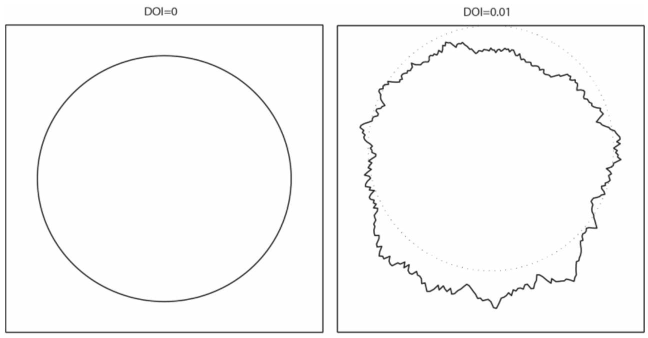
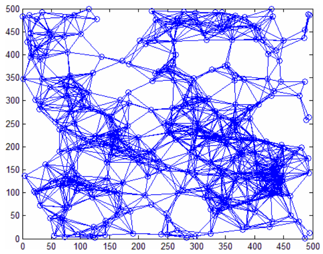
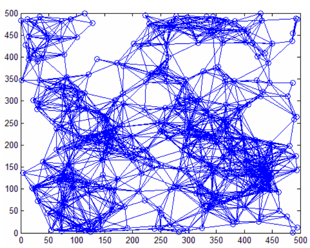


Figure 5. Irregular degree of the signal transmission

Figure 6 shows that under the same node distribution, the node connection is different because of the different DOI values.



(a) Regular transmission

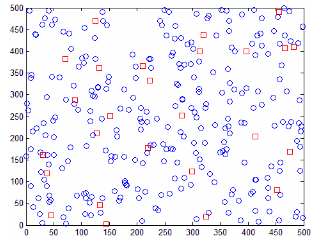


(b) DOI=0.01, irregular transmission

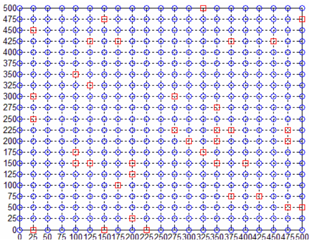
Figure 6. Comparison between the regular transmission and the irregular transmission

In this section, we set up DOI=0.01, and assume that nodes are randomly or regularly distributed in the area of 500×500 with no obstacles. There are 300 nodes deployed in the random deployment; a total of 441 nodes are in the regular deployment, and the distance between nodes is 25. The increase of the reference nodes in the random deployment scene is similar to the previous section. In order to maintain the ratio of the reference node in the random deployment, the number of the reference nodes in the regular deployment experiment increases from 26 to 36 by 2 as the length of the step.

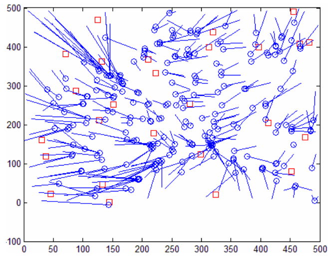
Figure 7(a) and Figure 7(b) show the deployments of the experiment. The localization results of DV-hop, Amorphous, PDM and WNLT are shown from Figure 7(c-f), and their RMS errors are 63.8248, 97.0551, 45.8464, 36.7001, respectively. Uneven node distribution still has high impact on the multi-hop range-free method that adopts fixed matching coefficient. In accordance with Figure 7, we can see that the RMS errors of DV-hop and Amorphous that adopt fixed matching coefficient are significantly higher than the multi-hop range-free methods PDM and WNLT which do not use fixed matching coefficient. In accordance with Figure 7, we can also find that in this set of experiments, the WNLT method proposed in this paper still has higher localization precision.



