Cluster Validity Indexes to Uncertain Data for Multi-Attribute Decision-Making Datasets

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Abstract

This paper proposes a novel function which is designated as the multi-attribute (MA) index function (derived from the conventional PBMF-index function), is used to evaluate the quality of the clustering solution in terms of the number of clusters assigned to each attribute and the accuracy of the corresponding Rough Set (RS) classification. The MA-index function processes a set of parameter values obtained from the Fuzzy C Mean method, Fuzzy Set theory, and RS theory. The MA-index function is embedded within an iterative procedure designated as a multi-attribute decision-making index method, which optimizes both the number of clusters per attribute in the dataset and the accuracy of the corresponding classification. In other words, the clustering/ classification outcome obtained from the multi-attribute decision making index method provides a suitable basis for the formation of reliable decision-making rules. On the whole, the outcomes reveal that the suggested technique not simply generates a much better clustering efficiency as compared to the single-attribute decision-making (SADM) and also PBMF techniques however additionally supplies a much more trustworthy basis for the removal of decision-making policies.

Keywords: MADM-index, PBMF-based index, Cluster vector index, Rough set

1 Introduction

Data mining is a process used by companies to turn raw data into useful information. By using software to look for patterns in large batches of data, businesses can learn more about their customers and develop more effective marketing strategies as well as increase sales and decrease costs. Data mining depends on effective data collection and warehousing as well as computer processing.When applying classification schemes to constant valued datasets that lack class information, it is first required to discretize the real-value features right into discrete partitions [1].

The literature contains various techniques for clustering continuous value datasets, including interval analysis [2-4], equal-frequency [5] and Fuzzy C-means [6-7]. However, in practice, the instances within a dataset can be grouped in many different ways, and thus, a method is required to assess the validity of each possible clustering solution. The problem of evaluating the optimality of the various clustering solutions for a specific dataset is known as the cluster validity problem [8-9]. Generally speaking, the quality of a clustering solution can be evaluated in terms of both the compactness of the clusters and the distance that separates them. Many methods have been proposed to assess the optimality of the clustering results obtained using fuzzy clustering schemes [9-19] and the most widely used of these methods are the partition coefficient (PC) [20-21], the classification entropy (PE) [22], the Xie-Beni index [19], and the PBMF-based index [16-17]. Because these methods are based simply on the membership values of the instances within the dataset, they are computationally straightforward.

Most existing clustering methods cluster the dataset according to the norms of the instances rather than according to the values of the individual attributes. Such methods yield acceptable results for simple datasets with a limited number of interrelated attributes but perform less well when applied to real-world datasets with a large number of attributes with complex inter-relationships. In practice, the optimal clustering solution for a specific dataset depends on the method used for classification purposes. Thus, when constructing a data clustering algorithm, it is desirable to apply some form of classification-defined knowledge to the attribute values of the dataset so that the complex inter-relationships between them can be taken into account.

References contain many classification-based mining algorithms, including decision-tree algorithms such as ID3 [23], rule-based algorithms such as CN2 [24], neural network classifiers [25], support vector machines (SVMs) [26-27], fuzzy classifiers [28], and Bayesian classifiers [29]. All of these schemes have

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their respective merits and have been widely used in a diverse range of applications, including weather prediction, manufacturing process planning, and medical diagnoses. However, these methods cannot deal effectively with datasets that have no specified output values or that are characterized by uncertain or missing information.

Rough Set (RS) theory was first introduced more than twenty years ago [30] and has emerged as a powerful technique for automatic classification of datasets in a diverse range of fields. As described above, real-world datasets typically consist of multiple attributes with complex inter-relationships. Furthermore, the data within such datasets are usually incomplete and/or uncertain. Therefore, "crisp" classification methods, such as the equal-width or equal-frequency methods [5] have only achieved limited success in classifying most real-world information systems. As a result, in recent years, decision-makers have increasingly turned to using fuzzy-preference-relation methods to classify multi-attribute decision-making (MADM) datasets [30-35]. In line with this trend, the present study proposes a new method for clustering and classifying uncertain data in a multi-attribute decision-making (MADM) dataset based on the aggregative membership function values obtained using the Fuzzy C-Means (FCM) clustering method [7] and related fuzzy set theories.

The remainder of this paper is organized as follows. Section 2 presents a comparison between the PBMF-index function and the MA-index function. The proposed MADM-index method and a simple example are illustrated in sction 3. Section 4 compares the performance of the MADM-index method with the performance of the PBMF-based index method and a single-attribute decision-making (SADM) method [7, 36]. Finally, this research provides some concluding remarks and directions for future research in section 5.

2 Comparison of PBMF- and MA-index functions

Table 1 summarizes the major components of the MA-index function and the PBMF-index function to highlight the differences between them. Four principal differences exist at a high level, namely, (i) the MA-index function is based on the individual attribute values of the instances within the dataset, whereas the PBMF-index function is based on the norms of the instances; (ii) the MA-index function is based on the aggregative membership function values associated with each DACV, whereas the PBMF-index function is based on the membership function values of each instance; (iii) the MA-index function is based on z'_c, i.e., the centroids of the lower approximate sets associated with each indiscernible DACV, whereas the PBMF-index function is based on z_k, i.e., the centroid of the k-th cluster obtained when clustering the dataset using the FCM method; and (iv) the MA-index function takes explicit account of the classification accuracy when evaluating the optimality of the clustering results, whereas the PBMF-index function only considers the optimal number of clusters within the dataset.

Table 1. Detailed definition of MADM-index and PBMF-index functions

	MADM-index function	PBMF-index function
Formulation	$MA(C, N_I, \alpha_c) = (\frac{1}{C} \times \frac{\overline{E_1}}{F'_{N_1}} \times D'_{N_1})$	$PBMF(K) = (\frac{1}{K} \times \frac{\overline{E_1}}{J_{m'}} \times D_K)$
How to cluster the data	Cluster all attribute values of dataset	Cluster all data in dataset
	C is the number of clusters per decision attribute	K is the number of clusters in the dataset

$F'_{N_I} = \sum_{c=1}^{N_I} E'_c$, $E'_c = \sum_{j=1}^{n} \overline{\mu}_{cj}^{m'}\left(x_j(d)\right)\left\|x_j - z'_c\right\| \Big/ \alpha_c$

(1) $\overline{\mu}_{cj}\left(x_j(d)\right)$ is the aggregative membership function of instance x_j in the clusters indicated by the c-th DACV.

(2) z'_c is the multi-dimensional centroid of the lower approximate sets associated with the clusters indicated by the c-th indiscernible DACV and is obtained by computing the mean values of the conditional and decision attributes of each instance within the corresponding sets.

(3.1) $\left\|x_j - z'_c\right\|$ is the length of the vector (norm) between the x_j datum and z'_c.

(3.2) $E'_c = \sum_{j=1}^{n} \left\|x_{jc}\right\|$, where $\left\|x_{jc}\right\| = \overline{\mu}_{cj}^{m'}\left(x_j(d)\right)\left\|x_j - z'_c\right\|$

(3.3) α_c is the classification accuracy and is equal to the cardinality ratio of the lower approximate sets to the upper approximate sets when evaluated in terms of the c-th indiscernible DACV.

$D'_{N_I} = \max_{i,j=1}^{N_I} \left\|z'_i - z'_j\right\|$ is the maximum separation distance between all possible pairs of centroids of the lower approximate sets associated with different indiscernible DACVs, where N_I is the number of indiscernible DACVs.

As a result, when integrated with the specified classification method (i.e., the RS classification model in the present study), the MA-index function provides a more suitable basis for the discretization/classification of complex, real-world datasets compared to the PBMF-index function. The detailed processing steps of the MADM-index method based on the MA-index function are described in the following section. The performance of the MADM-index method is then compared with that of the PBMF-based index method in section 4.

3 MADM-index Calculation Methodology

For each instance in the dataset (and each run in the iterative procedure), the MADM-index method used an FCM clustering scheme to obtain the membership function values of the instance within each cluster of each attribute and to assign each attribute of each instance to an appropriate cluster. The RS classification model was then applied to the clustering results to identify the lower and upper approximate sets and to compute the corresponding classification accuracy. Finally, the aggregative membership function value of each instance was computed using the standard minimized fuzzy set operator. The value of the MA-index function was then calculated in accordance with the formula given in Section 2.2. The various steps of the MADM-index method are described in the sub-sections below. A simple step-by-step example is then provided to illustrate the derivation of the value of the MA-index function for a hypothetical dataset containing just four data entries.

The details of each step in the MADM procedure are summarized in the following paragraphs.

Step 1: Specify the number of clusters per conditional and the decision attribute in the interval $[\mathrm{N}_{\min}, \mathrm{N}_{\max}]$**.** The MADM-index method utilizes an iterative procedure to optimize the number of clusters assigned to the conditional and decision attributes within the dataset of interest. (It should be noted that the number of conditional and decision attributes to be considered in the optimization process are specified in advance by the user.) The conditional and decision attributes are partitioned into an equal number of clusters, N, where N is bounded by the interval $[\mathrm{N}_{\min}, \mathrm{N}_{\max}]$, in which $\mathrm{N}_{\min}$ represents the minimum number of clusters per attribute and $\mathrm{N}_{\max}$ represents the maximum number of clusters per attribute. It should be noted that the values of $\mathrm{N}_{\min}$ and $\mathrm{N}_{\max}$ vary from case to case. However, $\mathrm{N}_{\min}$ has a default value of 2, while $\mathrm{N}_{\max}$ is equal to the number of entries divided by 10. It should be noted that the algorithm initializes the number of clusters per attribute to $\mathrm{N}_{\min}$ in the first run.