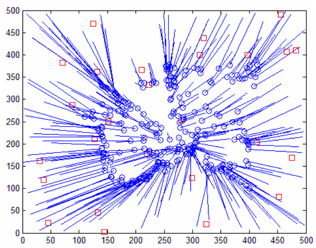
(a) Random deployment under radio irregularity



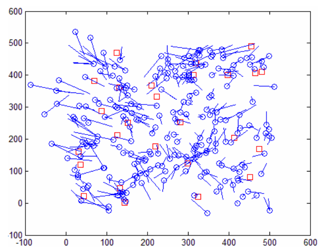
(b) Regular deployment under radio irregularity



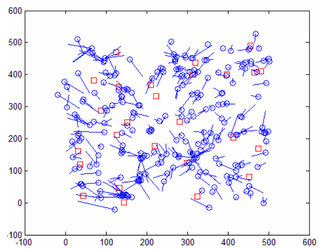
(c) Localization result of DV-hop



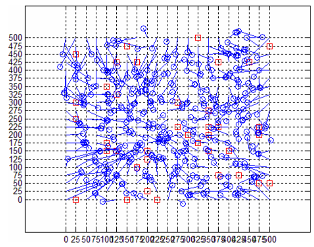
(d) Localization result of Amorphous



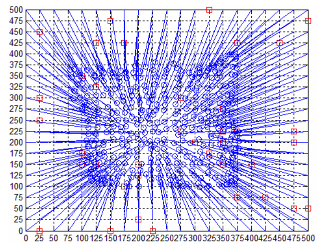
(e) Localization result of PDM



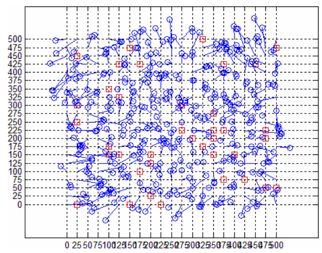
(f) Localization result of WNLT



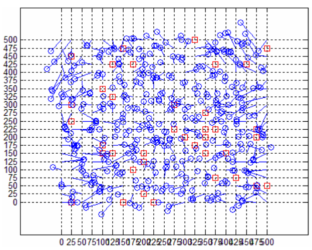
(g) Localization result of DV-hop



(h) Localization result of Amorphous



(i)Localization result of PDM



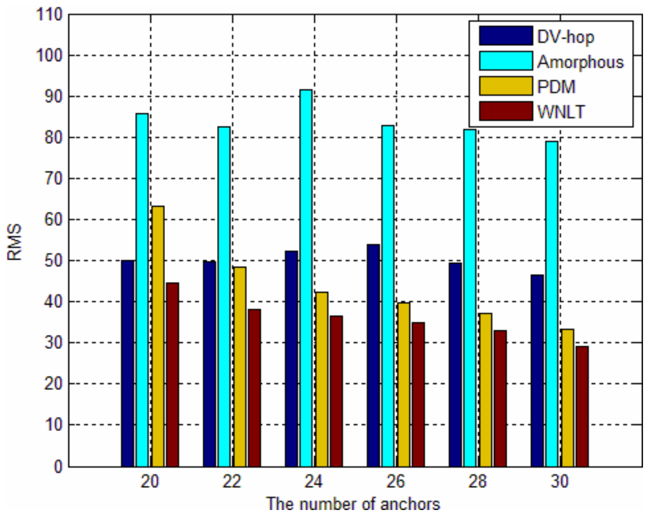
(j) Localization result of WNLT

Figure 7. Localization results of the random and regular deployments, DOI=0.01

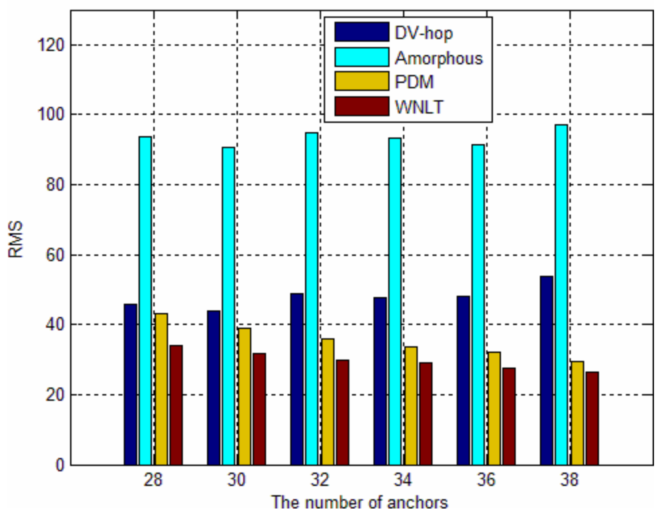
Figure 8 shows the effects of the number of anchors with DOI=0.01 on the localization performance. We can see that WNLT always obtains the best localization accuracy. Compared with DV-hop, Amorphous and PDM, the average localization accuracies of the proposed WNLT are improved by 28.5%, 57.1% and 18.2%, respectively, while in the regular deployment, they are improved by 33.9%, 66.4% and 17.7%, respectively.

5 Conclusion

We have presented a multi-hop range-free localization method based on Tikhonov regularization for anisotropy network. We first construct a hop-distance mapping model with the help of the Tikhonov regularization after centralized data. We show that the multi-hop localization can be formulated as a regression problem, which can effectively avoid the influence of anisotropy on localization. Compared to similar algorithms, our approach can simultaneously



(a) Random deployment



(b) Regular deployment

Figure 8. The bar graph of the change of RMS of four algorithms with the different number of reference nodes, DOI=0.01

handle two typical anisotropic networks, i.e., irregular distribution and radio irregularity. Simulation results demonstrate that the proposed approach has the characteristics of easiness of parameter setting and can effectively reduce distance estimation error in anisotropic network.

Acknowledgement

The paper is supported by National Natural Science Foundation of China (No. 61403080), National Natural Science Foundation of Jiangsu Province (BK20140641), China Postdoctoral Science Foundation (2016M601861), Jiangsu Postdoctoral Research Funds (1701049A), and the Open Project Program of Jiangsu Key Laboratory of Remote Measurement and Control (YCCK201603).

References

[1] P. G. V. Naranjo, M. Shojafar, H. Mostafaei, Z. Pooranian, E. Baccarelli, P-SEP: A Prolong Stable Election Routing Algorithm for Energy-limited Heterogeneous Fog-supported Wireless Sensor Networks, *The Journal of Supercomputing*, Vol. 73, No. 2, pp. 733-755, February, 2017.

[2] M. M. Umar, A. Mehmood, H. Song, SeCRoP: Secure Cluster Head Centered Multi-hop Routing Protocol for Mobile Ad Hoc Networks, *Security and Communication Networks*, Vol. 9, No. 16, pp. 3378-3387, November, 2016.

[3] A. Ahmadi, M. Shojafar, S. F. Hajeforosh, M. Dehghan, M. Singhal, An Efficient Routing Algorithm to Preserve K-coverage in Wireless Sensor Networks, *The Journal of Supercomputing*, Vol. 68, No. 2, pp. 599-623, May, 2014.

[4] S. Rani, R. Talwar, J. Malhotra, S. H. Ahmed, M. Sarkar, H. Song, A Novel Scheme for an Energy Efficient Internet of Things Based on Wireless Sensor Networks, *Sensors*, Vol. 15, No. 11, pp. 28603-28626, November, 2015.

[5] Z. Yang, C. S. Wu, Y. H. Liu, *Location-based Computing: Localization and Localizability of Wireless Networks*, Tsinghua University Press, 2014.

[6] D. M. Gao, H. F. Lin, X. F. Liu, Routing Protocol for K-anycast Communication in Rechargeable Wireless Sensor Networks, *Computer Standards & Interfaces*, Vol. 43, pp. 12-20, January, 2016.

[7] D. M. Gao, H. F. Lin, Y. F. Liu, G. X. Wu, Maximum Data Collection Rate in Rechargeable Wireless Sensor Networks with Multiple Sinks, *China Communications*, Vol. 13, No. 2, pp. 95-108, February, 2016.