Step 2: Fuzzify attribute values of instances using the FCM method. Usually, a continuous-value information system can only be converted into an equivalent fuzzy information system when a classified fuzzy set has been obtained. In the FCM clustering procedure performed in the MADM-index method, the interval values $[\alpha,\beta]$ of all of the conditional and decision attributes are assigned to p_l fuzzy clusters. The resulting continuous-value information system (U, A, V_q, f_q) is then converted into a fuzzy information system $(U, \tilde{A}, \Phi, d)$, where $\Phi = \{\tilde{A}_{lj} \mid l \le m, j \le p_l\}$, in which $\tilde{A}_{lj} = \mu_j(x_i(a_l))$ and indicates the value of the membership function associated with the l-th conditional attribute a_l of the i-th instance.

Step 3: Assign each attribute of each instance to the appropriate conditional or decision attribute cluster. The membership function values of each attribute of each instance are computed using the index function $C_{a_l}(x_i) = I_{\max}(\mu_j(x_i(a_l))) = Index(\max(\mu_j(x_i)))$ for $1 \le l \le m, 1 \le i \le n$ to determine the conditional or decision attribute cluster to which each attribute belongs and to obtain the corresponding DACV.

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Step 4: Identify the RS sets and compute the aggregative membership function value and the classification accuracy. Having mapped the attribute values of all of the instances to the appropriate conditional or decision attribute clusters, the lower approximate and upper approximate sets associated with each indiscernible DACV are extracted according to the definitions presented in Section 2.2. The classification accuracy associated with each indiscernible DACV is then obtained by computing the cardinality ratio of the corresponding lower approximate sets to the upper approximate sets. Finally, the aggregative membership function value of each instance is obtained using the minimized fuzzy set operator.

Step 5: Calculate the centroids of the lower approximate sets associated with each indiscernible DACV. The multi-dimensional centroids of the lower approximate sets associated with each indiscernible DACV are obtained by calculating the mean attribute values (both conditional and decision) of all of the instances within the corresponding sets.

Step 6: Determine the value of the MA cluster validity index. Having determined the aggregative membership function values and the classification accuracy and centroids of the lower approximate sets, the optimality of the clustering results is evaluated using the MA-index function.

Step 7: Check the termination criterion. Having computed the value of the MA-index function to obtain the current value of N (i.e., the number of clusters per attribute), a check is made to determine whether N is equal to the upper bound value ($N = \mathrm{N}_{\max}$). In the event that N is not equal to $\mathrm{N}_{\max}$, the value of N is incremented by 1, and the FCM, fuzzy set, RS and MA cluster validity index procedures are repeated. Conversely, if $N = \mathrm{N}_{\max}$, the computational process moves directly to Step 8.

Step 8: Identify the value of the MA cluster validity index. When the termination criterion has been satisfied, the values of the MA-index function obtained for $N = N_{\min} \sim \mathrm{N}_{\max}$ are compared to identify the clustering solution that yields the maximum index function value, i.e., the clustering solution that optimizes both the number of clusters per attribute and the overall classification accuracy of the dataset.

4 Performance Evaluation of A simple example of the uncertain data in MADM datasets

In this section, the performance of the proposed MADM-index method is evaluated by an example relates to a dataset in which the conditional and decision attributes are related via a hypothetical function The effectiveness of the MADM-index method is demonstrated by comparing the partitioning and classification results with those obtained from the PBMF-based index method and a single-attribute decision-making (SADM-) index method, respectively.

This example considers a hypothetical dataset in which each instance has two conditional attributes, i.e., a_1 and a_2 , and two decision attributes, i.e., d_1 and d_2 , related via the functions $g_{d_1}(x_j) = g_{d_2}(x_j) = f_{a_1}(x_j) + f_{a_2}(x_j)$, where $f_{a_1}(x_j) = (\cos(y_1) + \sin(y_1))$, ($y_1 \in [-\pi, \pi]$), and $f_{a_2}(x_j) = 100.0 \times (1.0 + \exp(y_2))$, ($y_2 \in [0, 2]$). (Note: $\exp(y_2)$ returns the exponential value of y_2). The dataset is assumed to contain 101 equally spaced instances. Taking the first instance x_1 for illustration purposes, the values of the first and second conditional attributes are assumed to be $1.0 + \sin(-\pi) = 1.0$ and $100.0 \times (1.0 + \exp(0)) = 200.0$, respectively. As a result, the two decision attributes have a value of $(1.0 + \sin(-\pi)) + 100.0 \ \times (1.0 + \exp(0))$ $= 1.0 + 200.0 = 201.0$. In other words, the first instance x_1 has attribute values of $x_1(1.0, 200.0, 201.0, 201.0)$. Significantly, it is noted that the order of magnitude of the second conditional attribute, a_2 , is much greater than that of the first conditional attribute, a_1 , i.e., $V_{a_2} >> V_{a_1}$. When performing the clustering/classification process, an assumption is made that all of the attributes (both conditional and decision) can be divided into three clusters. When clustering the dataset, the SADM- and MADM- index methods consider both conditional attributes, i.e., a_1 and a_2 . However, while the MADM-index method also considers both decision attributes, i.e., d_1 and d_2 , the SADM-index method only considers the first decision attribute, d_1 .