[8] D. M. Gao, H. F. Lin, F. Zhang, Y.F. Liu, Maximum Network Throughput Based on Distributed Algorithm for Rechargeable Wireless Sensor Networks, *Adhoc & Sensor Wireless Networks*, Vol. 35, No. 3-4, pp. 193-215, 2017.

[9] W. Zheng, X. Y. Yan, W. Zhao, C. S Qian, A Large-Scale Multi-Hop Localization Algorithm Based on Regularized Extreme Learning for Wireless Networks, *Sensors*, Vol. 17, No. 12, 2959, pp. 1-18, December, 2017.

[10] S. Shekhar, S. K. Feiner, W. G. Aref, Spatial Computing, *Communications of the ACM*, Vol. 59, No. 1, pp. 72-81, January, 2016.

[11] D. Synstelien, U.S. Environmental Protection Agency, *The Inside Story: A Guide to Indoor Air Quality*, EPA-230-01-90-073, January, 1990.

[12] X. Y. Yan, Z. Yang, A. G. Song, W. K. Yang, Y. Liu, R. H. Zhu, A Novel Multihop Range-Free Localization Based on Kernel Learning Approach for the Internet of Things. *Wireless Personal Communications*, Vol. 87, No. 1, pp. 269-292, March, 2016.

[13] X. Y. Yan, A. G. Song, Z. Yang, W. K. Yang, An Improved Multihop-based Localization Algorithm for Wireless Sensor Network using Learning Approach, *Computers & Electrical Engineering*, Vol. 48, pp. 247-257, November, 2015.

[14] X. Y. Yan, Z. Yang, Y. Liu, X. D. Xu, H. J. Li, Incremental Localization Algorithm Based On Regularized Iteratively Reweighted Least Square, *Foundations of Computing and*

Decision Sciences, Vol. 41, No. 3, pp. 183-196, September, 2016.

[15] Q. J. Xiao, B. Xiao, K. Bu, J. N. Cao, Iterative Localization of Wireless Sensor Networks: An Accurate and Robust Approach, *IEEE/ACM Transactions on Networking*, Vol. 22, No. 2, pp. 608-621, April, 2014.

[16] J. Zhao, L. Mo, X. Wu, G. Wang, E. Liu, D. Dai, Error Estimation of Iterative Maximum Likelihood Localization in Wireless Sensor Networks, *Adhoc & Sensor Wireless Networks*, Vol. 23, No. 3-4, pp. 277-295, December, 2014.

[17] D. Niculescu, B. Nath, DV Based Positioning in Ad Hoc Networks, *Telecommunication Systems*, Vol. 22, No. 1-4, pp. 267-280, January, 2003.

[18] H. Lim, J. C. Hou, Distributed Localization for Anisotropic Sensor Networks, *ACM Transactions on Sensor Networks*, Vol. 5, No. 2, p. 11, March, 2009.

[19] R. Nagpal, H. Shrobe, J. Bachrach, Organizing a Global Coordinate System from Local Information on an Ad Hoc Sensor Network, *Proceedings of the 2nd International Conference on Information Processing in Sensor Networks*, Palo Alto, CA, 2003, pp. 333-348.

[20] J. Lee, W. Chung, E. Kim, A New Kernelized Approach to Wireless Sensor Network Localization, *Information Sciences*, Vol. 243, pp. 20-38, September, 2013.

[21] Z. Wang, Y. Chen, C. Li, PSR: A Lightweight Proactive Source Routing Protocol For Mobile Ad Hoc Networks, *IEEE Transactions on Vehicular Technology*, Vol. 63, No. 2, pp. 859-868, February, 2014.

[22] J. Liu, J. Wan, Q. Wang, P. Deng, K. Zhou, Y. Qiao, A Survey on Position-based Routing for Vehicular Ad Hoc Networks, *Telecommunication Systems*, Vol. 62, No. 1, pp. 15-30, May, 2016.

[23] L. Kleinrock, J. Silvester, Optimum Transmission Radii for Packet Radio Networks or Why Six Is A Magic Number, *Proceedings of the IEEE National Telecommunications Conference*, Birmingham, AL, 1978, pp. 431-435.