First, the above reasoning shows that, under certain conditions, the grouping results obtained from the MADM- and SADM-index methods are the same as those obtained when clustering the dataset using the PBMF-index method on the basis of the norms of the instances rather than on the basis of the values of individual attributes. The clustering results obtained by the PBMF-index method and the MADM-index method are shown in Figure 1(a) and Figure 1(b), respectively, where parameters 1 and 2 represent the values of the first and second conditional attributes, respectively, while parameter 3 represents the values of the first and second decision attributes. It should be noted that the values of the two decision attributes of data point x_j vary as the same function of the conditional attributes of the data point x_j (i.e., $g_{d_1}(x_j) = g_{d_2}(x_j) = f_{a_1}(x_j) + f_{a_2}(x_j)$), and thus, only one parameter needs to be shown here. It should also be noted that the results shown in Figure 1(b) relate to both the MADM-index method and the SADM-index method because the values of the first and second decision attributes of all of the data points are the same. In Figure 1(a), the red, green, and blue symbols correspond to the data points assigned by the PBMF-index method to the first, second and third clusters in the dataset, respectively, while in Figure 1(b), the red, green and blue symbols correspond to the data points within the lower approximate sets associated with the three indiscernible DACVs, i.e., $[1,1]$, $[2,2]$, and $[3,3]$.

For the MADM-index method, the number of cluster indices scales as the product of the number of clusters per attribute, whereas in the SADM-index method, the number of cluster indices remains equal to the number of clusters per attribute at all times. Thus, as the number of decision attributes is increased, the inter-relationships among the various conditional and decision attributes of the dataset becomes more complex. In other words, the MADM-index method provides a more comprehensive model of the information system but gives a more highly complex result.

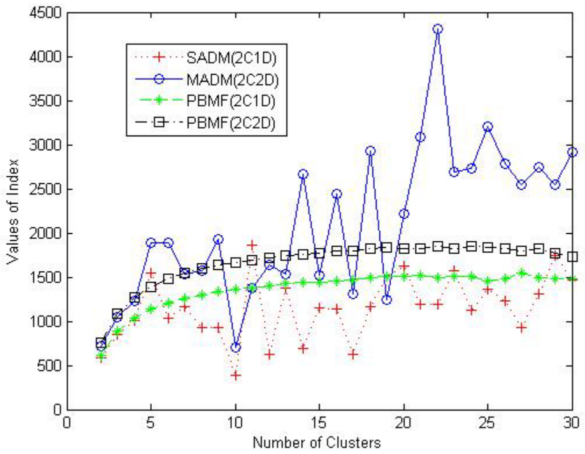

Figure 1. Variation of MADM-, SADM- and PBMF-index values as a function of the number of clusters per attribute and decision attributes for the hypothetical function $g_{d_1}(x_j)=g_{d_2}(x_j)=(\cos(y_1)+\sin(y_1))+100.0\times(1.0+\exp(y_2))$

5 Conclusions

The proposed method is based on the FCM clustering scheme, fuzzy set theory, RS theory and a modified form of the PBMF-index function designated as the MA-index function for the partitioning and classification of complex, real-world multi-attribute decision-making (MADM) datasets. This research provides the means to determine the optimal number of attribute clusters within the dataset and the optimal classification accuracy. The effectiveness of the proposed method has been confirmed by comparing the results obtained for the clustering and classification of a real-world stock and a hypothetical dataset market dataset, respectively, with the corresponding results obtained using the conventional PBMF-based index and a single-attribute decision-making (SADM) index method.

Overall, the results have shown that the MADM-index method provides an effective means of optimizing both the number of attribute clusters and the classification accuracy of complex, real-world datasets. Accordingly, in a future study, the MADM-index method will be incorporated with the Variable Precision Rough Set (VPRS) theory to construct an efficient and reliable mechanism for the classification of complex MADM datasets.

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