[24] Z. Wu, S. Bian, C. Xiang, Y. Tong, A New Method for TSVD Regularization Truncated Parameter Selection, *Mathematical Problems in Engineering*, Vol. 2013, Article ID 161834, 2013.

[25] J. Lee, B. Choi, E. Kim, Novel Range-Free Localization Based on Multidimensional Support Vector Regression Trained in the Primal Space, *IEEE Transactions on Neural Networks and Learning Systems*, Vol. 24, No. 7, pp. 1099-1113, July, 2013.

[26] T. Ma, Y. Zhang, J. Cao, J. Shen, M. Tang, Y. Tian, A. Al-Dhelaan, M. Al-Rodhaan, KDVEM: A K-degree Anonymity with Vertex and Edge Modification Algorithm, *Computing*, Vol. 97, No. 12, pp. 1165-1184, December, 2015.

[27] J. Chen, C. Wang, Y. Sun, X. Shen, Semi-supervised Laplacian Regularized Least Squares Algorithm for Localization in Wireless Sensor Networks, *Computer Networks*, Vol. 55, No. 10, pp. 2481-2491, July, 2011.

[28] P. J. Bickel, B. Li, A. B. Tsybakov, S. A. van de Geer, B. Yu, T. Valdes, C. Rivero, J. Fan, A. van der Vaart, Regularization in Statistics, *Test*, Vol. 15, No. 2, pp. 271-344, September, 2006.

[29] J. B. Bell, Reviewed Work: Solutions of Ill-Posed Problems, *Mathematics of Computation*, Vol. 32, No. 144, pp. 1320-1322, October, 1978.

[30] P. C. Hansen, Regularization Tools: A Matlab Package for Analysis and Solution of Discrete Ill-posed Problems, *Numerical Algorithms*, Vol. 6, No. 1, pp. 1-35, March, 1994.

[31] J. Chen, X. Chen, *Special Matrix*, Tsinghua University Press, 2001.

[32] G. Zhou, T. He, S. Krishnamurthy, J. A. Stankovic, Models and Solutions for Radio Irregularity in Wireless Sensor Networks, *ACM Transactions on Sensor Networks*, Vol. 2, No. 2, pp. 221-262, May, 2006.

Biographies

Jiandong Yao received the B.S. degree in school of computer science & technology from the Soochow University, and M.S. degree in school of computer science & technology from the Soochow University. Now he is a lecturer at Jinling Institute of Technology. His current research interests include wireless sensor networks, intelligent control and Internet of Things.

Xiaoyong Yan He is an associate professor at the School of Modern Posts & Institute of Modern Posts, Nanjing University of Posts and Telecommunications, China. He received the B.S. degree in mechanical electronic engineering from Nanjing Agricultural University in 2000 and received the M.S. and Ph.D. Degree in computer application technology from Nanjing University of Science and Technology in 2006 and 2013, respectively. Now he is a Postdoctoral Fellow at School of Computer, Nanjing University of Posts and Telecommunications, China. His current interests are in the areas of wireless sensor networks, pattern recognition and machine learning.

Chengshan Qian received the M.S. degree in measuring and testing technologies and instruments from the Shandong University of Science and Technology and the Ph.D. degree in control theory and control engineering from the Nanjing University of Aeronautics and Astronautics, China, in 2003 and 2009, respectively. He is currently a Professor with the School of Computer and Software, Nanjing University of Information Science and Technology, Nanjing. His current research interests include intelligent control, nonlinear system control, and Internet of Things.

Huijun Li received her B.S. degree in Measurement and Control in 1999, and M.S. degree in Measurement and Control in 2002 from Zhengzhou University, People's Republic of China, and Ph.D. degree in Measurement and Control from Southeast University, Nanjing, People's Republic of China in 2005. She is currently an Associate Professor with the School of Instrument Science and Engineering, Southeast University, People's Republic of China. Her research interests are on teleoperation, space robot and rehabilitation robot.